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MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - 4f'(t) = \cos(2t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y$ i $y = x - 2$. 20

4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje. 20

5. Izracunati $\int_{\overset{\frown}{ABC}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\overset{\frown}{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: 60

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1.) $f'''(t)-4f'(t)=\cos(2t)$, $f(0)=f'(0)=f''(0)=0$

$$s^3F(s)-s^2\underset{0}{f(0)}-s\underset{0}{f'(0)}-\underset{0}{f''(0)}-4(sF(s)-\underset{0}{f(0)})=\frac{s}{s^2+4}$$

$$s^3F(s)-4sF(s)=\frac{s}{s^2+4}$$

$$F(s)\cdot(s^3-4s)=\frac{s}{s^2+4}$$

$$F(s)=\frac{s}{s^2+4}\cdot\frac{1}{s^3-4s}=\frac{s}{(s^2+4)s(s^2-4)}=\frac{\cancel{s}}{\cancel{s}(s+2)(s-2)(s^2+4)}$$

$$\frac{s}{s(s+2)(s-2)(s^2+4)}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s-2}+\frac{Ds+E}{s^2+4}\ /\cdot s(s+2)(s-2)(s^2+4)$$

$$s=A(s^2-4)(s^2+4)+Bs(s-2)(s^2+4)+Cs(s+2)(s^2+4)+(Ds+E)s(s^2-4)$$

$$s=A(s^4-16)+Bs(s^3+4s-2s^2-8)+Cs(s^3+4s+2s^2+8)+(Ds+E)(s^3-4s)$$

$$s=As^4-16A+Bs^4+4Bs^2-2Bs^3-8Bs+Cs^4+4Cs^2+2Cs^3+8Cs+Ds^4-4Ds^2+Es^3-4Es$$

$$s=(A+B+C+D)s^4+(-2B+2C+E)s^3+(4B+4C-4D)s^2+(-8B+8C-4E)s-16A$$

$A+B+C+D=0$

$-2B+2C+E=0$

$B+C-D=0$

$-8B+8C-4E=1$

$-16A=0$

$\boxed{A=0}$

$-2B+2C+E=0\ /\cdot(-4)$

$-8B+8C-4E=1$

$\cancel{8B}-\cancel{8C}-4E=0$

$-\cancel{8B}+\cancel{8C}-4E=1$ } +

$-8E=1\ /:(-8)$

$\boxed{E=-\frac{1}{8}}$

$B+C-D=0$

$B=-C+D$, $B=-C$

$-\cancel{C}+D+\cancel{C}+D=0$

$2D=0\ /:2$

$\boxed{D=0}$

$2C+2C-\frac{1}{8}=0$

$4C=\frac{1}{8}\ /:4$

$C=\frac{1}{8}\cdot\frac{1}{4}$

$\boxed{C=\frac{1}{32}}$ $\boxed{B=-\frac{1}{32}}$

$$F(s)=0\cdot\frac{1}{s}-\frac{1}{32}\cdot\frac{1}{s+2}+\frac{1}{32}\cdot\frac{1}{s-2}+\frac{0\cdot s-\frac{1}{8}}{s^2+4}$$

$$F(s)=-\frac{1}{32}\cdot\frac{1}{s+2}+\frac{1}{32}\cdot\frac{1}{s-2}-\frac{1}{8}\cdot\frac{1}{s^2+4}$$

$$f(t)=-\frac{1}{32}e^{-2t}+\frac{1}{32}e^{2t}-\frac{1}{8}\cdot\frac{1}{2}\left(\frac{2}{s^2+2^2}\right)$$

$$f(t)=-\frac{1}{32}e^{-2t}+\frac{1}{32}e^{2t}-\frac{1}{16}\sin(2t)$$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

4.) $x=\cos 2t$ $\qquad t\in[0,6\pi]$

$y=\sin 2t$

$z=t$

$$r(t)=\begin{pmatrix}\cos 2t\\ \sin 2t\\ t\end{pmatrix}$$

$$r'(t)=\begin{pmatrix}-2\sin 2t\\ 2\cos 2t\\ 1\end{pmatrix} \qquad \sin^2 2t+\cos^2 2t=1$$

$$\|r'(t)\|=\sqrt{(-2\sin 2t)^2+(2\cos 2t)^2+1^2}=\sqrt{4\sin^2 2t+4\cos^2 2t+1}$$

$$=\sqrt{4(\sin^2 2t+\cos^2 2t)+1}=\sqrt{4\cdot 1+1}=\sqrt{5}$$

$$d=\int_0^{6\pi}\sqrt{5}\,dt=\sqrt{5}\,t\Big|_0^{6\pi}=\sqrt{5}(6\pi-0)=6\sqrt{5}\,\pi$$

3.) $x^2+z^2=1$, $z=y$, $y=x-2$,

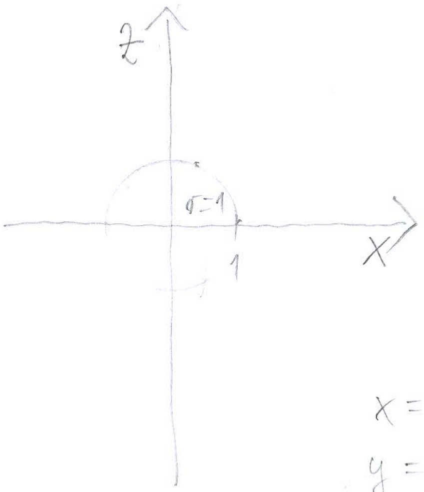
$x^2+y^2=r^2$

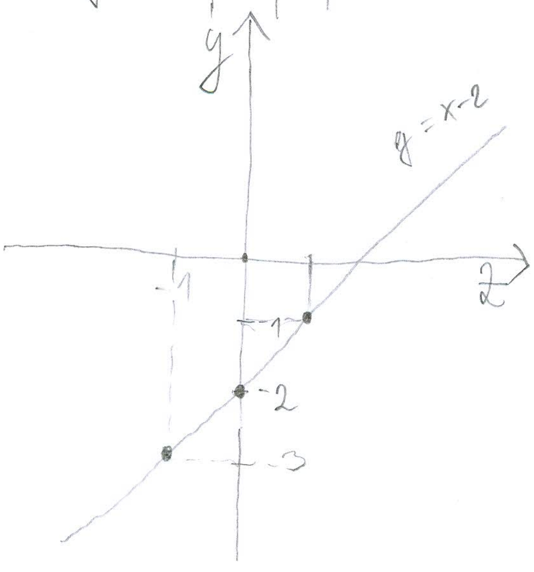
x	0	1	-1
y=x-2	-2	-1	-3

$r^2=1$ $\quad \varphi\in[0,2\pi]$

$r=\sqrt{1}$

$\boxed{r=1}$ $\quad r\in[0,1]$





$x=r\cos\varphi$

$y=r\sin\varphi$

$z=z$

$dx\,dy\,dz=r\,dr\,d\varphi\,dz$

$z\in[r\cos\varphi-2,\ r\sin\varphi]$

$$V=\int_0^{2\pi}\int_0^1\int_{r\cos\varphi-2}^{r\sin\varphi} r\,dz\,dr\,d\varphi=\int_0^{2\pi}\int_0^1 r\,(r\sin\varphi-r\cos\varphi+2)\,dr\,d\varphi$$

$$=\int_0^{2\pi}\int_0^1 (r^2\sin\varphi-r^2\cos\varphi+2r)\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\left(\frac{r^3}{3}\sin\varphi-\frac{r^3}{3}\cos\varphi+\cancel{2}\cdot\frac{r^2}{\cancel{2}}\right)\Big|_0^1 d\varphi=\int_0^{2\pi}\left(\frac{1^3}{3}\sin\varphi-\frac{1^3}{3}\cos\varphi+1^2\right)d\varphi$$

$$=\int_0^{2\pi}\left(\frac{1}{3}\sin\varphi-\frac{1}{3}\cos\varphi+1\right)d\varphi=-\frac{1}{3}\cos\varphi-\frac{1}{3}\sin\varphi+\varphi\Big|_0^{2\pi}$$

$$=\left(-\frac{1}{3}\cos 2\pi-\frac{1}{3}\sin 2\pi+2\pi\right)-\left(-\frac{1}{3}\cos 0-\frac{1}{3}\sin 0+0\right)=$$

$$=-\frac{1}{3}\cdot 1-\frac{1}{3}\cdot 0+2\pi+\frac{1}{3}\cdot 1-\frac{1}{3}\cdot 0=$$

$$=-\cancel{\frac{1}{3}}+2\pi+\cancel{\frac{1}{3}}=2\pi$$

2.) $F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$

$r \in [0,1]$

$\varphi \in [0, 2\pi]$

$\iint\limits_{\partial K} \langle F, ds \rangle = \iiint\limits_{K} \operatorname{div} F$

$\operatorname{div} W = 2x + 0 + 0$

$\operatorname{div} W = 2x$

$\int_0^{2\pi}\int_0^1 2r\cos\varphi \cdot r \, dr\, d\varphi = \int_0^{2\pi}\int_0^1 (2r^2\cos\varphi)\, dr\, d\varphi$

$= \int_0^{2\pi} 2 \cdot \frac{r^3}{3}\cos\varphi \Big|_0^1 d\varphi = \int_0^{2\pi}\left(\frac{2}{3}\cdot 1^3 \cos\varphi - \frac{2}{3}\cdot 0^3 \cos\varphi\right) d\varphi$

$= \int_0^{2\pi} \frac{2}{3}\cos\varphi \, d\varphi = \frac{2}{3}\sin\varphi \Big|_0^{2\pi} = \frac{2}{3}\sin(2\pi - 0) = \frac{2}{3}\sin 2\pi = \frac{2}{3}\cdot 0 = 0$ //

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3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y$ i $y = x - 2$. ~~20~~ 15

4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje. 20

5. Izracunati $\int_{\widehat{ABC}} y dx + y dy$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0, 0, 0)$, $B(1, 0, 0)$, $C(0, 1, 0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. ~~20~~

Ukupno: 55

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int \frac{dx}{x} = \ln \|x\| + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\| x + \sqrt{x^2 \pm a^2} \right\| + C$
$\int \sin x\, dx = -\cos x + C$	$\int \tanh x\, dx = \ln \|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\, dx = \sin x + C$	$\int \coth x\, dx = \ln \|\sinh x\|$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \tan x\, dx = -\ln \|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\, dx = \ln \|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a}\right)\right] + C$

1.

$$x'''(t) - 4x'(t) = \cos(2t) \qquad x(0) = x'(0) = x''(0) = 0$$

$$s^3 X(s) - s^2 \cancel{x(0)} - s\cancel{x'(0)} - \cancel{x''(0)} - 4(sX(s) - \cancel{x(0)}) = \frac{s}{s^2+4}$$

$$s^3 X(s) - 4s X(s) = \frac{s}{s^2+4}$$

$$X(s)(s^3 - 4s) = \frac{s}{s^2+4} \quad \Big| : (s^3-4s)$$

$$X(s) = \frac{s}{(s^2+4)(s^3-4s)} = \frac{s}{s(s^2+4)(s^2-4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4} + \frac{Ds+E}{s^2-4} \quad \Big| \cdot s(s^2+4)(s^2-4)$$

$$s = A(s^2+4)(s^2-4) + (Bs+C)\cdot s(s^2-4) + (Ds+E)\, s\cdot(s^2+4)$$

$$s = A(s^4-16) + (Bs+C)\cdot(s^3-4s) + (Ds+E)(s^3+4s)$$

$$s = As^4 - 16A + Bs^4 - 4Bs^2 + Cs^3 - 4Cs + Ds^4 + 4Ds^2 + Es^3 + 4Es$$

$$0 = A + B + D \Rightarrow 0 = \cancel{A} + B + B \Rightarrow 0 = 2B \Rightarrow \boxed{B = 0} \qquad \boxed{D = 0}$$

$$0 = C + E \Rightarrow C = -E \Rightarrow \boxed{C = -\frac{1}{8}}$$

$$0 = -4B + 4D \Rightarrow 4D = 4B \Rightarrow \boxed{D = B}$$

$$1 = -4C + 4E \Rightarrow 1 = -4(-E) + 4E$$

$$1 = 4E + 4E$$

$$1 = 8E \quad | : 8$$

$$\boxed{E = \frac{1}{8}}$$

$$0 = -16A \Rightarrow \boxed{A = 0}$$

$$X(s) = \cancel{\frac{0}{s}} - \frac{\frac{1}{8}}{s^2+4} + \frac{\frac{1}{8}}{s^2-4}$$

$$x(t) = -\frac{1}{8}\sin(2t) + \frac{1}{8}\sinh(2t)$$

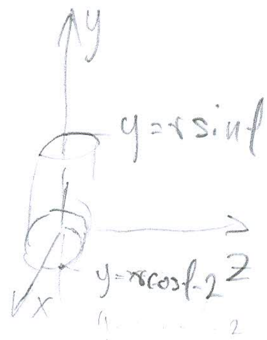
3. $x^2+z^2=1$ $\qquad z=y \qquad y=x-2$

$\Downarrow$

$r=\sqrt{1}$ $\qquad x=r\cos\varphi$

$r=1$ $\qquad z=r\sin\varphi$

$\varphi\in[0,2\pi] \quad r\in[0,1] \qquad y=y$



$y\in[x-2,\ z] \Rightarrow [r\cos\varphi-2,\ r\sin\varphi]$

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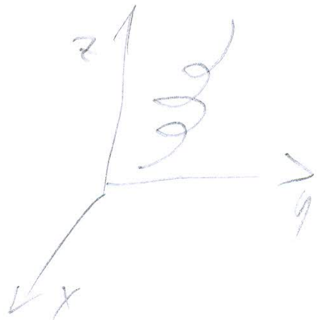
$$\int_0^{2\pi}\int_0^1\int_{r\cos\varphi-2}^{r\sin\varphi} r\,dy\,dr\,d\varphi=\int_0^{2\pi}\int_0^1 r(r\sin\varphi-r\cos\varphi+2)\,dr\,d\varphi=$$

$$=\int_0^{2\pi}\int_0^1 r^2\sin\varphi-r^2\cos\varphi+2r\,dr\,d\varphi=\int_0^{2\pi}\frac{r^3}{3}\sin\varphi-\frac{r^3}{3}\cos\varphi+\cancel{2}\cdot\frac{r^2}{\cancel{2}}\Big|_0^1\,d\varphi$$

$$=\int_0^{2\pi}\left(\frac{1}{3}\sin\varphi-\frac{1}{3}\cos\varphi+1\right)d\varphi=-\frac{1}{3}\cos(\overset{1}{\cancel{2\pi}})-\frac{1}{3}\sin(\overset{0}{\cancel{2\pi}})+2\pi=-\frac{1}{3}+2\pi=$$

$=5.95$

4. $x=\cos(2t)$ $y=\sin(2t)$ $z=t$ $t\in[0,6\pi]$



$$\|r'\|=\sqrt{((\cos(2t))')^2+((\sin(2t))')^2+(t')^2}$$

$$\|r'\|=\sqrt{(-2\sin(2t))^2+(2\cos(2t))^2+1^2}$$

$$\|r'\|=\sqrt{4\sin^2(2t)+4\cos^2(2t)+1}$$

$$\|r'\|=\sqrt{4\underbrace{(\sin^2(2t)+\cos^2(2t))}_{1}+1}$$

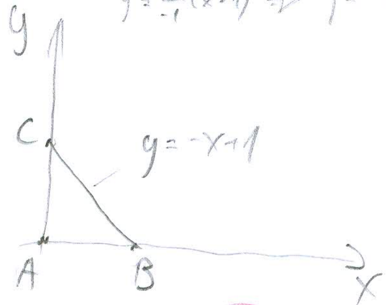
$$\|r'\|=\sqrt{4+1}$$

$$\|r'\|=\sqrt{5}$$

$$\int_{t_1}^{t_2}\|r'\|\,dt=\int_0^{6\pi}\sqrt{5}\,dt=\sqrt{5}\cdot 6\pi=42.14$$

5. $\int_{\overrightarrow{ABC}} y\,dx+y\,dy$ $A(0,0,0)$ $B(1,0,0)$ $C(0,1,0)$

BC:
$y=\frac{1}{-1}(x-1) \Rightarrow y=-x+1$



$$\mathrm{rot}\,\vec{w}=\begin{vmatrix}dx\\dy\\dz\end{vmatrix}\times\begin{vmatrix}0\\y\\y\end{vmatrix}=\begin{vmatrix}1\\0\\0\end{vmatrix}$$

$$\vec{n}=B\times C=\begin{vmatrix}0\\0\\1\end{vmatrix}\times\begin{vmatrix}0\\1\\0\end{vmatrix}=\begin{vmatrix}-1\\0\\0\end{vmatrix}$$

$$\int_0^1\int_0^{-x+1}\begin{bmatrix}1\\0\\0\end{bmatrix}\cdot\begin{bmatrix}-1\\0\\0\end{bmatrix}dx\,dy=\int_0^1\int_0^{-x+1}((-1)^2+0+0)\,dx\,dy=\int_0^1\int_0^{-x+1}dx\,dy=\int_0^1 -x+1\,dx=-1+1$$

$$=0$$

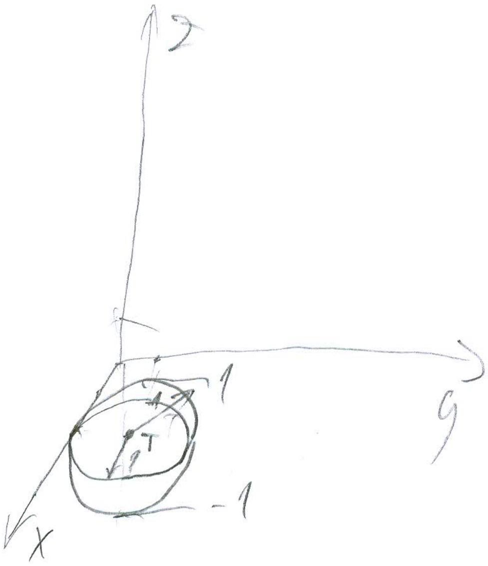
VIDI NAPOMENU KOD LJUBICA BARAĆ

2. $F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ $r = 1$ $T(2,1,0)$

$z \in [-1,1]$

$r \in [0,1]$

$\varphi \in [0, 2\pi]$



$$\operatorname{div} F = \begin{vmatrix} 2x \\ 0 \\ 0 \end{vmatrix} = 2x$$

$x = r\cos\varphi$
$y = r\sin\varphi$
$z = z$

$$\int_0^{2\pi}\int_{-1}^{1}\int_0^{1} 2x\, r\, dr\, dz\, d\varphi = \int_0^{2\pi}\int_{-1}^{1}\int_0^{1} r(2 r\cos\varphi)\, dr\, dz\, d\varphi$$

$$= \int_0^{2\pi}\int_{-1}^{1}\int_0^{1} 2r^2\cos\varphi\, dr\, dz\, d\varphi = \int_0^{2\pi}\int_{-1}^{1} 2\cdot\frac{r^3}{3}\cos\varphi\Big|_0^1 dz\, d\varphi = \int_0^{2\pi}\int_{-1}^{1} \frac{2}{3}\cos\varphi\, dz\, d\varphi =$$

$$= \int_0^{2\pi} \frac{2}{3}\cos\varphi\,(1-(-1))\,d\varphi = \int_0^{2\pi} \frac{2}{3}\cos\varphi\cdot 2\, d\varphi = \int_0^{2\pi} \frac{4}{3}\cos\varphi\, d\varphi = \frac{4}{3}\cancelto{1}{\cos(2\pi)}$$

$$= \frac{4}{3}$$

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IME I PREZIME: Lovre Gojdanić — **BROJ INDEKSA:** 57097	POPUNJAVA NASTAVNIK

	Broj ↓ bodova
+ 1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $$f'''(t) - 4\,f'(t) = \cos(2t)\,, \quad f(0) = f'(0) = f''(0) = 0.$$	~~20~~
2. Izračunati $\iint\limits_{\partial K} \mathbf{F}\cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom.	20
3. Izracunati volumen tijela omeđenog valjkom $x^2+z^2=1$ i ravninama $z=y$ i $y=x-2$.	~~20~~
4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x=\cos 2t$, $y=\sin 2t$ i $z=t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje.	20
+ 5. Izracunati $\int_{\overset{\frown}{ABC}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\overset{\frown}{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu.	~~20~~
	Ukupno: 20

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1. $f'''(t) - 4f'(t) = \cos(2t)$ $\qquad f(0) = f'(0) = f''(0) = 0$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) - 4(sF(s) - f(0)) = \frac{s}{s^2+4}$$

$$s^3F(s) - 4sF(s) = \frac{s}{s^2+4}$$

$$F(s)(s^3 - 4s) = \frac{s}{s^2+4}$$

$$F(s)\,s(s^2-4) = \frac{s}{s^2+4}$$

$$\frac{s}{s(s^2-4)(s^2+4)} = \frac{s}{s(s-2)(s+2)(s^2+4)}$$

$$\frac{s}{s(s-2)(s+2)(s^2+4)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+4}$$

$$s = A(s^2-4)(s^2+4) + B(s^2+2s)(s^2+4) + C(s^2-2s)(s^2+4) + (Ds+E)(s^3-4s)$$

$$s = A(s^4+4\cancel{s^2}-4\cancel{s^2}-16) + B(s^4+4s^2+2s^3+8s) + C(s^4+4s^2-2s^3-8s) + (Ds+E)(s^3-4s)$$

$$s = As^4 - 16A + Bs^4 + 4Bs^2 + 2Bs^3 + 8Bs + Cs^4 + 4Cs^2 - 2Cs^3 - 8Cs + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$0 = A + B + C + D$

$0 = 2B - 2C + E$

$0 = 4B + 4C - 4D$

$1 = 8B - 8C - 4E$

$0 = -16A$

$0 = B + C + D \quad / \cdot 4$

$0 = 4B + 4C - 4D$

$0 = 4B + 4C + 4D$

$0 = 4B + 4C - 4D$

$0 = 8B + 8C$

$0 = 2B - 2C + E \quad / \cdot 4$

$1 = 8B - 8C - 4E$

$0 = 8B - 8C + 4E$

$1 = 8B - 8C - 4E$

$1 = 8B - 16C$

$0 = 8B + 8C \quad \cdot 2$

$1 = 8B - 16C$

$0 = 16B + 16C$

$1 = 24B$

$24B = 1$

$B = \frac{1}{24}$

$-16A = 0$

$A = 0$

$C = -\frac{1}{48}$

$D = -\frac{1}{48}$

$E = -\frac{1}{8}$

VIDI KRISTIJAN KOVAČ

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

$$f(x)=\frac{1}{24}\cdot\frac{1}{s-2}-\frac{1}{48}\cdot\frac{1}{s+2}-\frac{\frac{1}{48}s-\frac{1}{8}}{s^2+4}$$

$$f(x)=\frac{1}{24}\cdot\frac{1}{s-2}-\frac{1}{48}\cdot\frac{1}{s+2}-\frac{1}{48}\cdot\frac{s}{s^2+4}-\frac{1}{8}\cdot\frac{2}{s^2+4}\cdot\frac{1}{2}$$

$$f(t)\ \mathcal{L}^{-1}\{f(x)\}=\frac{1}{24}e^{2t}-\frac{1}{48}e^{-2t}-\frac{1}{48}\cdot\cos(2t)-\frac{1}{18}\cdot\sin(2t)\ //$$

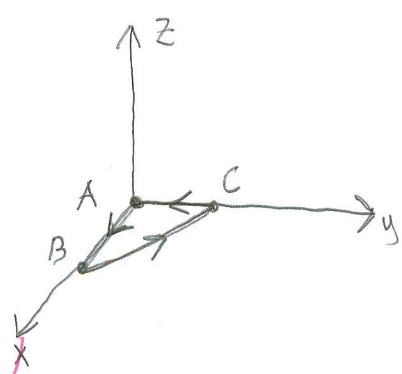
5. $\int\limits_{\overrightarrow{ABC}} y\,dx+y\,dy$

$A(0,0,0)$
$B(1,0,0)$
$C(0,1,0)$

$$W=\begin{bmatrix}y\\y\\0\end{bmatrix}\ \text{rot}\,W\ \begin{bmatrix}\cancel{\partial x}\\\partial y\\\partial z\end{bmatrix}\times\begin{bmatrix}\cancel{y}\\y\\0\end{bmatrix}$$

$$\begin{matrix}\partial x & \times & y\\ \partial y & & y\end{matrix}$$

$$=\begin{bmatrix}0-y\partial z\\ y\partial z-0\\ y\partial x-y\partial y\end{bmatrix}=\begin{bmatrix}0-0\\0-0\\0-1\end{bmatrix}=\begin{bmatrix}0\\0\\-1\end{bmatrix}$$



$$ds=\begin{bmatrix}\cancel{0\quad 0}\\1\times 0\\0\times 1\end{bmatrix}\quad\begin{matrix}0\times 0\\1\quad 0\end{matrix}=\begin{bmatrix}1-0\\0-0\\0-0\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}\ //$$

VIDI NAPOMENU BARAĆ GJOBICA

$$\iint\begin{bmatrix}0\\0\\-1\end{bmatrix}\times\begin{bmatrix}1\\0\\0\end{bmatrix}dx\,dy=-\int_0^1\int_1^0 dx\,dy$$

$$-\int_0^1 dx\int_1^2 dy=-\int_0^1 dx\,(1)\Big|_1^2=-\int_0^1 dx=-x\Big|_0^1=-1\ //$$

$[0, 6\pi]$

4. $x = \cos 2t$
$y = \sin 2t$
$z = t$

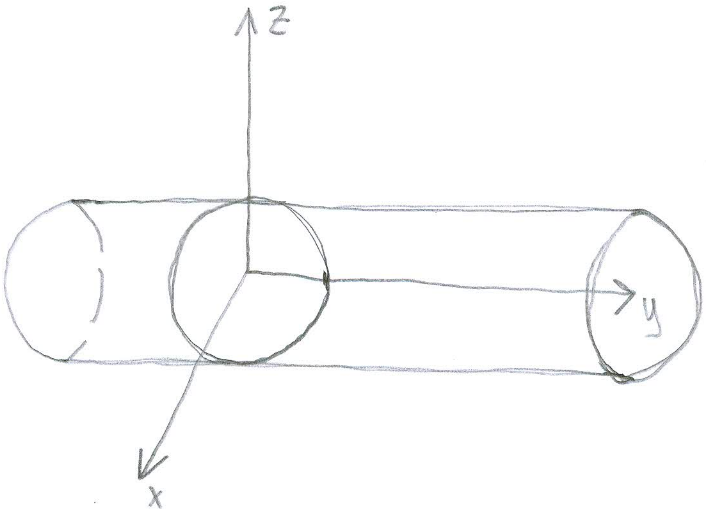
$x = -2\sin 2t$
$y = 2\cos 2t$
$z = 1$

$$||r(t)|| = \sqrt{(-2\sin 2t)^2 + (2\cos 2t)^2 + (1)^2}$$
$$= \sqrt{4+1}$$
$$= \sqrt{5}$$

$$\int_0^{6\pi} \sqrt{5}\, dt = \sqrt{5}\, t \Big|_0^{6\pi} = 6\pi\sqrt{5}$$

3.
$x^2 + z^2 = 1$
$r^2 = 1$
$r = \sqrt{1}$

$z = y \quad y = x - 2$



$\varphi \in [0, 2\pi]$
$r \in [0, 1]$
$y \in [0, x-2]$

$x = r\cos\varphi$
$z = r\sin\varphi$
$y = y$

$$V = \int_0^{2\pi} d\varphi \int_0^1 r\, dr \int_0^{x-2} dy$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: JOSIP MIJALIĆ **BROJ INDEKSA:** 57154	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) - 4\,f'(t) = \cos(2t)\,, \quad f(0) = f'(0) = f''(0) = 0.$$

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3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y$ i $y = x - 2$. ~~20~~ 15+5

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Ukupno: 20

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$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1) $f'''(t) \Rightarrow s^3F(s) - \cancel{s^2\cdot 0 - 0 - 0}$

$4\,f'(t) \Rightarrow sF(s) - \cancel{0}$

$\cos(2t) \Rightarrow \frac{s}{s^2+4}$

$s^3F(s) - 4\cdot sF(s) = \frac{s}{s^2+4}$

$f(s)=$

$f(s) = \frac{s}{(s-2)(s+2)(s^2+4)} = \frac{s}{(s^2+4)(s-2)(s+2)} =$

$= \frac{A}{s} + \frac{Bs+C}{s^2+4} + \frac{D}{s-2} + \frac{E}{s+2} \;/\cdot s(s^2+4)(s-2)(s+2)$

$s = A(s^2+4)(s-2)(s+2) + (Bs+C) + s(s^2-4) + D(s)(s^2-4)(s+2) +$
$Es(s^2+4)(s-2)$

$s = A(s^4-16) + (Bs^2+C)(s^2-4) + D(s^4\,2s)(s+2) + E(s^3+4)(s-2)$

$s = As^4 - 16A + Bs^4 - Bs^2 - 4 + Cs^3 - 4Cs + D(s^4+2s^3+4s^2)$

$+E(s^3-2s^2+4s-8)$

$$S = S^4\left(A+B+D+E\right) + S^3\left(C+2E-2D\right) - S^2\left(-4B+4B+4E\right)$$
$$+ S\left(4C+8D-8E\right) - 16A$$

$$A+B+D+E=0$$
$$C+2E-2D=0$$
$$-4B+4B+4E=0$$
$$-4C+8D-8E=1$$
$$-16A = 0 \qquad A=0$$

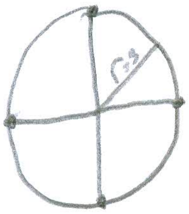
$\boxed{A=0}$ $\boxed{D=\frac{1}{32}}$ $\boxed{E=-\frac{1}{32}}$ $\boxed{B=0}$ $\boxed{C=0}$

$$f(s) = \frac{0}{S} + \frac{0.S+0}{S^2+4} + \frac{\frac{1}{32}}{S-2} + \frac{-\frac{1}{32}}{S+2}$$

$$f(s) = \frac{1}{32(S-2)} - \frac{1}{32(S+2)}$$

$$f(t) = \frac{1}{32}e^{2t} - \frac{1}{32}e^{-2t}$$

3) $x^2+z^2=1 \qquad z=y \qquad y=x-2$



$x = r\cos f$
$z = r\sin f$
$y = y$

15

$$\int_0^{2\pi}\delta f\int_0^1 r\,dr\int_{r\cos f-2}^{r\sin f}dy = \int_0^{2\pi}\delta f\int_0^1 r\,dr\,(r\sin f - r\cos f+2)$$

$$= \int_0^{2\pi}\delta f\int_0^1 (r^2\sin f - r^2\cos f + 2r)\,dr$$

$$= \int_0^{2\pi}\delta f\left(\int_0^1 r^2\sin f - r^2\cos f + 2r\right)dr$$

$$= \int_0^{2\pi}\delta f\left[\frac{r^3}{3}\sin f\Big|_0^1 - \frac{r^3}{3}\cos f\Big|_0^1 + 2\frac{r^2}{2}\Big|_0^1\right]$$

$$= \int_0^{2\pi}\delta f\left[\frac{1^3}{3}\sin f - \frac{1}{3}\cos f + \cancel{2}\cdot\frac{1}{\cancel{2}}\right]$$

$$= \frac{1}{3}\int_0^{2\pi}\sin f\,dy - \frac{1}{3}\int_0^{2\pi}\cos f\,dy + \int_0^{2\pi}dy$$

$$= -\frac{1}{3}\cos f\int_0^{2\pi} - \frac{1}{3}\sin f\,dy + \int_0^{2\pi}dy =$$

?

$$= \left(-\frac{1}{3}\cos 2\pi - \cos 0\right) - \left(\frac{1}{3}\sin 2\pi - \cos 0\right) + 2\pi - 0$$

$$= -\frac{1}{3}$$

$$= -\frac{1}{3}\left(\cancel{1-1}^{\,0}\right) - \frac{1}{3}\left(\cancel{0-0}\right) + 2\pi - 0 = 2\pi$$

5

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** BARAĆ LJUBICA **BROJ INDEKSA:**	Grupa XXOXO POPUNJAVA NASTAVNIK Broj ↓ bodova

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3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y$ i $y = x - 2$. 20

4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje. 20

5. Izracunati $\int_{\overset{\frown}{ABC}} y dx + y dy$ gdje je $\overset{\frown}{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0, 0, 0)$, $B(1, 0, 0)$, $C(0, 1, 0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: 0

Tablica integrala

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$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x \,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \tanh x \,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \coth x \,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1. $f'''(t)-4f'(t)=\cos(2t)$ $\qquad f(0)=f'(0)=f''(0)=0$

$$s^3F(s)-s^2f(0)-sf'(0)-f''(0)-4\left[sF(s)-f(0)\right]=\frac{s}{s^2+4}$$

$$s^3F(s)-s^2\cancel{f(0)}-s\cancel{f'(0)}-\cancel{f''(0)}-4sF(s)+4\cancel{f(0)}=\frac{s}{s^2+4}$$

$$s^3F(s)-4s\,F(s)=\frac{s}{s^2+4}$$

$$F(s)(s^3-4s)=\frac{s}{s^2+4}\qquad \Big|:(s^3-4s)$$

$$F(s)=\frac{s}{s(s^2-4)(s^2+4)}=\qquad \boxed{s^2-4=(s-2)(s+2)}$$

$$F(s)=\frac{s}{s(s-2)(s+2)(s^2+4)}=$$

$$\frac{s}{s(s-2)(s+2)(s^2+4)}=\frac{A}{s}+\frac{B}{s-2}+\frac{C}{s+2}+\frac{Ds+E}{s^2+4}$$

$$s=A(s^2-4)(s^2+4)+Bs(s+2)(s^2+4)+Cs(s-2)(s^2+4)+Ds^2(s^2-4)+Es(s^2-4)$$

$$s=A(s^4+\cancel{4s^2}-\cancel{4s^2}-16)+Bs(s^3+4s+2s^2+8)+Cs(s^3+4s-2s^2-8)+Ds^4-4Ds^2+Es^3-4Es$$

$$s=As^4-16A+Bs^4+4Bs^2+2Bs^3+8Bs+Cs^4+4Cs^2-2Cs^3-8Cs+Ds^4-4Ds^2+Es^3-4Es$$

$$s=(A+B+C+D)s^4+(2B-2C+E)s^3+(4B+4C-4D)s^2+(8B-8C-4E)s+(-16A)$$

$0 = A + B + C + D$

$0 = 2B - 2C + E$

$0 = 4B + 4C - 4D$

$1 = 8B - 8C - 4E$

$0 = -16A$

$8B - 8C - 4E = 0$

$-8E - 8 \cdot \frac{15}{16} B - 4E = 0$

$-8E - \frac{120}{16} B - 4E = 0 \mid \cdot 16$

$-128E - 120B - 64E = 0$

$-192E - 120B = 0$

$-120B = 192E \mid :120$

$\boxed{B = \frac{8}{5}}$

$-16A = 0$

$\boxed{A = 0}$

$2B - 2C + E = 0 \mid \cdot 4$

$8B - 8C - 4E = 1$

$8B - 8C + \cancel{4E} = 0$

$8B - 8C - \cancel{4E} = 1$

$16B - 16C = 1$

$-16C = 1 - 16B \mid (-16)$

$C = -\frac{1-16}{16} B$

$\boxed{C = \frac{15}{16} B}$

$C = \frac{15}{16} \cdot \frac{8}{5}$

$C = \frac{120}{80}$

$\boxed{C = \frac{3}{2}}$

$2B - 2C + E = 0$

$2B - \cancel{2} \cdot \frac{15}{\cancel{16}_{8}} B + E = 0$

$2B - \frac{15}{8} B + E = 0 \mid \cdot 8$

$16B - 15B + E = 0$

$1B + E = 0$

$\boxed{B = -E}$

$\frac{8}{5} E = -1E \mid \cdot 5$

$8E = -5E \mid :8$

$\boxed{E = -\frac{5}{8}}$

$A + B + C + D = 0$

$0 + \frac{8}{5} + \frac{3}{2} + D = 0$

$\frac{16+15}{10} + D = 0$

$\frac{31}{10} + D = 0$

$D = -\frac{31}{10}$

$X(s) = \frac{8}{5} \cdot \frac{1}{s-2} + \frac{3}{2} \cdot \frac{1}{s+2} - \frac{31}{10} \cdot \frac{s}{s^2+4} - \frac{5}{8} \cdot \frac{1}{s^2+4}$

$x(t) = L^{-1} \{ x(s) \}$

$x(t) = \frac{8}{5} \cdot$

$t \in [0, 6\pi]$

(4) $x = \cos 2t$

$y = \sin 2t$

$z = t$

$$r(t) = \begin{bmatrix} \cos 2t \\ \sin 2t \\ t \end{bmatrix} \qquad r'(t) \begin{bmatrix} \cancel{-\frac{\sin 2t}{2}} \\ \cancel{\frac{\cos 2t}{2}} \\ 1 \end{bmatrix}$$

$$|r'(t)| = \sqrt{\left(-\frac{\sin 2t}{2}\right)^2 + \left(\frac{\cos 2t}{2}\right)^2 + 1^2}$$

$$= \sqrt{\frac{\sin 4t^2}{4} + \frac{\cos 4t^2}{4} + 1}$$

$$= \sqrt{\frac{1}{4}\left(\sin 4t^2 + \cos 4t^2\right) + 1}$$

$$= \sqrt{\cancel{\frac{1}{4}} \cdot \cancel{4}\left(\sin^2 t + \cos^2 t\right) + 1}$$

$$= \sqrt{2}$$

$$\int_0^{6\pi} \sqrt{2}\,dt = \sqrt{2} \cdot (t)\Big|_0^{6\pi} = \sqrt{2} \cdot 6\pi - 0 = 6\pi\sqrt{2}$$

5. $\oint_{ABC} y\,dx + y\,dy$

BOLJE PREKO GREENOVE FORMULE

A $(0,0,0)$

B $(1,0,0)$

C $(0,1,0)$

$$W = \begin{bmatrix} y \\ y \\ 0 \end{bmatrix} \qquad \operatorname{rot} W = \begin{bmatrix} \partial x \\ \partial y \\ \partial z \end{bmatrix} \times \begin{bmatrix} y \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 - y\partial z \\ y\partial z - 0 \\ y\partial x - y\partial y \end{bmatrix} =$$

$$= \begin{bmatrix} 0 - 0 \\ 0 - 0 \\ 0 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

MOŽE I OVAKO PREKO STOKESOVE FORMULE

<u>ALI</u>

$$ds = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 - 0 \\ 0 - 0 \\ 0 - 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

OVO NIJE VEKTOR NORMALE

$$\circledast \iint \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

③ $\underbrace{x^2+z^2}=1$ $\quad z=y$ $\quad y=x-2$ $\quad$ VALJAK

$r^2=1$ $\quad y=z$

$r=\pm 1$ $\quad y=r\sin\varphi$

$$\boxed{\begin{array}{l} x=r\cos\varphi \\ z=r\sin\varphi \\ y=y \\ dxdy=rdrd\varphi \end{array}}$$

$r\in[0,1]$

$\varphi\in[0,2\pi]$

$y\in[r\sin\varphi, x-2]$

$$\int_0^{2\pi}\int_0^1\int_{r\sin\varphi}^{x-2} dy\, rdrd\varphi = \int_0^{2\pi}\int_0^1 \left(y\right)\Big|_{r\sin\varphi}^{x-2} rdrd\varphi =$$

$$= \int_0^{2\pi}\int_0^1 (x-2)-(r\sin\varphi)\; rdrd\varphi$$

VAJAK

(3) $\underbrace{x^2+z^2}_{}=1$ $\qquad z=y \qquad y=x-2$

$r^2=1 \qquad y=r\sin\varphi$

$r=\pm1$

$$\boxed{\begin{aligned} x&=r\cos\varphi\\ z&=r\sin\varphi\\ y&=y\\ dxdy&=rdrd\varphi\end{aligned}}$$

$r\in[0,1]$
$\varphi\in[0,2\pi]$
$y\in[r\sin\varphi, x-2]$

$$\int_0^{2\pi}\int_0^{1}\int_{r\sin\varphi}^{x-2} dy\, rdrd\varphi = \int_0^{2\pi}\int_0^1 (y)_{r\sin\varphi}^{x-2}\, rdrd\varphi =$$

$$=\int_0^{2\pi}\int_0^1 (x-2)-(r\sin\varphi)\, rdrd\varphi =$$

(1) $$\int_0^{2\pi}\int_0^1 x-2\ rdrd\varphi = \int_0^{2\pi}\left(x-2\cdot\frac{r^2}{2}\right)_0^1 d\varphi =$$

$$=\int_0^{2\pi} -\frac{1}{2}d\varphi = -\frac{1}{2}\cdot 2\pi = -\pi$$

(2) $$\int_0^{2\pi}\int_0^1 r\sin\varphi\, rdrd\varphi = \int_0^{2\pi}\int_0^1 r^2\sin\varphi\, drd\varphi =$$

$$=\int_0^{2\pi}\left(\frac{r^3}{3}\cdot\sin\varphi\right)_0^1 d\varphi = \int_0^{2\pi}\left(\frac{1}{3}\cdot 0\right)d\varphi =$$

$$=\int_0^{2\pi} d\varphi = (\varphi)_0^{2\pi} = 2\pi$$

$$*\iint -\pi-2\pi = -3\pi = -9.42$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ANTE GRUBIŠA **BROJ INDEKSA:** 57831

Grupa xxoxo
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20
$$f'''(t) - 4\,f'(t) = \cos(2t)\,, \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint\limits_{\partial K} \mathbf{F}\cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2+z^2=1$ i ravninama $z=y$ i $y=x-2$. 20

4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x=\cos 2t$, $y=\sin 2t$ i $z=t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje. 20

5. Izracunati $\int_{\overset{\frown}{ABC}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\overset{\frown}{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
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5 $\iint y\,dx \; y\,dy$

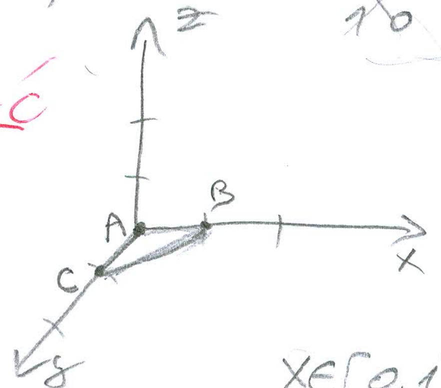
$\overrightarrow{ABC}$

$A\overset{x\ y\ z}{(0,0,0)}$

$B(1,0,0)$

$C(0,1,0)$

$\begin{matrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{matrix} \qquad \begin{matrix} 1-0 \\ 0-0 \\ 0-0 \end{matrix}$



$w = \begin{bmatrix} y \\ y \\ 0 \end{bmatrix}$ rot w

VIDI NAPOMENU KOD LJUBICA BARAĆ

$$\begin{bmatrix} dx & y \\ dy & y \\ dz & 0 \\ dx & y \\ dy & y \end{bmatrix} = \begin{bmatrix} 0 - ydz \\ ydz - 0 \\ ydx - ydy \end{bmatrix} = \begin{bmatrix} 0-0 \\ 0-0 \\ 0-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$x \in [0,1]$

$y \in [0, -x+1]$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$= \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1-0 \\ 0-0 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

OVO NIJE VEKTOR NORMALE

$BC: y - 0 = \frac{1-0}{0-1}(x-1)$

$y = -1x + 1$

$y = -x + 1$

$$\int_0^1 \int_0^{-x+1} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} dx\,dy = \cancel{0}$$

---

1. $f'''(t) - 4f''(t) = \cos(2t)$

$f''(0) = 0$

$f'(0) = 0$

$f(0) = 0$

$$s^3F(s) - \cancel{s^2 f(0)} - \cancel{s f'(0)} - \cancel{f''(0)} - 4\left(s^2F(s) - \cancel{s f(0)} - \cancel{f'(0)}\right) = \frac{s}{s^2+2^2}$$

$$s^3F(s) - 4s^2F(s) = \frac{s}{s^2+4}$$

$\frac{A}{s^3}$

$$F(s)(s^3 - 4s^2) = \frac{s}{s^2+4}$$

$$F(s) = \frac{s}{s^2+4} \cdot \frac{1}{s^3 - 4s^2}$$

$$F(s) = \frac{s}{s^5 - 4s^4 + 4s^3 - 16s^2}$$

$\cancel{0}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: MATIJA ŠKIBOLA **BROJ INDEKSA:**

Grupa xxoxo
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

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3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y$ i $y = x - 2$. 20

4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje. 20

5. Izracunati $\int_{\widehat{ABC}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
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MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: LOVRE KOLEGA **BROJ INDEKSA:** 58143	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

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Ukupno: 0

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$\int \frac{\mathrm{d}x}{x} = \ln \vert x \vert + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\vert \frac{x-a}{x+a} \right\vert + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x \,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\vert x + \sqrt{x^2 \pm a^2} \right\vert + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \tanh x \,\mathrm{d}x = \ln \vert \cosh x \vert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \coth x \,\mathrm{d}x = \ln \vert \sinh x \vert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \tan x \,\mathrm{d}x = -\ln \vert \cos x \vert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \,\mathrm{d}x = \ln \vert \sin x \vert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

1. $f'''(t) - 4f'(t) = \cos(2t)$ $f(0) = f'(0) = f''(0) = 0$

$f'''(t) \circ\!\!- \ s^3 X(s) - s^2 X'(0) - s X''(0) - 4f'(x)(0)$

$s^3 X(s) - s^2 - 1$

$f'(t) \circ\!\!- \ sX(s) - X(0)$

$4(sX(s) - 1)$

$4sX(s) - 4$

$s^3 X(s) - s^2 - 1 - 4sX(s) + 4 = \frac{s}{s^2+4}$

$X(s)(s^3 - 4s) - s^2 + 3 = \frac{s}{s^2+4}$

$X(s) = \frac{\frac{s}{s^2+4}}{(s^3 - 4s) - s^2 + 3}$

$X(s) = \cancel{\frac{s}{s^5 + 3s^2 - s^4}}$

$\boxed{X(s) = \frac{s}{s^5 - s^4 - s^2 - 16s + 12}}$

$(s^2+4)\cdot(s^3 - 4s - s^2 + 3)$

$s^5 - \cancel{4s^3} - s^4 + 3s^2 + \cancel{4s^3} - 16s - 4s^2 + 12$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: ANĐELO UGRINIĆ **BROJ INDEKSA:** 55581	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - 4\,f'(t) = \cos(2t)\,, \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F}\cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2+z^2=1$ i ravninama $z=y$ i $y=x-2$. 20

4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x=\cos 2t$, $y=\sin 2t$ i $z=t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje. 20

5. Izracunati $\int_{\widehat{ABC}} y\,dx + y\,dy$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: 0

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1. $f'''(t)-4f'(t)=\cos(2t)$ $\qquad f(0)=f'(0)=f''(0)=0$

$$s^3F(s)-s^2f(0)-sf'(0)-f''(0)-4\big(sF(s)-f(0)\big)=\frac{s}{s^2+2^2}$$

$$s^3F(s)-\cancel{s^2f(0)}-\cancel{sf'(0)}-\cancel{f''(0)}-4sF(s)+\cancel{4f(0)}=\frac{s}{s^2+2^2}$$

$$s^3F(s)-4sF(s)=\frac{s}{s^2+4}$$

$$F(s)(s^3-4s)=\frac{s}{s^2+4}$$

$$F(s)=\frac{s}{s(s^2-4)(s^2+4)}=\frac{s}{s(s-2)(s+2)(s^2+4)}$$

$$\frac{s}{s(s-2)(s+2)(s^2+4)}=\frac{A}{s}+\frac{B}{s-2}+\frac{C}{s+2}+\frac{Ds+E}{s^2+4}$$

$$s=A(s^2-4)(s^2+4)+Bs(s+2)(s^2+4)+Cs(s-2)(s^2+4)+(Ds+E)(s^3-4s)$$

$$s=A(s^4-16)+Bs(s^3+4s+2s^2+8)+Cs(s^3-4s-2s^2-8)+Ds^4-4Ds^2+Es^3-4Es$$

$$s=As^4-16A+Bs^4+4Bs^2+2Bs^3+8Bs+Cs^4-4Cs^2-2Cs^3-8Cs+Ds^4-4Ds^2+Es^3-4Es$$

$0=A+B+C+D$

$0=2B-2C+E$

$0=4B-4C-4D$

$1=8B-8C-4E$

$0=-16A \Rightarrow \boxed{A=0}$

$0=A+B+C+D$

$0=4B-4C-4D \;/:4$

$0=B+C+D$

$0=B-C-D$ $]+$

$0=2B \Rightarrow \boxed{B=0}$

$0=C+D$

$0=-4C-4D \;/:4$

$0=C+D$

$0=-C-D$ $]+$

$0=0$

$\boxed{C=0}$ $\boxed{D=0}$

$1=0-0-4E$

$1=-4E \;/:4$

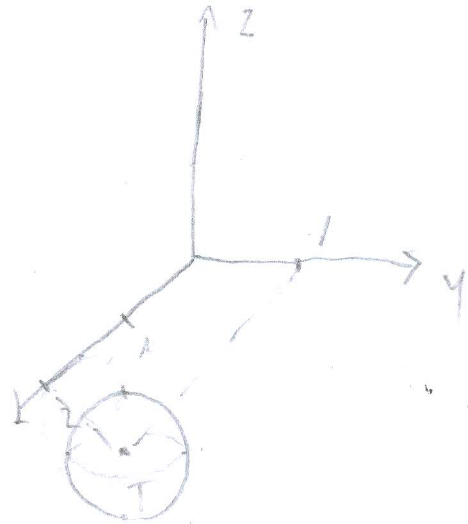
$\boxed{E=-\frac{1}{4}}$

$$F(s)=-\frac{1}{4}\cdot\frac{1}{s^2+4}$$

$$f(t)=\mathcal{L}^{-1}\{F(s)\}$$

$$f(t)=-\frac{1}{4}\cdot\frac{1}{2}\cdot\frac{2\cdot 1}{s^2+2^2}=\boxed{-\frac{1}{8}\sin(2t)}$$

② $\iint_{\delta K} F \cdot dS$ $\quad F = \begin{bmatrix} x^2+y^2 \\ z \\ 1 \end{bmatrix}$ $\quad T(2,1,0)$



$\emptyset$

③ $\underbrace{x^2+z^2}_{r^2} = 1$ $\quad z = y \quad y = x-2$

$r^2 = 1 \quad / \sqrt{\ }$

$r = \sqrt{1}$

$r = 1$

$x \in (0,1)$

$y \in (0, 2\pi)$

$z \in (x, x-2)$

$$V = \int_0^{2\pi} dy \int_0^1 dx \int_x^{x-2} dz = \int_0^{2\pi} dy \int_0^1 x-2-x \, dx = \int_0 \frac{x^2}{2} - 2x - \frac{x^2}{2} \Big|_0^1 = \int_0^{2\pi} -2 \, dy$$

$$= -2 \int_0^{2\pi} dy$$

$$= -2 \frac{y^2}{2} \Big|_0^{2\pi}$$

$$= -y^2 \Big|_0^{2\pi}$$

$$= -2\pi^2$$

$\emptyset$

(4) $x = \cos 2t \qquad y = \sin 2t \qquad z = t$

$$r'(t) = \begin{bmatrix} -\sin 2t \\ \cos 2t \\ 1 \end{bmatrix}$$

$$\|r'(t)\| = \sqrt{(-\sin(t))^2 + (\cos 2t)^2 + 1^2}$$

$$= \sqrt{\sin^2 2t + \cos^2 2t + 1}$$

$$= \sqrt{2t\,(\underbrace{\sin^2 + \cos^2}_{1}) + 1}$$

$$= \sqrt{2t + 1}$$

$$\int \sqrt{2t+1}\, dt$$

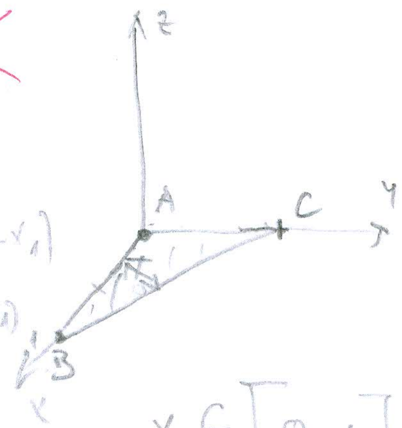
⑤ $\int\limits_{\overset{\frown}{ABC}} y\,dx + y\,dy$

VIDI NAPOMENU KOD LJUBICA BARAĆ

	x	y	z
A	0	0	0
B	1	0	0
C	0	1	0

$W \begin{bmatrix} y \\ y \\ 0 \end{bmatrix} \Rightarrow \text{rot}\, W \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \times \begin{bmatrix} y \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 - y\,dz \\ y\,dz - 0 \\ y\,dx - y\,dy \end{bmatrix}$

$= \begin{bmatrix} 0 - 0 \\ 0 - 0 \\ 0 - y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -y \end{bmatrix}$



$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{1-0}{0-1}(x-1)$

$y = -1(x-1)$

$y = -x+1$

$y = 1-x$

$x \in [0, 1]$

$y \in [x, 1-x]$

$dS = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1-0 \\ 0-0 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\iint\limits_{\overset{\frown}{ABC}} \begin{bmatrix} 0 \\ 0 \\ -y \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} dx\,dy = \iint\limits_{\overset{\frown}{ABC}} -y\,dx\,dy = \int_0^1\int_x^{1-x} -y\,dx\,dy = \int_0^1 dx \int_x^{1-x} -y\,dy$

$= \int -(1-x) + x\,dx = \int^1 -1 + x + x\,dx = \int^1 -1 + 2x\,dx = -1 + 2x\Big|^1 = -1 + 2 = 1$