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MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - 4f'(t) = \cos(2t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y$ i $y = x - 2$. 20

4. Zadana je kruzna uzvojnica (spirala) s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Skiciraj krivulju. Izraèunati duljinu 3 namotaja ove krivulje. 20

5. Izracunati $\int_{\widehat{ABC}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: 60

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1.) $f'''(t) - 4f'(t) = \cos(2t)$, $f(0) = f'(0) = f''(0) = 0$

$$s^3F(s) - s^2\underbrace{f(0)}_{0} - s\underbrace{f'(0)}_{0} - \underbrace{f''(0)}_{0} - 4\left(sF(s) - \underbrace{f(0)}_{0}\right) = \frac{s}{s^2+4}$$

$$s^3F(s) - 4sF(s) = \frac{s}{s^2+4}$$

$$F(s)\left(s^3 - 4s\right) = \frac{s}{s^2+4}$$

$$F(s) = \frac{s}{s^2+4} \cdot \frac{1}{s^3-4s} = \frac{s}{(s^2+4)s(s^2-4)} = \frac{s}{s(s+2)(s-2)(s^2+4)}$$

$$\frac{s}{s(s+2)(s-2)(s^2+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s-2} + \frac{Ds+E}{s^2+4} \quad / \cdot s(s+2)(s-2)(s^2+4)$$

$$s = A(s^2-4)(s^2+4) + Bs(s-2)(s^2+4) + Cs(s+2)(s^2+4) + (Ds+E)s(s^2-4)$$

$$s = A(s^4-16) + Bs(s^3+4s-2s^2-8) + Cs(s^3+4s+2s^2+8) + (Ds+E)(s^3-4s)$$

$$s = As^4 - 16A + Bs^4 + 4Bs^2 - 2Bs^3 - 8Bs + Cs^4 + 4Cs^2 + 2Cs^3 + 8Cs + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$$s = (A+B+C+D)s^4 + (-2B+2C+E)s^3 + (4B+4C-4D)s^2 + (-8B+8C-4E)s - 16A$$

$A+B+C+D=0$

$-2B+2C+E=0$

$B+C-D=0$

$-8B+8C-4E=1$

$-16A=0$

$\boxed{A=0}$

$-2B+2C+E=0 \;/\cdot(-4)$

$-8B+8C-4E=1$

$8B-8C-4E=0$

$-8B+8C-4E=1$ $\}+$

$-8E=1 \;/:(-8)$

$\boxed{E=-\frac{1}{8}}$

$B+C-D=0$

$B=-C+D$, $B=-C$

$-C+D+C+D=0$

$2D=0 \;/:2$

$\boxed{D=0}$

$2C+2E+\frac{1}{8}=0$

$4C=\frac{1}{8} \;/:4$

$C=\frac{1}{8}\cdot\frac{1}{4}$

$\boxed{C=\frac{1}{32}}$

$\boxed{B=-\frac{1}{32}}$

$$F(s) = 0\cdot\frac{1}{s} - \frac{1}{32}\cdot\frac{1}{s+2} + \frac{1}{32}\cdot\frac{1}{s-2} + \frac{0\cdot s - \frac{1}{8}}{s^2+4}$$

$$F(s) = -\frac{1}{32}\cdot\frac{1}{s+2} + \frac{1}{32}\cdot\frac{1}{s-2} - \frac{1}{8}\cdot\frac{1}{s^2+4}$$

$$f(t) = -\frac{1}{32}e^{-2t} + \frac{1}{32}e^{2t} - \frac{1}{8}\cdot\frac{1}{2}\left(\frac{2}{s^2+2^2}\right)$$

$$f(t) = -\frac{1}{32}e^{-2t} + \frac{1}{32}e^{2t} - \frac{1}{16}\sin(2t)$$