Popuniti odmah!

Ime i prezime: LUKA SJAUŠ **Broj indeksa:** 57680 - 2009

Datum: 21.02.2012. **Vrijeme: od** 13:13 **do** .

50

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

✓ 1. Izračunati $\int \frac{x^2+2x+2}{x^2+x-2}\,dx$. 15

✓ 2. Izračunati $\int x^2 \sin(x)\,dx$. 15

3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala:
$$\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,dx$$ 15

4. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$. 20

✓ 5. Riješiti zadanu diferencijalnu jednadžbu uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. Uvrstiti rješenje u jednadžbu i provjeriti zadovoljenje jednakosti. 20
$$y'' + 2y' + y = 0$$

✓ 6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = 2x\cos x$ oko točke $x_0 = \frac{\pi}{2}$. $\cancel{15}$

1.
$$\int \frac{x^2+2x+2}{x^2+x-2}\,dx = \int\left(1+\frac{x+4}{x^2+x-2}\right)dx$$

$$(x^2+2x+2):(x^2+x-2) = 1 + \frac{x+4}{x^2+x-2}$$
$$\underline{-x^2 \mp x \pm 2}$$
$$x+4$$

$$= \int 1\cdot dx + \underbrace{\int \frac{x+4}{x^2+x-2}\,dx}_{I_1} = x + I_1 = x + \frac{5}{3}\ln|x-1| - \frac{2}{3}\ln|x+2| + C$$ ✓

x^2+x-2

$a=1 \quad b=1 \quad c=-2$

$$x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x_{1,2} = \frac{-1\pm\sqrt{1^2+8}}{2}$$
$$x_1 = \frac{-1+3}{2} = 1$$
$$x_2 = \frac{-1-3}{2} = -2$$

$$I_1 = \int \frac{x+4}{(x-1)(x+2)}\,dx = \int \frac{A}{x-1}\,dx + \int \frac{B}{x+2}\,dx$$

$$\frac{x+4}{(x-1)(x+2)} \equiv \frac{A}{x-1} + \frac{B}{x+2} \quad /(x-1)(x+2)$$

$$x+4 \equiv A(x+2) + B(x-1)$$
$$x+4 \equiv Ax + 2A + Bx - B$$
$$x+4 \equiv (A+B)x + 2A - B$$

$$A+B = 1 \quad / \pm \quad \Rightarrow \frac{5}{3} + B = 1$$
$$2A - B = 4 \qquad B = 1 - \frac{5}{3}$$
$$3A = 5 \;/:3 \qquad \boxed{B = -\frac{2}{3}}$$
$$\boxed{A = \frac{5}{3}}$$

$$I_1 = \int \frac{\frac{5}{3}}{x-1}\,dx + \int \frac{-\frac{2}{3}}{x+2}\,dx \qquad \left\{\begin{matrix} x-1 = t & x+2 = t \\ dx = dt & dx = dt \end{matrix}\right\}$$

$$= \frac{5}{3}\int \frac{dx}{x-1} - \frac{2}{3}\int\frac{dx}{x+2}$$

$$= \frac{5}{3}\int \frac{dt}{t} - \frac{2}{3}\int \frac{dt}{t}$$

$$= \frac{5}{3}\ln|t| - \frac{2}{3}\ln|t| = \frac{5}{3}\ln|x-1| - \frac{2}{3}\ln|x+2| + C$$

2. $\int x^2 \sin(x)dx = u \cdot v - \int v \cdot du$

$u = x^2 \; / ^d$ $\qquad dv = \sin(x) \cdot dx \; / ^\int$

$du = 2x \cdot dx$ $\qquad V = \int \sin x \cdot dx$

$V = -\cos x$

$$\int x^2 \sin(x)dx = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x \cdot dx$$

$$= -x^2 \cos x + 2\int \cos x \cdot x \cdot dx$$

NOVA PARCIJALNA:

$u = x \; / ^d$ $\qquad dv = \cos x \cdot dx \; / ^\int$

$du = dx$ $\qquad v = \int \cos x \cdot dx$

$V = \sin x$

$$= -x^2 \cos x + 2\left(x \cdot \sin x - \int \sin x \cdot dx\right)$$

$$= -x^2 \cos x + 2(x \sin x - (-\cos x))$$

$$\underline{\underline{= -x^2 \cos x + 2x \sin x + 2\cos x + C}}$$

5. $y'' + 2y' + y = 0$ $\qquad$ POČETNI UVJETI $\quad y(0) = 1 \quad y'(0) = 0$

$r^2 + 2r + 1 = 0$

$a = 1 \quad b = 2 \quad c = 1$

$r_{1,2} = \dfrac{-2 \pm \sqrt{4-4}}{2}$

$r_{1,2} = \dfrac{-2 \pm \sqrt{0}}{2}$

$r_1 = r_2 = r = \frac{-2}{2} = -1$

$y_H = C_1 e^{rx} + x C_2 e^{rx}$

$y_x = C_1 e^{-x} + x C_2 e^{-x}$

Y JE POLINOM NULTOG STUPNJA

$Y = a_0$

$Y' = 0$

$Y'' = 0$

$0 + 2 \cdot 0 + a_0 = 0$

$a_0 = 0$

$Y = 0$

$y = y_H + Y$

$\underline{\underline{y = C_1 e^{-x} + x C_2 e^{-x}}}$

$y(0) = 1 \Rightarrow x = 0, \; y = 1$

$y'(0) = 0 \Rightarrow x = 0, \; y' = 0$

$1 = C_1 \cdot e^{-0} + 0 \cdot C_2 \cdot e^{-0}$

$1 = C_1 \cdot 1 + 0$

$\boxed{C_1 = 1}$

$y' = C_1 \cdot e^{-x} \cdot (-1) + C_2 \cdot x \cdot e^{-x}$

$y' = -C_1 e^{-x} + C_2 \cdot (x' \cdot e^{-x} + x \cdot (e^{-x})')$

$y' = -C_1 e^{-x} + C_2 \cdot e^{-x} - x \cdot e^{-x}$

$0 = -C_1 \cdot e^{-0} + C_2 \cdot e^{-0} - 0 \cdot e^{-0}$

$0 = -C_1 \cdot 1 + C_2 \cdot 1$

$-C_1 + C_2 = 0$

$-1 + C_2 = 0$

$\boxed{C_2 = 1}$

$y = 1 \cdot e^{-x} + x \cdot 1 \cdot e^{-x}$

$y = e^{-x} + x \cdot e^{-x}$

$x' \cdot e^{-x} + x \cdot (e^{-x})'$

$1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1)$

$e^{-x} - x \cdot e^{-x}$

$y' = \cancel{-e^{-x}} \cancel{+e^{-x}} - xe^{-x}$

$y = e^{-x} + x \cdot e^{-x}$

$y' = e^{-x} \cdot (-1) + \left(\left(x' \cdot e^{-x} + x \cdot (e^{-x})'\right)\right)$

$y' = -e^{-x} + \left(1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1)\right)$

$y' = -e^{-x} + \left(e^{-x} - x \cdot e^{-x}\right)$

$y' = \cancel{-e^{-x}} + \cancel{e^{-x}} - x \cdot e^{-x}$

$y' = -x \cdot e^{-x}$

$y'' = -e^{-x} - x \cdot e^{-x}$

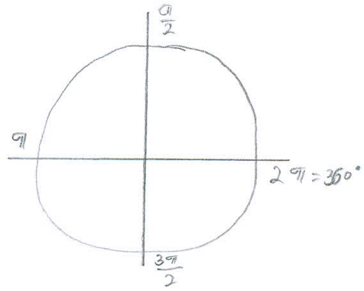
$-e^{-x} - xe^{-x} + 2(-x \cdot e^{-x}) + e^{-x} + x \cdot e^{-x} = 0$

$\cancel{-e^{-x}} - \cancel{xe^{-x}} - 2xe^{-x} + \cancel{e^{-x}} + \cancel{xe^{-x}} = 0$

$(-x \cdot e^{-x})' = -x' \cdot e^{-x} + x \cdot (e^{-x})'$

$= -1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1)$

$= -e^{-x} - x \cdot e^{-x}$



6.) $f(x) = 2x \cos x$ $\quad x_0 = \frac{\pi}{2} = \frac{3.14}{2} = 1.57$ rad

$\cos \frac{\pi}{2} = 0$

$\sin \frac{\pi}{2} = 1$

$f(x) = 2x \cos x \quad f\left(\frac{\pi}{2}\right) = 2 \cdot \frac{\pi}{2} \cdot \cos \frac{\pi}{2} = 3.14 \cdot 1 = 3.14 \text{ rad} = 180°$

$f'(x) = (2x)' \cdot \cos x + 2x \cdot (\cos x)' = 2 \cdot \cos x - 2x \sin x \quad f'\left(\frac{\pi}{2}\right) = 2 \cdot \cos \frac{\pi}{2} - 2 \cdot \frac{\pi}{2} \cdot \sin \frac{\pi}{2} =$ ✓

$= 2 - 0.09 = 1.91$ rad ✗

$f''(x) = -2 \sin x - 2\left(x' \cdot \sin x + x \cdot (\sin x)'\right) = -2 \sin x - 2(\sin x + x \cos x)$

$= -2 \sin x - 2 \sin x + x \cos x = -4 \sin x + x \cos x \Rightarrow f''\left(\frac{\pi}{2}\right) = -4 \sin \frac{\pi}{2} + \frac{\pi}{2} \cdot \cos \frac{\pi}{2}$

$= -4 \cdot 0.03 + 1.57 \cdot 1$

$= 1.57 - 0.12$

$= 1.45$ rad

$f'''(x) = -4 \cos x + \left(x' \cos x + x \cdot (\cos x)'\right)$

$= -4 \cos x + \cos x - x \sin x$

$= -3 \cos x - x \sin x \Rightarrow f'''\left(\frac{\pi}{2}\right) = -3 \cos \frac{\pi}{2} - \frac{\pi}{2} \cdot \sin \frac{\pi}{2}$

$= -3 \cdot 1 - 0.04$

$= -3.04$ rad

TAYLOROVA FORMULA:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$f(x) = 3.14 + \frac{1.91}{1}(x - 1.57) + \frac{1.45}{1\cdot 2}(x - 1.57)^2 + \frac{-3.04}{1\cdot 2\cdot 3}(x-1.57)^3$$

Popuniti odmah!

Ime i prezime: LUKA KURILIĆ **Broj indeksa:** 58076

Datum: Vrijeme: od do .

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2 $\int x^2 \sin(x)\,dx$

$u = x^2 /'$ $\quad dv = \sin x\,dx /\int$

$du = 2x\,dx$ $\quad v = \int \sin x\,dx$

$v = -\cos x + c$

$uv - \int v\,du$

$x^2 \cdot (-\cos x) - \int(-\cos x)\cdot 2x\,dx$ ✓

$x^2(-\cos x) - [-\int \cos x]2x\,dx$

$x^2(-\cos x) - [-\sin x]2x + c$ ✗

$x^2(-\cos x) + \sin x\, 2x + c$

I_1 Nova I_2 parcijalna

$I_1 = \int x^2(-\cos x)\,dx$

$u = x^2$ $\quad dv = -\cos x\,dx /\int$

$du = 2x\,dx$ $\quad v = -\int \cos x\,dx$

$v = -\sin x + c$

$uv - \int v\,du$

$x^2 \cdot (-\sin x) - \int(-\sin x)\,2x\,dx$

$x^2(-\sin x) - [-\int \sin x]2x\,dx$

$x^2(-\sin x) - [\cos x]2x + c$

$x^2(-\sin x) - \cos x\, 2x + C$

$I_2 = \int \sin x\, 2x\,dx$

$u = 2x /'$ $\quad dv = \sin x\,dx /\int$

$du = 2\,dx$ $\quad v = -\cos x + c$

$uv - \int v\,du$

$2x \cdot (-\cos x) - \int(-\cos x)\,2\,dx$

$2x(-\cos x) - [-\int \cos x]\,2\,dx$

$2x(-\cos x) - [-\sin x]2 + c$

$2x(-\cos x) + \sin x \cdot 2 + c$

$I = I_1 + I_2$

$I = x^2(-\sin x) - \cos x\, 2x + 2x(-\cos x) + \sin x \cdot 2 + c$

$\int \cos x\, 2x \neq \left[\int \cos x\right]\cdot 2x$

✗

$$\int \frac{x^2+2x+2}{x^2+x-2}\,dx$$

$$= \int \left(1+\frac{x+4}{x^2+x-2}\right) dx$$

$$= \underbrace{\int dx}_{I_1} + \underbrace{\int \frac{x+4}{(x-1)(x+2)}\,dx}_{I_2}$$

$$(x-1)(x+2) = x^2+2x-x-2$$
$$= x^2+x-2$$
$$(x-1)(x+2) = x^2+x-2$$

$$x^2+2x+2 : x^2+x-2 = 1+\frac{x+4}{x^2+x-2}$$
$$-(x^2+x-2)$$
$$x+4$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x_{1,2} = \frac{-1 \pm \sqrt{1-4\cdot 1\cdot(-2)}}{2}$$
$$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$$
$$x_{1,2} = \frac{-1 \pm 3}{2}$$

$$I_1 = \int dx$$
$$I_1 = x$$

$$I_2 = \int \frac{x+4}{(x-1)(x+2)}\,dx$$

$$\frac{x+4}{(x-1)(x+2)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x+2)} \quad / (x-1)(x+2)$$

$$x-1 = t \quad / '$$
$$dx = dt$$

$$x+2 = z \quad / '$$
$$dx = dz$$

$$x+4 \equiv A(x+2) + Bx(x-1) + C(x-1)$$
$$x+4 \equiv \underline{Ax}+2A+\underline{Bx^2}-\underline{Bx}+\underline{Cx}-C$$
$$x+4 \equiv Bx^2 = 0 \Rightarrow B=0$$
$$x+4 \equiv x(A-B+C) = 1 \Rightarrow A-B+C = 1$$
$$x+4 \equiv 2A-C = 4$$
$$\boxed{2A = 4C} \times$$
$$A = 2C \quad / \cdot \tfrac{1}{4}$$
$$1 = 2C$$
$$-2C = -1$$
$$2C = 1$$
$$\boxed{C = \tfrac{1}{2}}$$

$$A-B+C = 1$$
$$A-0+\tfrac{1}{2} = 1$$
$$A = 1-\tfrac{1}{2}$$
$$\boxed{A = \tfrac{1}{2}}$$

$$\frac{x+4}{(x-1)(x+2)} = \frac{\frac{1}{2}}{(x-1)} + \frac{0+\frac{1}{2}}{(x+2)}$$

$$= \frac{1}{2}\int \frac{dx}{(x-1)} + \frac{1}{2}\int \frac{dx}{(x+2)}$$
$$= \frac{1}{2}\int \frac{dt}{t} + \frac{1}{2}\int \frac{dx}{(x+2)}$$
$$= \frac{1}{2}\ln|x-1| + \frac{1}{2}\int \frac{dz}{z}$$
$$= \frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+2|$$

④ $f(x,y) = y^3 - 3xy + x^2$

$z_x = -3y + 2x \Rightarrow -3y + 2x = 0$

$z_y = 3y^2 - 3x$

$-3y + 2x = 0$

$2x = 3y$

$\boxed{x = \frac{3}{2}y}$

$3\cdot\left(\frac{3}{2}y\right)^2 - 3\cdot\frac{3}{2}y = 0$

$3\cdot\frac{9}{4}y^2 - \frac{9}{2}y = 0$

$\frac{27}{4}y^2 - \frac{9}{2}y = 0$

$y\left(\frac{27}{4}y - \frac{9}{2}\right) = 0$

$y = 0$

$\frac{27}{4}y - \frac{9}{2} = 0$

$\frac{27}{4}y = \frac{9}{2} \quad / \cdot \frac{1}{4}$

$27y = \frac{9}{8}\cdot\frac{1}{27}$

$y = \frac{9}{216}$

$-3y + 2x = 0$

$-3y = -2x$

$3y = 2x$

$y = \frac{2}{3}x$

$\Downarrow$

$y = \frac{2}{3}\cdot\frac{1}{4}$

$y = \frac{2}{12}$

$\boxed{y = \frac{1}{6}}$

$3y^2 - 3x = 0$

$3\left(\frac{2}{3}x\right)^2 - 3x = 0$

$3\,\frac{4}{9}x^2 - 3x = 0$

$\frac{12}{9}x^2 - 3x = 0$

$\frac{4}{3}x^2 - 3x = 0$

$\frac{4}{3}x^2 = 3x \quad / : x$

$\frac{4}{3}x = 3 \quad / \cdot \frac{1}{3}$

$4x = 1$

$\boxed{x = \frac{1}{4}}$

$z(x,y) = y^3 - 3xy + x^2$

$$\boxed{\begin{aligned} z_x &= -3y + 2x \\ z_y &= 3y^2 - 3x \end{aligned}} \Rightarrow \begin{aligned} -3y + 2x &= 0 \\ -3y &= -2x \\ 2x &= 3y \\ \boxed{x = \tfrac{3}{2}y} \end{aligned}$$

$3y^2 - 3x = 0$

$3y^2 - 3\,\frac{3}{2}y = 0$

$3y^2 - \frac{9}{2}y = 0$

$y\left(3y - \frac{9}{2}\right) = 0$

$\swarrow$ $\searrow$

$Y = 0$

$T_1(0,0)$ ✓

$3y - \frac{9}{2} = 0$

$3y = \frac{9}{2} \Big/ \cdot \frac{1}{3}$

$y = \cancel{\frac{9}{6}}\frac{3}{2}$ ✓

$\boxed{y = \frac{3}{2}}$ ✓

$\Downarrow$

$x = \frac{3}{2} \cdot \frac{3}{2}$

$\boxed{x = \frac{9}{4}}$ ✓

$T_2\left(\frac{9}{4}, \frac{3}{2}\right)$ ✓

ZA T1

$Y^3 - 3xY + x^2 = 0$

$0^3 - 3 \cdot 0 \cdot 0 + 0^2$

$0 - 0 + 0 = 0$ ✓

$$\boxed{\begin{aligned} z_{xx} &= 2 \\ z_{xy} &= -3 \\ z_{yy} &= 6y \end{aligned}}$$ ✓

ZA T1 (0,0)

$\Delta = A \cdot C - B^2$

$\Delta = 2 \cdot (-3) - (6 \cdot 0)^2$

$2 \cdot 0 - (-3)^2$

$\Delta = -6 - (0)^2$

$\boxed{\Delta = -6}$ ✗

ZA T2

$\left(\frac{3}{2}\right)^3 - 3 \cdot \frac{9}{4} \cdot \frac{3}{2} + \left(\frac{9}{4}\right)^2$

$\cancel{\frac{27}{8} - \frac{27}{4} \cdot \frac{3}{2} + \frac{18}{16}}$

$\cancel{\frac{27}{8} - \frac{81}{8} + \frac{18}{16}}$

$\cancel{-5{,}62}$

$= -1{,}68$ ✓

ZA $T_2\left(\frac{9}{4}, \frac{3}{2}\right)$

$\Delta = A \cdot C - B^2$

$\Delta = 2 \cdot 6Y - (-3)^2$

$\cancel{\Delta = 2 \cdot 6 \cdot \frac{3}{2} - 8} =$

$\Delta = 2 \cdot 6 \cdot \frac{3}{2} - 9 = \boxed{9}$ ✓

DA LI SU T_1 ILI T_2 EKSTREMI I KAKVI?

5 $y'' + 2y' + y = 0$ $\quad y(0) = 1 \quad y'(0) = 0$

$r^2 + 2r + y = 0$

$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$r_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 1}}{2}$ $\quad \boxed{b \neq r_1 = r_2}$

$r_{1,2} = \frac{-2 \pm \sqrt{0}}{2}$

$\boxed{r_{1,2} = \frac{-2}{2} = -1}$

$Y_0 = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

$\boxed{Y_0 = C_1 e^{-x} + C_2 e^{-x}}$ X $C_1 e^{-x} + C_2 x e^{x}$

$Y'_0 = C_1 e^0 + C_2 e^0$

$Y'_0 = C_1 + C_2$

$Y_0 = C_1 e$

$P(b) = b - 3b + \cancel{0}$

$P(0) = 0 - 3 \cdot 0 + 0$

$P(0) = 0$

$\eta = \frac{k e^{x}}{?}$ X

⑥ $f(x) = 2x\cos x \qquad x_0 = \frac{\pi}{2}$

① $f(x) = 2x\cos x \qquad f(0) = \cancel{2}\cdot\frac{\pi}{\cancel{2}}\cdot\cos\pi = 0$

② $f'(x) = 2\cdot\cos x + 2x\cdot(-\sin x)$
$= 2\cos x + 2x(-\sin x)$

$f'(0) = \cancel{2}\cdot\cos\frac{\pi}{\cancel{2}} + \cancel{2}\cdot\frac{\pi}{2}\cdot(-\sin\frac{\pi}{2})$
$= \cos\pi + \pi(-\sin\frac{\pi}{2})$
$= 1 + \frac{3}{2}\pi(-\sin)$
$= 1 - \cancel{0{,}082}\ 0$
$= \cancel{0{,}92}\ 1$

③ $f''(x) = -2\sin x\ \cancel{-2x} + 2\cdot(-\sin x) + 2x\cdot(-\cos x)$
$\cancel{-2x\cos x}$
$= -2\sin x - 2\sin x - 2x\cos x$
$= -4\sin x - 2\cos x$ ✗
$-2x\cos x$

$f''(0) = -4\cdot\sin\frac{\pi}{2} - 2\cos\frac{\pi}{2}$
$= -4\cdot0 - 2\cdot(-1)$
$= 0 + 2$
$= 2$

④ $f'''(x) = -4\cos x + 2\sin x$ ✗

$f'''(0) = -4\cdot\cos\frac{\pi}{2} + 2\cdot\sin\frac{\pi}{2}$
$= -4\cdot(-1) + 2\cdot(-1)$
$= 4 - 2$
$= 2$

$f(x,y) = f(x)\,\frac{}{1!}$?

Popuniti odmah!
Ime i prezime: DINO KURIĆ **Broj indeksa:** 5619-2008
Datum: Vrijeme: od do

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1) $\int x^2 \cdot \sin x \, dx = u \cdot v - \int v\,du$

$x^2 = u$
$2x = du$

$(\sin x)\,dx = v$
$-\cos x = dv$

2) $f(x,y) = y^3 - 3xy + x^2$

$f_x = 3y + 2x$

$f_y = 3y^2 - 3x$

$3y^2 + 3y =$

$$y'' + 2y' + y = 0$$

$$r^2 + 2r + 1 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = -b \pm \frac{\sqrt{(2)^2 - 4 \cdot 1 \cdot c}}{2a}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2}$$

$$r_{1,2} = -2$$

Popuniti odmah!

IME I PREZIME: **BROJ INDEKSA:**

DATUM: VRIJEME: OD DO .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Izračunati $\int \frac{x^2+2x+2}{x^2+x-2}\,\mathrm{d}x.$ ~~15~~
2. Izračunati $\int x^2 \sin(x)\,\mathrm{d}x.$ 15
3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala:
$$\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$$ 15
4. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$. 20
5. Riješiti zadanu diferencijalnu jednadžbu uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. Uvrstiti rješenje u jednadžbu i provjeriti zadovoljenje jednakosti. ~~20~~
$$y'' + 2y' + y = 0$$
6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = 2x\cos x$ oko točke $x_0 = \frac{\pi}{2}$. ~~15~~

(5.) $y''+2y'+y=0$

$r^2+2r+1=0$

a b c

$x_{1,2}=\frac{-2\pm\sqrt{b^2-2ac}}{2c}=\frac{-2\pm\sqrt{2^2-2\cdot1\cdot1}}{2\cdot1}$

$=\frac{-2\pm\sqrt{4-2}}{2}$

$=\frac{-2\pm\sqrt{4}}{2}=\frac{-2\pm2}{2}$

$x_1=\frac{-2-2}{2}=\frac{-4}{2}=-2$

$x_2=\frac{-2+2}{2}=\frac{0}{2}$

$y=C_1x^{-2}+C_2x^{\frac{-2-2}{2}}$

(6.) $f(x)=2x\cos x$ $x_0=\frac{\pi}{2}$

$f(x_0)=2\frac{\pi}{2}\cos\frac{\pi}{2}$

$f'(x)=x\cos x+2x(-\sin x)$ $f(x_0)=\frac{\pi}{2}\cos\frac{\pi}{2}+2\frac{\pi}{2}(-\sin\frac{\pi}{2})=1{,}49$

$f''(x)=\cancel{\cos x}-\sin x+(-x\sin x)+(\cancel{-\cos x})$ $f(x_0)=\cos\frac{\pi}{2}-\sin\frac{\pi}{2}+(\frac{\pi}{2}\sin\frac{\pi}{2})+(-\cos\frac{\pi}{2})=0{,}95$

$f'''(x)=\cos x+(-\sin x)+\cos x$ $f(x_0)=\cos\frac{\pi}{2}+(-\sin\frac{\pi}{2})+\cos\frac{\pi}{2}=1{,}97$

$f^{IV}(x) = -\sin x + \cos x - \sin x$ $\quad f(x_0) = -\sin\frac{\pi}{2} + \cos\frac{\pi}{2} - \sin\frac{\pi}{2}$

$= 0{,}95$

I ONDA SE UVRSTI U ONU FORMULU

$\int x^2 \sin x\, dx = u \cdot v - \int v \cdot du$

$= x^2 \cdot (-\cos x) - \int -\cos x\, 2x\, dx$

$= -x^2\cos x - \int -2x\cos x\, dx$

$x^2 = u \,/ d$ $\quad \int v = \sin x \,/ \int$

$2x\,dy = du \,/ 2$ $\quad \int v\,dx = \int \sin x$

$2x\,dx = du$ $\quad v = -\cos x$

$= -x^2\cos x + 2\int x \cos x\, dx$

$x = u \,/ d$ $\quad v = \cos x \,/ \int$

$dx = du$ $\quad \int v\,dx = \int \cos x$

$v = -\sin x$

$= -x^2\cos x + 2\int x \cdot (-\sin x) - \int -\sin x\, dx$

$= -x^2\cos x + 2\int -x\sin x + \sin x$

$= -x^2\cos x + 2$

...

$f(x,y) = y^3 - 3xy + x^2$

$f_x = -3y + 2x$

$f_y = 3y^2 - 3x$

$-3y + 2x = 0 \;/ \cdot 3$

$3y^2 - 3x = 0 \;/ \cdot 2$

$-9y + \cancel{6x} = 0$

$6y^2 - \cancel{6x} = 0$ $\;/ +$

---

(1)

$$\int \frac{x^2+2x+2}{x^2+x-2}\, dx$$

$x^2 + 2 \cdot \frac{1}{2}x - \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - 2$

$$\int \frac{x^2+2x+2}{(x-\frac{1}{2})^2 \pm \frac{1}{4}} = \int \frac{x^2+2x+2}{(x-\frac{1}{2})^2 - (\frac{1}{2})^2} = \int x^2\, dx$$

$x^2 + 2x + 2 = t \;/ d$

$(2x+2)\,dx = dt \;/$

$dx = \frac{dt}{(2x+2)}$

A OVAKO NEŠTO

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + c \quad + \ln|2x + d + c$$

Popuniti odmah!

Ime i prezime: Igor Brajica **Broj indeksa:** 52803-2005

Datum: 21.02.2012 Vrijeme: od do

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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⑥ $f(x) = 2x \cdot \cos x \qquad x_0 = \frac{\pi}{2} \qquad = f(\frac{\pi}{2}) = 2 \cdot \frac{\pi}{2} \cdot \cos\frac{\pi}{2} =$

$f'(x) = 2\cdot\cos x + 2x \cdot (-\sin x)$ ✓

DALJE...

$\emptyset$

② $\int x^2 \sin x\,dx = \left| \begin{array}{ll} x^2 = u & \sin x\,dx = dv \\ 2x = du & -\cos x\,dx = v \end{array} \right. \qquad u\cdot v - \int v\,du$

$-x^2\cos x - 2\int x + \cos x\,dx$

$-2\int x\cos x\,dx$

$-x^2\cos x - 2 \cdot x^2 + \sin x\,dx$