

Popunite odmah!

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DO

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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Broj ↓
bodova

✓ 1. Izračunati $\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx$.

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✓ 2. Izračunati $\int x^2 \sin(x) dx$.

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3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala:

$$\int_{\pi}^{2\pi} \frac{\arctan x}{x} dx$$

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4. Istražite ekstreme funkcije $f(x, y) = y^3 - 3xy + x^2$.

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✓ 5. Riješiti zadanu diferencijalnu jednadžbu uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. Uvrstiti rješenje u jednadžbu i provjeriti zadovoljenje jednakosti.

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$$y'' + 2y' + y = 0$$

✓ 6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = 2x \cos x$ oko točke $x_0 = \frac{\pi}{2}$.

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① $\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx = \int \left(1 + \frac{x+4}{x^2+x-2} \right) dx$

$= \int 1 \cdot dx + \int \frac{x+4}{x^2+x-2} dx = x + I_1 = x + \frac{5}{3} \ln|x-1| - \frac{2}{3} \ln|x+2| + C$

$\frac{x^2+2x+2}{x^2+x-2} : (x^2+x-2) = 1 + \frac{x+4}{x^2+x-2}$

$\frac{x^2+x-2}{x^2+x-2} = 1$
 $\frac{x+4}{x^2+x-2}$

$I_1 = \int \frac{x+4}{(x-1)(x+2)} dx = \int \frac{A}{x-1} dx + \int \frac{B}{x+2} dx$

$\frac{x+4}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$x+4 = A(x+2) + B(x-1)$

$x+4 = Ax+2A+Bx-B$

$x+4 = (A+B)x + 2A-B$

$A+B=1 \quad | \Rightarrow \frac{5}{3} + B = 1$

$2A-B=4 \quad | \pm \quad B = 1 - \frac{5}{3}$

$3A = 5 \quad | :3 \quad B = -\frac{2}{3}$

$A = \frac{5}{3}$

x^2+x-2

$a=1 \quad b=1 \quad c=-2$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$

$x_1 = \frac{-1+3}{2} = 1$

$x_2 = \frac{-1-3}{2} = -2$

$\left. \begin{array}{l} x-1=t \\ dx=dt \end{array} \right\} \quad \left. \begin{array}{l} x+2=t \\ dx=dt \end{array} \right\}$

$I_1 = \int \frac{5}{x-1} dx + \int \frac{-2}{x+2} dx$

$= \frac{5}{3} \int \frac{dx}{x-1} - \frac{2}{3} \int \frac{dx}{x+2}$

$= \frac{5}{3} \int \frac{dt}{t} - \frac{2}{3} \int \frac{dt}{t}$

$= \frac{5}{3} \ln|t| - \frac{2}{3} \ln|t| = \frac{5}{3} \ln|x-1| - \frac{2}{3} \ln|x+2| + C$

$$(2) \int x^2 \sin(x) dx = u \cdot v - \int v \cdot du$$

$$u = x^2 / d \quad dv = \sin(x) \cdot dx / f$$

$$du = 2x \cdot dx \quad v = \int \sin x \cdot dx$$

$$v = -\cos x$$

$$\int x^2 \sin(x) dx = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x \cdot dx$$

$$= -x^2 \cos x + 2 \int \cos x \cdot x \cdot dx$$

NOVA PARCIJALNA:

$$u = x / d \quad dv = \cos x \cdot dx / f$$

$$du = dx \quad v = \int \cos x \cdot dx$$

$$v = \sin x$$

$$= -x^2 \cos x + 2(x \cdot \sin x - \int \sin x \cdot dx)$$

$$= -x^2 \cos x + 2(x \sin x - (-\cos x))$$

$$= \underline{\underline{-x^2 \cos x + 2x \sin x + 2 \cos x + C}} \quad \checkmark$$

$$(5) y'' + 2y' + y = 0$$

POČETNI UVJETI $y(0) = 1 \quad y'(0) = 0$

$$r^2 + 2r + 1 = 0$$

$$a=1 \quad b=2 \quad c=1$$

$$r_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2}$$

$$r_{1,2} = \frac{-2 \pm \sqrt{0}}{2}$$

$$r_1 = r_2 = r = \frac{-2}{2} = -1$$

$$y_H = C_1 e^{rx} + x C_2 e^{rx}$$

$$y_x = C_1 e^{-x} + x C_2 e^{-x} \quad \checkmark$$

Y JE POLINOM NULTOG STEPANJA

$$y = a_0$$

$$y' = 0$$

$$y'' = 0$$

$$0 + 2 \cdot 0 + a_0 = 0$$

$$a_0 = 0$$

$$y = 0$$

$$y = y_H + Y$$

$$\underline{\underline{y = C_1 e^{-x} + x C_2 e^{-x}}} \quad \checkmark$$

$$y(0) = 1 \Rightarrow x=0, y=1$$

$$y'(0) = 0 \Rightarrow x=0, y'=0$$

$$1 = C_1 \cdot e^{-0} + 0 \cdot C_2 \cdot e^{-0}$$

$$1 = C_1 \cdot 1 + 0$$

$$\boxed{C_1 = 1}$$

$$y' = C_1 \cdot e^{-x} \cdot (-1) + C_2 \cdot x \cdot e^{-x}$$

$$y' = -C_1 e^{-x} + C_2 \cdot (x' \cdot e^{-x} + x \cdot (e^{-x})')$$

$$y' = -C_1 e^{-x} + C_2 \cdot e^{-x} - x \cdot e^{-x}$$

$$0 = -C_1 \cdot e^{-0} + C_2 \cdot e^{-0} - 0 \cdot e^{-0}$$

$$0 = -C_1 \cdot 1 + C_2 \cdot 1$$

$$-C_1 + C_2 = 0$$

$$-1 + C_2 = 0$$

$$\boxed{C_2 = 1}$$

$$y = 1 \cdot e^{-x} + x \cdot 1 \cdot e^{-x}$$

$$y = e^{-x} + x \cdot e^{-x}$$

$$x' \cdot e^{-x} + x \cdot (e^{-x})'$$

$$1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1)$$

$$e^{-x} - x \cdot e^{-x}$$

$$y' = -e^{-x} + e^{-x} - x e^{-x}$$

$$y = e^{-x} + x \cdot e^{-x}$$

$$y' = e^{-x} \cdot (-1) + (x' \cdot e^{-x} + x \cdot e^{-x})'$$

$$y' = -e^{-x} + (1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1))$$

$$y' = -e^{-x} + (e^{-x} - x \cdot e^{-x})$$

$$y' = -e^{-x} + e^{-x} - x \cdot e^{-x}$$

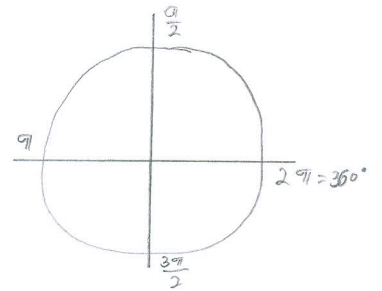
$$y' = -x \cdot e^{-x}$$

$$y'' = -e^{-x} - x \cdot e^{-x}$$

$$-e^{-x} - x e^{-x} + 2(-x \cdot e^{-x}) + e^{-x} + x \cdot e^{-x} = 0$$

$$-e^{-x} - x e^{-x} - 2x e^{-x} + e^{-x} + x e^{-x} = 0$$

$$\begin{aligned} (-x \cdot e^{-x})' &= -x' \cdot e^{-x} + x \cdot (e^{-x})' \\ &= -1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1) \\ &= -e^{-x} - x \cdot e^{-x} \end{aligned}$$



⑥. $f(x) = 2x \cos x$

$$x_0 = \frac{\pi}{2} = \frac{3.14}{2} = 1.57 \text{ rad}$$

$$f(x) = 2x \cos x \quad f\left(\frac{\pi}{2}\right) = 2 \cdot \frac{\pi}{2} \cdot \cos \frac{\pi}{2} = 3.14 \cdot 1 = 3.14 \text{ rad} = 180^\circ$$

$$f'(x) = (2x)' \cdot \cos x + 2x \cdot (\cos x)' = 2 \cdot \cos x - 2x \sin x \quad f'\left(\frac{\pi}{2}\right) = 2 \cdot \cos \frac{\pi}{2} - 2 \cdot \frac{\pi}{2} \cdot \sin \frac{\pi}{2} = \checkmark$$

$$= 2 - 0.09 = 1.91 \text{ rad} \quad \times$$

$$f''(x) = -2 \sin x - 2(x' \cdot \sin x + x \cdot (\sin x)') = -2 \sin x - 2(\sin x + x \cos x)$$

$$= -2 \sin x - 2 \sin x + x \cos x = -\{ \sin x + x \cos x \} \Rightarrow f''\left(\frac{\pi}{2}\right) = -\left\{ \sin \frac{\pi}{2} + \frac{\pi}{2} \cdot \cos \frac{\pi}{2} \right\}$$

$$= -\{ 0.03 + 1.57 \cdot 1 \}$$

$$= 1.57 - 0.12$$

$$= 1.45 \text{ rad}$$

$$f'''(x) = -4 \cos x + (x' \cdot \cos x + x \cdot (\cos x)')$$

$$= -4 \cos x + \cos x - x \sin x$$

$$= -3 \cos x - x \sin x \Rightarrow f'''\left(\frac{\pi}{2}\right) = -3 \cos \frac{\pi}{2} - \frac{\pi}{2} \cdot \sin \frac{\pi}{2}$$

$$= -3 \cdot 1 - 0.09$$

$$= -3.09 \text{ rad}$$

$$\begin{aligned} \cos \frac{\pi}{2} &= 0 \\ \sin \frac{\pi}{2} &= 1 \end{aligned}$$

