

Pečunite odmah!

IME I PREZIME:

LUKA BORZIC

BROJ INDEKSA: 17-2-0016-2010

DATUM: 21.2.2012. VRIJEME: OD

DO

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

5
Broj ↓
bodova

15+5

- 1. Izračunati inverz dane matrice (ako postoji) i provjeriti matričnim množenjem da je inverz dobro izračunat.

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

- 2. Ako su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - 2z + 2 = 0$, izracunati: $\overline{\left(\frac{z_1 - z_2}{z_2 - 2}\right)}$ i $Re\left(\overline{\left(\frac{z_2}{z_1}\right)}\right)$.

10+10

- 3. Zadana je funkcija $f(x) = e^{-x^2}$. Odrediti domenu, prvu derivaciju i sve asimptote funkcije.

5+5+5+5

- 4. Ispitati periodičnost, (ne)parnost i drugu derivaciju funkcije $g(x) = \arccos(3x)$.

5+5+10

- 5. Na temelju ispitivanja toka napraviti skicu grafa funkcije $h(x) = x - \frac{1}{x+1}$.

20

1.

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 \end{array} \right] \xrightarrow{(-1)} \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 \end{array} \right] \xrightarrow{+}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} :2 \\ :2 \end{array} \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & -1/2 & 0 & 1/2 & 0 \end{array} \right] \xrightarrow{+(-1)} \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} :1/2 \\ :1/2 \end{array} \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1/2 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & -1/2 & 0 & 1/2 & 0 \end{array} \right]$$

$$2. z^2 - 2z + 2 = 0$$

$$\overline{\left(\frac{z_1 - z_2}{z_2 - 2} \right)} \neq \operatorname{Re} \left(\overline{\left(\frac{z_2}{z_1} \right)} \right)$$

$$3. f(x) = e^{-x^2}$$

1) DOMENA $D_f = \mathbb{R}$ ✓

2) 1. DERIVACIJA

$$f(x) = e^{-x^2}$$

$$f'(x) = -2x e^{-x^2}$$
 ✓

3) VERTIKALNE ASIMPTOTE nema jer nema
kružimaka ✓

$$K_1 = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \frac{e^{-x^2}}{x} = \frac{0}{\infty} = 0$$

NEMA
KOSIH

$$K_2 = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \frac{e^{-x^2}}{x} = \frac{0}{-\infty} = 0$$

ASIMPTOTA

$$l = \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x-x} = \lim_{x \rightarrow \pm\infty} f(x) =$$

$$\lim_{x \rightarrow \pm\infty} e^{-x^2} = e^{-\infty} = 0$$
 ✓

HORIZONTALNA
ASIMPTOTA

$y = 0$ // ✓

$$4. f(x) = \arccos(3x)$$

funkcija nije periodična, nije ni parna
niti neparna ✓

$$f(x) = \arccos(3x)$$

$$f'(x) = -\frac{1}{\sqrt{1-(3x)^2}} \cdot 3 = \frac{-3}{\sqrt{1-9x^2}} \quad \checkmark$$

$$f''(x) = \frac{(-3)'(\sqrt{1-9x^2}) - (-3)(\sqrt{1-9x^2})'}{(\sqrt{1-9x^2})^2}$$

$$f''(x) = \frac{3 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1-9x^2}} \cdot (-18x)}{1-9x^2}$$

$$f''(x) = \frac{-27x}{(\sqrt{1-9x^2})^3} \quad \checkmark$$

$$5. f(x) = x - \frac{1}{x+1} \quad x+1 \neq 0$$

$$x \neq -1$$

$$\lim_{x \rightarrow -1^-} \frac{x^2+x-1}{x+1} = \frac{-1}{0^-} = +\infty \quad D_f = \mathbb{R} \setminus \{-1\}$$

$$\lim_{x \rightarrow -1^+} \frac{x^2+x-1}{x+1} = \frac{-1}{0^+} = -\infty$$

$$x = -1$$

↓
VERTIKALNA
ASIMPTOTA

$$K \Rightarrow \lim_{x \rightarrow \pm\infty} \frac{x^2+x-1 \overset{!}{:} x^2}{x^2+x \overset{!}{:} x^2} = 1$$

$$l = \lim_{x \rightarrow \infty} x - \frac{1}{x+1} \approx x$$

$$y = x$$

KOJA
ASIMPTOTA

$$\lim_{x \rightarrow \pm\infty} \frac{-1}{x+1} = 0$$

$$f'(x) = \frac{x^2+x-1}{x+1} = \frac{(2x+1)(x+1) - (x^2+x-1) \cdot 1}{(x+1)^2}$$

$$= \frac{2x^2 + 2x + x + 1 - x^2 - x + 1}{(x+1)^2} =$$

$$f'(x) = \frac{x^2 + 2x + 2}{(x+1)^2} = x^2 + 2x - 2 = 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4-8}}{2}$$

↓
NEMA
MINIMUMA
MAKSIMUMA

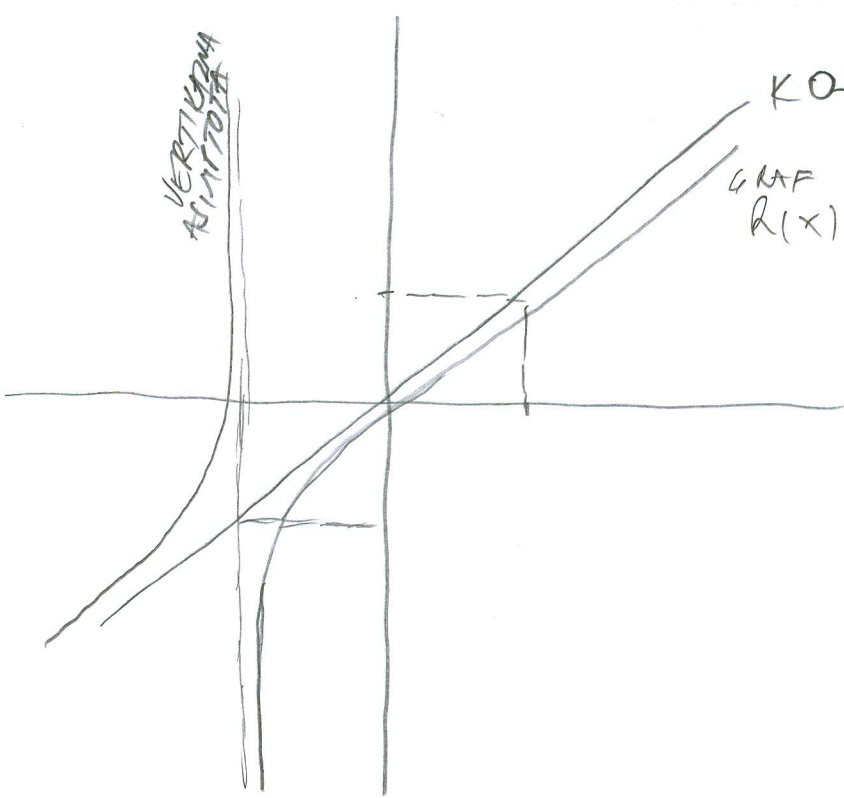
	$-\infty$	-1	$+\infty$
$f'(x)$	+	-	+
$f(x)$	↗	↘	↗

NEMA
EKSTREMA

VERTIKALNA
ASIMPTOTA

KOJA ASIMPTOTA

GRAF
 $R(x) = x - \frac{1}{x+1}$



Popunite odmah!

IME I PREZIME: MATEO MAVAR

BROJ INDEKSA: 17-2-0087-2011

DATUM: 21.2.2012. VRIJEME: OD DO

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

5
Broj ↓
bodova

- 1. Izračunati inverz dane matrice (ako postoji) i provjeriti matričnim množenjem da je inverz dobro izračunat.

15+5

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

- 2. Ako su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - 2z + 2 = 0$, izracunati: $\overline{\left(\frac{z_1 - z_2}{z_2 - 2}\right)}$ i $Re\left(\overline{\left(\frac{z_2}{z_1}\right)}\right)$.

10+10

- 3. Zadana je funkcija $f(x) = e^{-x^2}$. Odrediti domenu, prvu derivaciju i sve asimptote funkcije.

5+5+5+5

- 4. Ispitati periodičnost, (ne)parnost i drugu derivaciju funkcije $g(x) = \arccos(3x)$.

5+5+10

- 5. Na temelju ispitivanja toka napraviti skicu grafa funkcije $h(x) = x - \frac{1}{x+1}$.

20

5. $h(x) = x - \frac{1}{x+1}$

$x+1 \neq 0$

$x \neq -1$

$x \in \mathbb{R} \setminus \{-1\}$

GRAF?

ASIMPTOTE

V. A

$$\lim_{x \rightarrow -1} x - \frac{1}{x+1} = -1 - \frac{1}{-1+1} = \frac{-1-1}{0} = \infty$$

$$\lim_{x \rightarrow -1^+} x - \frac{1}{x+1} = -1^+ - \frac{1}{-1^++1} = \pm\infty$$

$$\lim_{x \rightarrow -1^-} x - \frac{1}{x+1} = -1^- - \frac{1}{-1^-+1} = -\infty$$

H. A.

$$\lim_{x \rightarrow \infty} x - \frac{1}{x+1} = \infty - \frac{1}{\infty} = \infty - 0 = \infty$$

K. A

$y = kx + l$

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x - \frac{1}{x+1}}{x} = \lim_{x \rightarrow \infty} \frac{x+1 - \frac{1}{x+1}}{x+1} = \frac{x}{x+1} = \frac{x/x}{x/x+1/x} = \frac{1}{1+0} = 1$$

$$= \frac{1}{1+0} = 1$$

$$l = \lim_{x \rightarrow \infty} f(x) - kx$$

$$= \lim_{x \rightarrow \infty} x - \frac{1}{x+1} - 1x = \infty$$

$$\textcircled{1} \left[\begin{array}{cccc|cccc} 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{array} \xrightarrow{T_2} \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 \end{array} \right] \begin{array}{l} (-1) \\ \uparrow \\ \uparrow \\ \uparrow \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 \end{array} \right] \begin{array}{l} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{array} \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 & -1 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \end{array} \right] \begin{array}{l} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{array} \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \end{array} \right] \begin{array}{l} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{array}$$

3. $f(x) = e^{-x^2}$

a) DOMENA $Df = \mathbb{R}$ ✓

b) $f(x) = e^{-x^2}$

$$f'(x) = (e^{-x^2})'$$

$$= -2x e^{-x^2} \quad \checkmark$$

c) NEMA VERTIKALNE ASIMPTOTE ✓ (PREKIDA DOMENU)



$$H.A. \quad L_1 = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \frac{e^{-x^2}}{x} = \frac{0}{\infty} = 0$$

$$L_2 = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \frac{e^{-x^2}}{x} = \frac{0}{-\infty} = 0$$

HEMA KOSE ASIMPTOTE

$$l = \lim_{x \rightarrow \pm\infty} f(x) - lx = \lim_{x \rightarrow \pm\infty} f(x) =$$

$$\lim_{x \rightarrow \pm\infty} e^{-x^2} = e^{-\infty} = 0$$

$$H.A. \quad y = 0 \quad \checkmark$$

4) $\arccos(3x)$

funkcija nije periodična, nije ni parna ni neparna ✓

$$g(x) = \arccos(3x)$$

$$g'(x) = -\frac{1}{\sqrt{1-(3x)^2}} \cdot 3 = -\frac{3}{\sqrt{1-9x^2}}$$

$$g''(x) = \frac{(-3)' \cdot (\sqrt{1-9x^2}) - (-3) \cdot (\sqrt{1-9x^2})'}{(\sqrt{1-9x^2})^2}$$

$$g''(x) = \frac{3 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1-9x^2}} \cdot (-18x)}{1-9x^2}$$

$$g''(x) = \frac{-27x}{(1-9x^2)^{3/2}}$$

