puniti odmah!

Ime i prezime: LOVRE KEREŠ **Broj indeksa:** 57933

25

Datum: 21.2.2012. Vrijeme: od do .

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. 8 Broj ↓ bodova

1. Na temelju ispitivanja toka funkcije nacrtati skicu grafa za $f(x) = \dfrac{x^2-2}{x^2+2}$. ~~20~~
2. Ispitati domenu, (ne)parnost i pronaći lokalne minimume i maksimume funkcije $g(x) = \sqrt{7-x^2}$. (skica grafa funkcije se ne boduje, ali ako je nacrtate odmah će vam sve biti jasno.) ~~5+5+5+5~~
3. Riješiti među kompleksnim brojevima $\left|\dfrac{z}{2}\right| = z + 7i$. Možete koristiti formulu za nultočke kvadratne funkcije. ~~20~~
4. Odrediti sva koja postoje riješenja sustava linearnih jednadžbi i provjeriti: 15+5

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 5 \\ 2x_1 + 2x_2 + 2x_3 &= 6 \\ -x_1 - 2x_2 - 4x_3 &= -7 \\ -4x_1 - x_2 - 9x_3 &= -14 \end{aligned}$$

5. Ispitati konvergenciju reda $\sum n(\sqrt{n} - \sqrt{n-1})$ ~~20~~

2) $g(x) = \sqrt{7-x^2}$

DOMENA

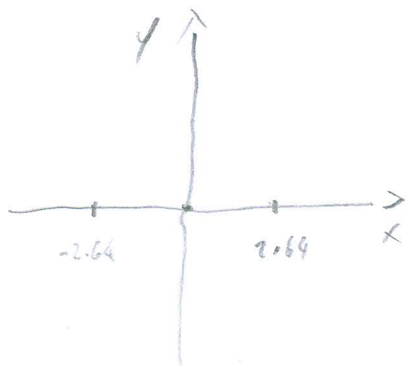
$D_g = \mathbb{R}\ \langle -\infty, +\infty \rangle$

Parnost i neparnost

$g(-x) = g(-x)$

$g(-x) = \sqrt{7-(-x)^2} = \sqrt{7-x^2} = g(x)$

Funkcija je parna



K.A.

$k_1 = \lim\limits_{x\to-\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{\sqrt{7+x^2}\ /:x}{x\ /:x} = \dfrac{1}{1} = 1$

$l_1 = \lim\limits_{x\to\infty} f(x) - k_1 x = \lim\limits_{x\to-\infty} \sqrt{7+x^2} - x =$

$= \lim\limits_{x\to\infty} \sqrt{7+x^2} + x \cdot \dfrac{\sqrt{7+x^2}-x}{\sqrt{7+x^2}-x} = \dfrac{7+x^2-x^2}{\sqrt{7+x^2}-x} = \dfrac{7\ /:x}{\sqrt{7+x^2}-x\ /:x} = \dfrac{0}{0}$

$\lim\limits_{x\to\infty} \sqrt{7-x^2} = -\infty$

$\lim\limits_{x\to-\infty} \sqrt{7-x^2} = |-x \to x| \lim\limits_{x\to\infty} \sqrt{7+x^2} = +\infty$

K.A.

$k_1 = \lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{\sqrt{7-x^2}\ /:x}{x\ /:x} = \dfrac{\infty}{\infty} = \dfrac{\sqrt{\frac{7}{x^2}-\frac{x^2}{x^2}}}{\frac{x}{x}} = \dfrac{1}{1}$

$l_1 = \lim\limits_{x\to\infty} f(x) - k_1 x = \lim\limits_{x\to\infty} \sqrt{7-x^2} - x = 1$

$\lim\limits_{x\to\infty} \sqrt{7-x^2} - x \cdot \dfrac{\sqrt{7-x^2}+x}{\sqrt{7-x^2}+x} = \dfrac{7-x^2-x^2\ /:x}{\sqrt{7-x^2}+x\ /:x} = \dfrac{0}{0}$

a = 1
b = 0
c = 7

$x_{1,2} = \dfrac{-b \pm \sqrt{b^2+4ac}}{2a} = \dfrac{0 \pm \sqrt{0+4\cdot 1\cdot 7}}{2} = \dfrac{\pm\sqrt{28}}{2}$

$x_1 = \dfrac{5.29}{2} = 2.64$

$x_2 = \dfrac{-5.29}{2} = -2.64$

3.

$$\left|\frac{z}{2}\right| = x+7i = \frac{x^2+iy^2}{2} = x+7i \quad /\cdot 2$$

$$x^2+yi^2 = 2x+14i$$

$$x^2-2x = -yi^2+14i$$

Realni brojevi: x^2-2x

Imaginarni brojevi: yi^2+14i

4.

$$x_1+2x_2+x_3+x_4 = 5$$
$$2x_1+2x_2+2x_3 = 6$$
$$-x_1-2x_2-4x_3 = -7$$
$$-4x_1-x_2-9x_3 = -14$$

PROVJERA

$$\begin{bmatrix}1&2&1&1\\2&2&2&0\\-1&-2&-4&0\\-4&-1&-9&0\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \begin{bmatrix}1+2+1+1\\2+2+2+0\\-1-2-4+0\\-4-1-9+0\end{bmatrix} = \begin{bmatrix}5\\6\\-7\\-14\end{bmatrix}$$

$$\begin{bmatrix}1&2&1&1\\2&2&2&0\\-1&-2&-4&0\\-4&-1&-9&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}5\\6\\-7\\-14\end{bmatrix}$$

$$\frac{1}{3}-1 = \frac{1-3}{3} = -\frac{2}{3} \qquad \frac{1}{3}+\frac{2}{3} = \frac{3}{3} = 1$$
$$-\frac{2}{3}+1 = \frac{-2+3}{3} = \frac{1}{3} \qquad -1+2 = 1$$
$$-\frac{5}{3}-3 = \frac{-5-9}{3} = -\frac{14}{3} \qquad \frac{2}{3}+\frac{1}{3} = 1$$
$$\frac{10}{3}-8 = \frac{10-24}{3} = -\frac{14}{3}$$

$$\left[\begin{array}{cccc|c}1&2&1&1&5\\2&2&2&0&6\\-1&-2&-4&0&-7\\-4&-1&-9&0&-14\end{array}\right](-2),(1),(4) \sim \left[\begin{array}{cccc|c}1&2&1&1&5\\0&-2&0&-2&-4\\0&0&-3&1&-2\\0&7&-5&4&6\end{array}\right]\cdot\left(-\frac{1}{2}\right) \sim \left[\begin{array}{cccc|c}1&2&1&1&5\\0&1&0&1&2\\0&0&-3&1&-2\\0&7&-5&4&6\end{array}\right](-2),(-7) \sim \left[\begin{array}{cccc|c}1&0&1&-1&1\\0&1&0&1&2\\0&0&-3&1&-2\\0&0&-5&-3&-8\end{array}\right]\cdot\left(-\frac{1}{3}\right) \sim$$

$$\sim \left[\begin{array}{cccc|c}1&0&1&-1&1\\0&1&0&1&2\\0&0&1&-\frac{1}{3}&\frac{2}{3}\\0&0&-5&-3&-8\end{array}\right](-1),(5) \sim \left[\begin{array}{cccc|c}1&0&0&-\frac{2}{3}&\frac{1}{3}\\0&1&0&1&2\\0&0&1&-\frac{1}{3}&\frac{2}{3}\\0&0&0&-\frac{14}{3}&-\frac{14}{3}\end{array}\right]\cdot\left(-\frac{3}{14}\right) \sim \left[\begin{array}{cccc|c}1&0&0&-\frac{2}{3}&\frac{1}{3}\\0&1&0&1&2\\0&0&1&-\frac{1}{3}&\frac{2}{3}\\0&0&0&1&1\end{array}\right]\cdot\left(+\frac{1}{3}\right),(-1),\left(\frac{2}{3}\right) \left[\begin{array}{cccc|c}1&0&0&0&1\\0&1&0&0&1\\0&0&1&0&1\\0&0&0&1&1\end{array}\right]$$

(5)

$$\sum n(\sqrt{n}-\sqrt{n-1})$$

Nužan uvjet

$$\lim_{n\to\infty} n(\sqrt{n}-\sqrt{n-1}) = \infty(\infty-\infty)$$

MOŽE SE KRATITI n^2 IZ BROJNIKA SA n IZ NAZIVNIKA

$$\lim_{n\to\infty} n(\sqrt{n}-\sqrt{n-1})\cdot\frac{n(\sqrt{n}+\sqrt{n-1})}{n(\sqrt{n}+\sqrt{n-1})} = \lim_{n\to\infty}\frac{n^2(n-n-1)}{\cancel{n(\sqrt{n}+\sqrt{n-1})}} = \lim_{n\to\infty}\frac{n^2}{\sqrt{n^2}+\sqrt{n^2-1}}$$

$+$ ×

$$\lim_{n\to\infty}\frac{n^2 \;|:n^2}{\sqrt{n^2}+\sqrt{n^2-1}\;|:n^2} = \frac{1}{2}$$ ×

Red DIV. ✓

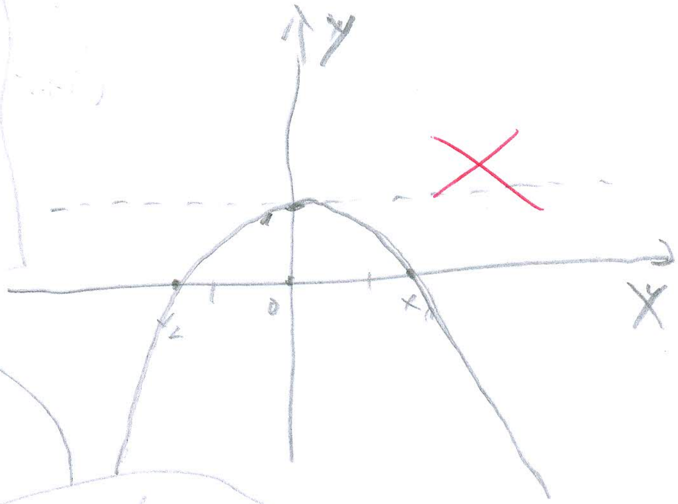
Nema kosih asimptota?

Nul točke $x^2+2=0$

$x_{1,2} = \frac{\pm\sqrt{4\cdot 1\cdot 2}}{2\cdot 1}$

$x_1 = \frac{\sqrt{8}}{2} = 1,41$

$x_2 = -\frac{\sqrt{8}}{2} = -1,41$



(1)

$$f(x) = \frac{x^2-2}{x^2+2}$$

DOMENA: $D(f) = \mathbb{R}\ \langle -\infty, +\infty\rangle$

$x^2+2=0$

$$\lim_{x\to-\infty}\frac{x^2-2}{x^2+2} = (x\to x) = \lim_{x\to-\infty}\frac{x^2-2\;|:x^2}{x^2+2\;|:x^2} = \frac{\infty}{\infty} = \frac{\frac{x^2}{x^2}-\frac{2}{x^2}}{\frac{x^2}{x^2}+\frac{2}{x^2}} = \frac{1}{1} = 1$$

$$\lim_{x\to\infty}\frac{x^2-2}{x^2+2} = \lim_{x\to\infty}\frac{x^2-2\;|:x^2}{x^2+2\;|:x^2} = \frac{\infty}{\infty} = \frac{1}{1} = 1$$

$\boxed{H.A = y = 1}$

Derivacija:

$$f'(x) = \frac{x^2-2}{x^2+2} = \frac{(x^2-2)'\cdot(x^2+2)+(x^2-2)\cdot(x^2+2)'}{(x^2+2)^2} = \frac{(2x-0)(x^2+2x)+(x^2-2)\cdot(2x+0)}{(x^2+2)^2} = \frac{2x(x^2+2x)+(x^2-2)2x}{(x^2+2)^2}$$

Parna ili neparna

$f(-x) = f(x)$

$$f(-x) = \frac{(-x)^2-2}{(-x)^2+2} = \frac{x^2-2}{x^2+2} = f(x)$$

Funkcija je parna!!!

Popuniti odmah!

IME I PREZIME: DARIAN RADMAN **BROJ INDEKSA:** 57635

20

DATUM: 21.2.2012. **VRIJEME:** OD 13:00 DO .

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. 8

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5. Ispitati konvergenciju reda $\sum n(\sqrt{n} - \sqrt{n-1})$ ~~20~~

4

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 2 & 2 & 2 & 0 & 6 \\ -1 & -2 & -4 & 0 & -7 \\ -4 & -1 & -9 & 0 & -14 \end{array}\right] \times(-2)\ \times(1)\ \times(4) \sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 0 & -2 & -4 \\ 0 & 0 & -3 & 1 & -2 \\ 0 & 7 & -5 & 4 & 6 \end{array}\right] :(-2) \quad \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & -3 & 1 & -2 \\ 0 & 7 & -5 & 4 & 6 \end{array}\right] \times(-7)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & -3 & 1 & -2 \\ 0 & 0 & -5 & -3 & -8 \end{array}\right] \times(-1) \quad \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & -5 & -3 & -8 \end{array}\right] :2 \quad \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -5 & -3 & -8 \end{array}\right] \times(-1)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -3 & -2 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -5 & -3 & -8 \end{array}\right] \times(5) \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -3 & -2 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 7 & 7 \end{array}\right] \cdot\left(\frac{1}{7}\right) \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -3 & -2 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \times(3)\ \times(-1)\ \times(-2)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \quad \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 & 1 \\ -1 & -2 & -4 & 0 & 1 \\ -4 & -1 & -9 & 0 & 1 \end{array}\right] \text{TOČNO} \begin{bmatrix} 5 \\ 6 \\ -7 \\ -14 \end{bmatrix}$$

$|z| = \sqrt{x^2+y^2}$

② $\left|\frac{z}{2}\right| = z + 7i$

$\frac{|z|}{|2|} = z + 7i$

$\frac{|z|}{2} = z + 7i \quad / \cdot 2$

$|z| = 2z + 14i$

$\sqrt{x^2+y^2} = 2 \cdot (x+yi) + 14i$

$\sqrt{(x+yi)^2} = 2x + 2yi + 14i$

$x^2+yi = 2x + 2yi + 14i$ ×

$x^2+y^2 \neq (x+y)^2$

$-1x^2 + 3yi = 14i \quad / : i$

$\frac{-1x^2+3y\cancel{i}}{\cancel{i}} = \frac{14\cancel{i}}{\cancel{i}}$

$-1x^2 + 3y = 14$

a b c

$-1x^2 + 3y - 14 = 0$

$x_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot (-1) \cdot (-14)}}{-2}$

$x_{1,2} = \frac{-3 \pm \sqrt{9-56}}{-2}$

$x_{1,2} = \frac{-3 \pm (-47i)}{-2}$

$x_1 = \frac{-3+47i}{-2}$

$x_2 = \frac{-3-47i}{-2}$

5 $\sum n(\sqrt{n} - \sqrt{n-1})$

$\lim_{n\to\infty} \sqrt[\cancel{n}]{(\sqrt{n}-\sqrt{n-1})^{\cancel{n}}} = \sqrt{n} - \sqrt{n-1} = n^{\frac{1}{2}} - (n-1)^{\frac{1}{2}} \; / : n$

$= \dfrac{\cancel{n}^{\frac{1}{2}} - \cancel{n}^{\frac{1}{2}} + 1^{\frac{1}{2}}}{} = \left(\dfrac{1}{n}\right)^{\frac{1}{2}} = 0 < 1$ – KONVERGIRA

$1^{\frac{1}{2}} = 1 \qquad 1-1=0$

2 $g(x) = \sqrt{7 - x^2}$ DOMENA, NEPARNOST, EKS.

$7 - x^2 \geq 0$

$-x^2 \geq -7 \; / : (-1)$

$x^2 \leq 7$

$x \leq \sqrt{7}$

$D_f \in R / \{-\sqrt{7}, \sqrt{7}\}$

$g(x) = \sqrt{7-x^2}$

$g'(x) = \dfrac{1}{2\sqrt{7-x^2}}$

$g''(x) = \dfrac{2\sqrt{7-x^2} - 4\dfrac{1}{\sqrt{7-x^2}}}{(2\sqrt{7-x^2})^2}$

PAR

$f(x) \neq f(-x)$

NEP

$f(-x) = f(x)$

$f(x) = \sqrt{7+x^2}$

FUNKCIJA NIJE PARNA

$f(x) = -\sqrt{7-x^2}$

FUNKCIJA NIJE NEPARNA

Popuniti odmah!

IME I PREZIME: KRISTIAN MARTINOVIĆ **BROJ INDEKSA:** 17-2-0110-2011

DATUM: 21.2.2012. **VRIJEME:** OD DO .

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. 8

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4.

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 2 & 2 & 2 & 0 & 6 \\ -1 & -2 & -4 & 0 & -7 \\ -4 & -1 & -9 & 0 & -14 \end{array}\right] \begin{array}{l} \\ \\ /\cdot(-1) \\ /\cdot(-1) \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 2 & 2 & 2 & 0 & 6 \\ 1 & 2 & 4 & 0 & 7 \\ 4 & 1 & 9 & 0 & 14 \end{array}\right] \begin{array}{l} \\ /:2 \\ /:4 \\ \\ \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 1 & 1 & 1 & 0 & 3 \\ \frac{1}{4} & \frac{1}{2} & 1 & 0 & \frac{7}{4} \\ 4 & 1 & 9 & 0 & 14 \end{array}\right] \sim \left[\begin{array}{cccc|c} 4 & 1 & 9 & 0 & 14 \\ 1 & 1 & 1 & 0 & 3 \\ \frac{1}{4} & \frac{1}{2} & 1 & 0 & \frac{7}{4} \\ 1 & 2 & 1 & 1 & 5 \end{array}\right] \begin{array}{l} /:4 \\ \\ \\ \\ \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & \frac{1}{4} & \frac{9}{4} & 0 & \frac{14}{4} \\ 1 & 1 & 1 & 0 & 3 \\ \frac{1}{4} & \frac{1}{2} & 1 & 0 & \frac{7}{4} \\ 1 & 2 & 1 & 1 & 5 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & \frac{1}{4} & \frac{9}{4} & 0 & \frac{14}{4} \\ 0 & 1 & \frac{5}{3} & 0 & -\frac{2}{3} \\ & & & & \\ & & & & \end{array}\right]$$

$R_1 - R_2$

$$\begin{array}{rrrrrl} & 1 & \frac{1}{4} & \frac{9}{4} & 0 & \frac{14}{4} \\ + & -1 & -1 & -1 & -0 & -3 \\ \hline & 0 & -\frac{3}{4} & \frac{5}{4} & 0 & \frac{1}{2} \quad / : \left(-\frac{3}{4}\right) \\ & 0 & 1 & -\frac{5}{3} & 0 & -\frac{2}{3} \end{array}$$

Popuniti odmah!
Ime i prezime: ANĐELA UHODA **Broj indeksa:** 17-2-0106-2011 ~~0~~
Datum: 21.2.2012. **Vrijeme:** od 13:05 do .
MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. 8

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1. Tok funkcije $f(x) = \dfrac{x^2-2}{x^2+2}$

NUL TOČKE

$x^2+2=0$
$x^2=-2 \,/\sqrt{\ }$
$x \in R$

$x^2-2=0$
$x^2=2 \,/\sqrt{\ }$
$x_{1,2}=\pm\sqrt{2}$

$-\infty \quad -\sqrt{2} \quad \sqrt{2} \quad +\infty$
$+ \quad 0 \quad - \quad 0 \quad +$

V.A. $\cancel{\lim_{x\to\sqrt{2}} f(x)\dfrac{x^2-2}{x^2+2} = \lim_{x\to\sqrt{2}} \dfrac{2-2}{2+2} = \dfrac{0}{4} = 0}$

$\cancel{\lim_{x\to-\sqrt{2}} f(x)\dfrac{(-\sqrt{2})^2-2}{(-\sqrt{2})^2+2} = \dfrac{-2-2}{-2+2} = \dfrac{-4}{0} = \infty}$

H.A. $\lim_{x\to\infty} f(x)\dfrac{x^2-2 \,/:x^2}{x^2+2\,/:x^2} = \dfrac{\frac{x^2}{x^2}-\frac{2}{x^2}}{\frac{x^2}{x^2}+2} = \boxed{1}$

$f(0) = \dfrac{x^2-2}{x^2+2} = \boxed{-1}$

$\boxed{y=1}$ $\boxed{y=-1}$

$f(-x) = \dfrac{(-x)^2-2}{(-x)^2+2} = f(-x) = \dfrac{x^2-2}{x^2+2}$ ~~PARNA~~

NIJE PERIODIČNA

$\left(\dfrac{u}{v}\right)' = \dfrac{u'\cdot v - u\cdot v'}{v^2}$

$f'(x) = \dfrac{(x^2-2)'\cdot(x^2+2)-(x^2-2)\cdot(x^2+2)'}{(x^2+2)^2} = \dfrac{(2x)\cdot(x^2+2)-(x^2-2)\cdot 2x}{(x^2+2)^2}$

$f'(x) = \dfrac{2x^3+4x-(2x^3-4x)}{(x^2+2)^2} = \dfrac{\cancel{2x^3}+4x-\cancel{2x^3}+4x}{(x^2+2)^2} = \dfrac{8x}{(x^2+2)^2}$

$8x = 0 \,/:8$
$\boxed{x=0}$

$f''(x) = \dfrac{(8x)'\cdot(x^2+2)^2-(8x)\cdot((x^2+2)^2)'}{(x^2+2)^4}$

$f''(x) = \dfrac{8\cdot(x^2+2)^2-(8x)\cdot(2(2x+0))}{x^2+2^4}$

$$f''(x) = \frac{8\cdot(x^2+2)^2 - (8x)\cdot(2(2x+0))}{(x^2+2)^4}$$

$$f'(x) = \frac{8\cdot(x^2+2)^2-(8x)\cdot 4x}{(x^2+2)^4} = \frac{8\cdot(x^2+2)^2-32x^2}{(x^2+2)^4}$$

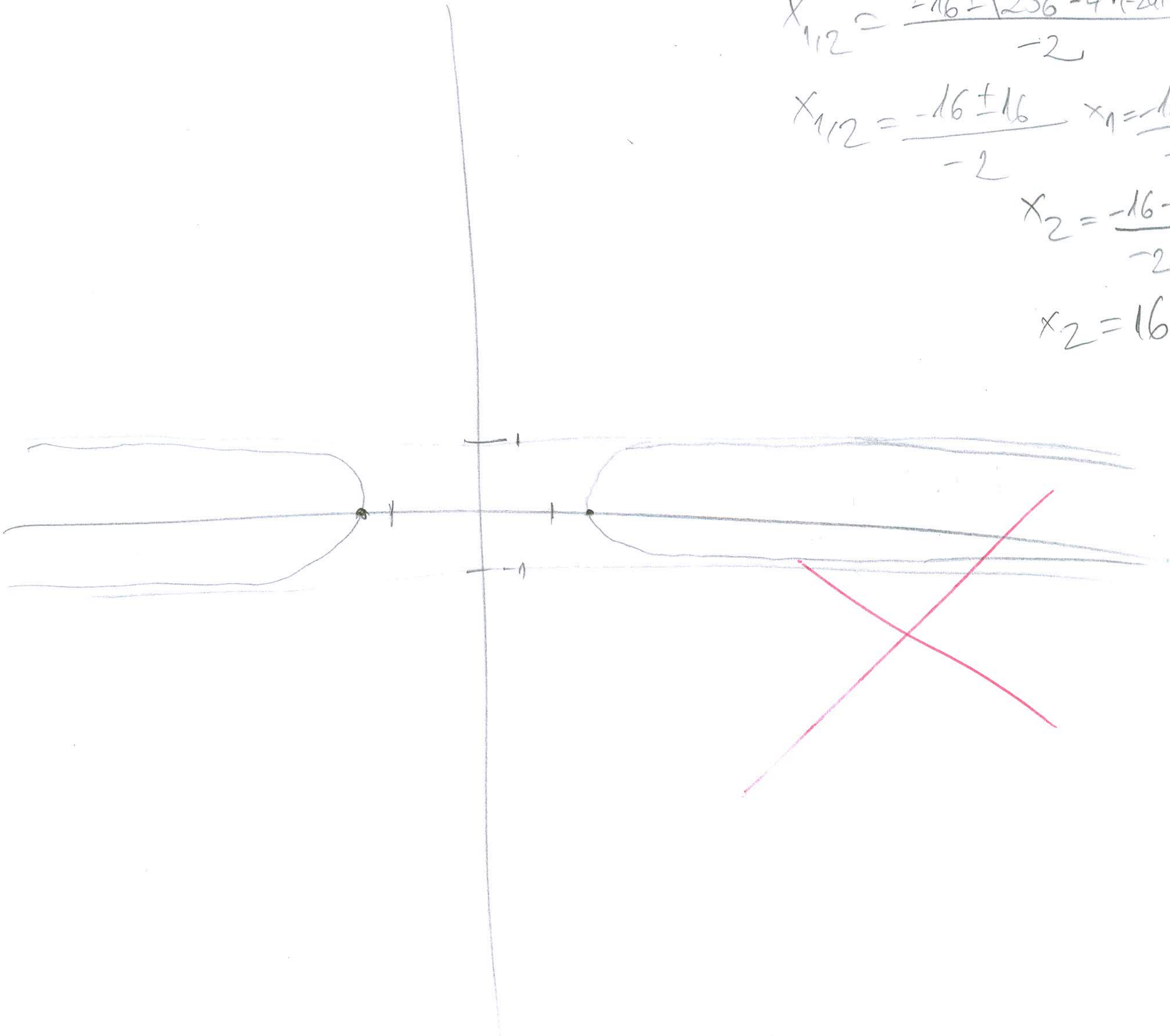
$$f''(x) = \frac{\cancel{(x^2+2)}\cdot(8x^2+16-32x^2)}{(x^2+2)^{\cancel{4}3}} = \frac{-24x^2+16}{(x^2+2)^3}$$

$$x_{1,2} = \frac{-16\pm\sqrt{256-4\cdot(-24)\cdot 0}}{-2}$$

$$x_{1,2} = \frac{-16\pm 16}{-2} \qquad x_1 = \frac{-16+16}{-2} = 0$$

$$x_2 = \frac{-16-16}{-2} = \frac{+32}{2}$$

$$x_2 = 16$$



4. $x_1 + 2x_2 + x_3 + x_4 = 5$

$2x_1 + 2x_2 + 2x_3 = 6$

$-x_1 - 2x_2 - 4x_3 = -7$

$-4x_1 - x_2 - 9x_3 = -14$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 2 & 2 & 2 & 0 & 6 \\ -1 & -2 & -4 & 0 & -7 \\ 4 & -1 & -9 & 0 & -14 \end{array}\right] \xrightarrow{I\cdot(-2)+II,\ I\cdot(1)+III,\ I\cdot(-4)+IV} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 0 & -2 & -4 \\ 0 & 0 & -3 & 0 & -2 \\ 0 & -9 & -13 & 6 & -34 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \end{array}\right]$$

Popuniti odmah!
Ime i prezime: TINO BRAJKOVIĆ **Broj indeksa:** 17-2-0100-2011

Datum: 21.2.2012. **Vrijeme:** od do .

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. 8

Broj ↓ bodova

1. Na temelju ispitivanja toka funkcije nacrtati skicu grafa za $f(x) = \dfrac{x^2-2}{x^2+2}$. 20

2. Ispitati domenu, (ne)parnost i pronaći lokalne minimume i maksimume funkcije $g(x) = \sqrt{7-x^2}$. (skica grafa funkcije se ne boduje, ali ako je nacrtate odmah će vam sve biti jasno.) 5+5+5+5

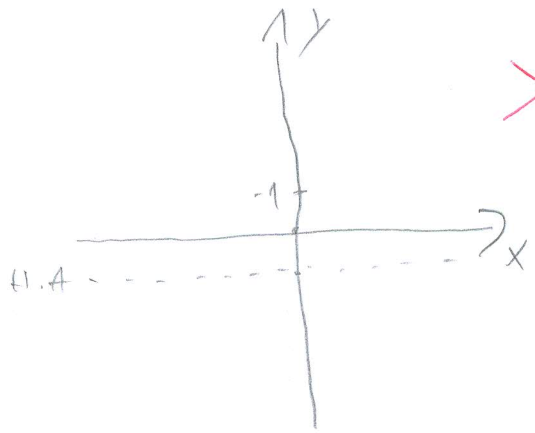
3. Riješiti među kompleksnim brojevima $\left|\frac{z}{2}\right| = z + 7i$. Možete koristiti formulu za nultočke kvadratne funkcije. 20

4. Odrediti sva koja postoje riješenja sustava linearnih jednadžbi i provjeriti: 15+5

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 5 \\ 2x_1 + 2x_2 + 2x_3 &= 6 \\ -x_1 - 2x_2 - 4x_3 &= -7 \\ -4x_1 - x_2 - 9x_3 &= -14 \end{aligned}$$

5. Ispitati konvergenciju reda $\sum n(\sqrt{n} - \sqrt{n-1})$ 20

1. $f(x) = \dfrac{x^2-2}{x^2+2}$



Domena

$x^2+2 \neq 0$

$x \neq \sqrt{2}$ Domene nema

Sjecište na osi y

$f(0) = \dfrac{-2}{2} = -1$ $S(0,-1)$

Nult točke

$x^2-2=0$

$x = \sqrt{2}$

Nema nult točke

2. $g(x)\sqrt{7-x^2}$

Domene nema

ekstremi

$f'(x) = \dfrac{2x}{2x}$

Asimptote

H.A. $\lim\limits_{x\to\infty} \dfrac{x^2-2 \;/:x^2}{x^2+2 \;/:x^2} = \dfrac{\frac{x^2}{x^2} - \overset{0}{\frac{2}{x^2}}}{\frac{x^2}{x^2} + \underset{0}{\frac{2}{x^2}}} = 1$

$y = 1$

Kose nema

4.

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ & 2 & 2 & 2 & 6 \\ & -1 & -2 & -4 & -7 \\ & -4 & -1 & -9 & -14 \end{array}\right] \begin{array}{l} | \cdot -1 \\ + \end{array} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 1 & 1 & 1 \\ & -1 & -2 & -4 & -7 \\ & -4 & -1 & -9 & -14 \end{array}\right] \begin{array}{l} \cdot 1 \\ + \end{array} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & -3 & -1 & -3 & -6 \\ & -4 & -1 & -9 & -14 \end{array}\right] |(-1)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & -3 & -1 & -3 & -6 \\ 0 & -1 & 0 & -6 & -8 \end{array}\right) | \cdot 3 \; + \left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -1 & 15 & 18 \\ 0 & -1 & 0 & -6 & -8 \end{array}\right) | \cdot 2 \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -1 & 15 & 18 \\ 0 & -2 & 0 & -12 & -8 \end{array}\right] \begin{array}{l} \cdot(-1) \\ + \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -1 & 15 & 18 \\ 0 & 0 & 0 & -13 & -9 \end{array}\right) | \cdot \frac{1}{15} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -\frac{1}{15} & 1 & \frac{18}{15} \\ 0 & 0 & 0 & -13 & -9 \end{array}\right) |(-1) + \left(\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 4 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -\frac{1}{15} & 1 & \frac{18}{15} \\ 0 & 0 & 0 & -13 & -9 \end{array}\right) \cdot 15$$

$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 4 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -1 & 15 & 18 \\ 0 & 0 & 0 & -13 & -9 \end{array}\right] | \cdot 1 \left(\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 4 \\ 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & -1 & 15 & 18 \\ 0 & 0 & 0 & 2 & 9 \end{array}\right)$$