

...uniti odmah!

IME I PREZIME:

ŠIME NEKIĆ

BRJ INDEKSA:

17-1-0102-2011

DATUM: 21.2.2012. VRIJEME: OD

DO

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

4
Broj ↓
bodova

50

1. Odrediti determinantu matrice $A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

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2. Ispitati konvergenciju reda $\sum \left(\frac{n+2}{n+1}\right)^n$

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3. Ispitati sve asimptote funkcije $f(x) = \frac{1-x^2}{2x^2-x-3}$.

5+5+5+5

4. Odrediti domenu i prvu derivaciju funkcije $g(x) = x + \sqrt{1-x^2}$.

7+13

5. Na temelju ispitivanja toka funkcije ispitati da li funkcija g iz zadatka 4 ima globalni minimum. Ako ima koliko iznosi? Najlakše je odgovoriti na ovo pitanje ako skicirate graf funkcije.

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4. $g(x) = x + \sqrt{1-x^2}$

1) DOMENA

$$\sqrt{1-x^2} \geq 0$$

$$1-x^2 \geq 0$$

$$1-x < 0$$

$$1+x < 0$$

$$-x \leq -1$$

$$x \leq -1$$

$$x \leq 1$$

	$-\infty$	-1	1	$+\infty$
$1-x$		+	+	-
$1+x$		-	+	+
		-	(+)	-

DEFER $[-1, 1]$ ✓

$$g(x) = x + \sqrt{1-x^2} \Rightarrow x + [(1-x)^2]^{\frac{1}{2}}$$

$$g'(x) = 1 + \frac{1}{2} (1-x^2)^{-\frac{1}{2}} \cdot (-2x)$$

$$g'(x) = 1 - x (1-x^2)^{-\frac{1}{2}} \Leftrightarrow \frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}}$$



$$5.) g(x) = x + \sqrt{1-x^2}$$

ASIMPTOTE

$$VA = 0 \Rightarrow \text{NEMA}$$

H.A. i K.A.

$$y = kx + p$$

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x}$$

$$k = \lim_{x \rightarrow \infty} \frac{x + \sqrt{1-x^2}}{x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$k = \lim_{x \rightarrow \infty} \frac{\frac{x}{x} + \sqrt{\frac{1}{x^2} - \frac{x^2}{x^2}}}{\frac{x}{x}} = \frac{1 + \sqrt{\frac{1}{x^2} - 1}}{1}$$

$$k \Rightarrow \frac{1+1}{1} \Rightarrow \text{NEMA ASIMPTOTE}$$

PERIODIČNOST.

Nije periodična, jer nema nijednu od TRIGONOMETRIJSKIH FUNKCIJA

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6. LOKALNI EKSTREMI

$$g'(x) = \frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}} \Leftrightarrow \leq 0$$

$$\sqrt{1-x^2} - x = 0 \quad |^2$$

$$1-x^2-x^2=0$$

$$1-2x^2=0$$

$$-2x^2 = -1$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \frac{1}{\sqrt{2}}$$

$$g''(x) = \frac{1}{4} (1-x^2)^{-\frac{3}{2}} \cdot 2x \cdot (-2x) \cdot \frac{1}{2} (1-x^2)^{-\frac{1}{2}} \cdot (-2)$$

$$g''(x) = -\frac{1}{4} \cdot \frac{1}{\sqrt{(1-x^2)^3}} \cdot \frac{4x^2 + (-2)}{2\sqrt{1-x^2}}$$

$$g''(x) = -\frac{1}{\sqrt{(1-x^2)^3}} \cdot \frac{x^2 - 2}{2\sqrt{1-x^2}}$$

$$g''\left(x = \frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}} \cdot \frac{\left(\frac{1}{\sqrt{2}}\right)^2 - 2}{2\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}} = < 0 \Rightarrow \text{MAX}$$

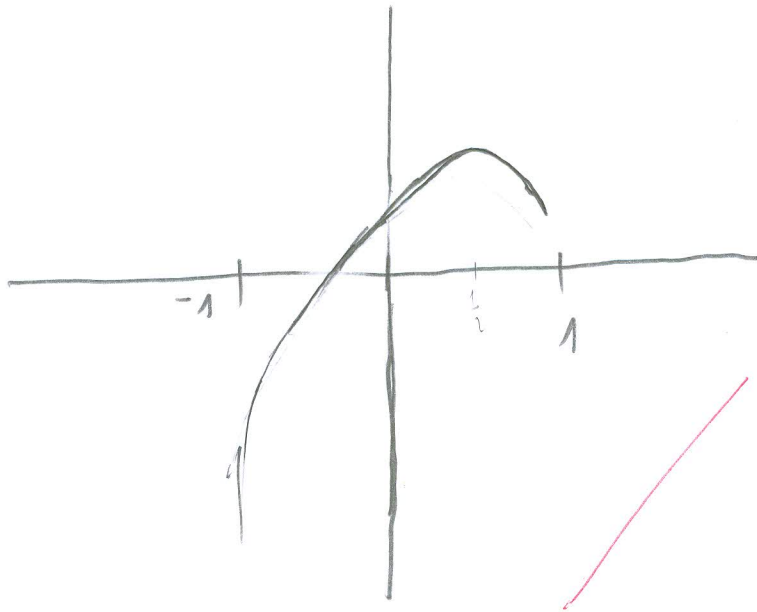
$$g\left(x = \frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} + \sqrt{1 - \frac{1}{4}} = \left(\frac{1}{\sqrt{2}}, 1.366\right)$$

$$g\left(x = \frac{1}{2}\right) = \frac{1}{2} + \sqrt{\frac{3}{4}}$$

$$g\left(x = \frac{1}{2}\right) = \frac{1 + \sqrt{3}}{2} = 1.366$$

$$g\left(x = \frac{1}{2}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}$$

GRAF



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