Popuniti odmah!

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DATUM: 21.2.2012. VRIJEME: OD 13:05 DO 14:42

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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	Broj ↓ bodova
1. Da li postoji i ako postoji koji je inverz dane matrice? Ako postoji inverz tada matričnim množenjem provjeriti da je dobro izračunat. $$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}.$$	15+5
2. Pronaći sve kompleksne brojeve z takve da je $z^3 + \lvert 3+4i \rvert = \frac{5}{i}$.	20
3. Odrediti domenu i ispitati ponašanje na rubovima domene (asimptote) funkcije $f(x) = \ln(2-3x)$.	6+7+7
4. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije $g(x) = \arcsin(2x)$.	5+5+5+5
5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2}{x-1}$.	20

(1)

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \quad \det A = \begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -3 \end{vmatrix} =$$

1.r·(-2)+4.r

računamo po 1 stupcu

$$\det A = (-1)^{1+1} \cdot 1 \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & -3 \end{vmatrix} = 1 \cdot (-1)^{3+3} \cdot (-3) \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = -3 \cdot (4-1) = -3 \cdot 3 = -9$$

računamo po 3. stupcu

$\det A = -9 \neq 0 \Rightarrow$ matrica je regularna i ima inverz ✓

$$A = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & -2 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & -2 & 0 & 0 & 1 \end{array}\right] \sim$$

1.r·(-2)+4.r

2.r ⟷ 3.r

2.r·(-2)+3.r

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & -3 & -2 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & 2/3 & 0 & 0 & -1/3 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1/3 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & 2/3 & 0 & 0 & -1/3 \end{array}\right] \sim$$

3.r:(-3)

4.r:(-3)

4.r·(-2)+1.r

$-\frac{4}{3} + 1 = \frac{-4+3}{3} = -\frac{1}{3}$

3.r·(-2)+2.r

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1/3 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & 0 & 0 & 2/3 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & 2/3 & 0 & 0 & -1/3 \end{array}\right]$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1/3 & 0 & 0 & 2/3 \\ 0 & 2/3 & -1/3 & 0 \\ 0 & -1/3 & 2/3 & 0 \\ 2/3 & 0 & 0 & -1/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

✓

PROVERA!

$-\frac{1}{3}+\frac{4}{3}=\frac{3}{3}=1$ ✓

① $1\cdot(-\frac{1}{3})+0\cdot0+0\cdot0+2\cdot\frac{2}{3}$, $1\cdot0+0\cdot\frac{1}{3}+0(-\frac{1}{3})+2\cdot0$, $1\cdot0+0\cdot(-\frac{1}{3})+0\cdot\frac{2}{3}+2\cdot0$, $1\cdot\frac{2}{3}+0\cdot0+0\cdot0+2\cdot(-\frac{1}{3})$ ✓

② $0\cdot(-\frac{1}{3})+2\cdot0+1\cdot0+0\cdot\frac{2}{3}$, $1\cdot0+2\cdot\frac{2}{3}+1\cdot(-\frac{1}{3})+0\cdot0$, $1\cdot0+0\cdot(-\frac{1}{3})+1\cdot\frac{2}{3}+0\cdot0$, $0\cdot\frac{2}{3}+2\cdot0+1\cdot0+0\cdot(-\frac{1}{3})$ ✓

③ $0\cdot(-\frac{1}{3})+1\cdot0+0\cdot\frac{2}{3}$, $0\cdot0+1\cdot\frac{2}{3}+2\cdot(-\frac{1}{3})+0\cdot0$, $0\cdot0+1\cdot(-\frac{1}{3})+2\cdot\frac{2}{3}+0\cdot0$, $0\cdot\frac{2}{3}+1\cdot0+2\cdot0+0\cdot(-\frac{1}{3})$ ✓

④ $2\cdot(-\frac{1}{3})+0\cdot0+0\cdot0+1\cdot\frac{2}{3}$, $2\cdot0+0\cdot\frac{2}{3}+0\cdot(-\frac{1}{3})+1\cdot0$, $2\cdot0+0\cdot(-\frac{1}{3})+0\cdot\frac{2}{3}+1\cdot0$, $2\cdot\frac{2}{3}+0\cdot0+0\cdot0+1\cdot(-\frac{1}{3})$ ✓

② $z^3+|3+4i|=\frac{5}{i}$

$|3+4i|=\sqrt{x^2+y^2}=\sqrt{3^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$ ✓

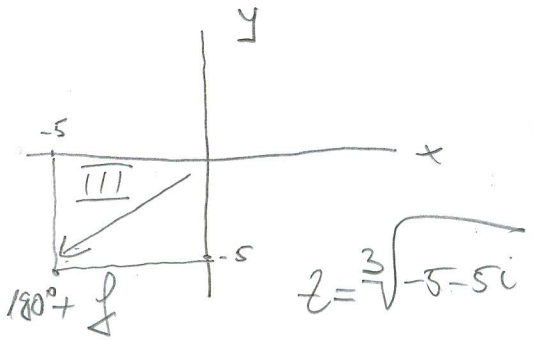
$z^3+5=-5i$

$\frac{5}{i}=\frac{5}{i}\cdot\frac{-i}{-i}=\frac{-5i}{-i^2}=\frac{-5i}{1}=-5i$

$z^3=-5i-5$

$z^3=-5-5i$

$|w|=\sqrt{x^2+y^2}=\sqrt{(-5)^2+(-5)^2}=\sqrt{25+25}=\sqrt{50}$



$tg\varphi=\frac{y}{x}=\frac{-5}{-5}=1\Rightarrow 45°$

$\varphi=180°+45°=225°$

$z=\sqrt[3]{-5-5i}$

$z=\sqrt[3]{\sqrt{50}}=\sqrt[6]{50}$

$k=0,1,2$

a) $k=0$

$z_1=\sqrt[6]{50}\left(\cos\frac{\varphi+k\cdot360°}{3}+i\sin\frac{\varphi+k\cdot360°}{3}\right)$

$z_1=\sqrt[6]{50}\left(\cos\frac{225°+0\cdot360°}{3}+i\sin\frac{225°+0\cdot360°}{3}\right)$

$z_1=\sqrt[6]{50}\left(\cos 75°+i\sin 75°\right)$

$z_1=\sqrt[6]{50}\left(0.258+i0.966\right)$

b) $k=1$

$z_2=\sqrt[6]{50}\left(\cos\frac{225°+1\cdot360°}{3}+i\sin\frac{225°+1\cdot360°}{3}\right)$

$z_2=\sqrt[6]{50}\left(\cos 195°+i\sin 195°\right)$

$z_2=\sqrt[6]{50}\left(-0.966+i0.258\right)$