

$$2.) \quad y'''(x) - y(x) = x$$

$$y(0) = 1$$

$$y''(0) = 2$$

$$y'(0) = 1$$

$$x^3 y(x) - x^2 \overset{1}{y'(0)} - x \overset{1}{y''(0)} - \overset{2}{y'''(0)} - y(x) = \frac{1}{x^2} \checkmark$$

$$x^3 y(x) - x^2 - x - 2 - y(x) = \frac{1}{x^2}$$

$$x^3 y(x) - y(x) = \frac{1}{x^2} + x^2 + x + 2$$

$$y(x)(x^3 - 1) = \frac{1 + x^4 + x^3 + 2x^2}{x^2}$$

$$y(x) = \frac{x^4 + x^3 + 2x^2 + 1}{(x^3 - 1) \cdot x^2} = \frac{x^4 + x^3 + 2x^2 + 1}{x^2 \cdot (x-1) \cdot (x^2 + x + 1)} \checkmark$$

$$\frac{x^4 + x^3 + 2x^2 + 1}{x^2(x-1)(x^2+x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{Dx+E}{x^2+x+1} \checkmark = \frac{\cancel{Ax^4} - \cancel{Ax^3} + \cancel{Bx^2} + \cancel{Cx} + \cancel{Dx^4} + \cancel{Ex^3} + \cancel{Cx^2} + \cancel{Dx} + \cancel{Ex}}{x^2 \cdot (x-1) \cdot (x^2+x+1)}$$

$$= \frac{\cancel{Dx^4} - \cancel{Dx^3} + \cancel{Ex^2} - \cancel{Cx}}{x^2 \cdot (x-1) \cdot (x^2+x+1)}$$

$$\text{N2 } x^4: 1 = A + C + D \Rightarrow 1 = C + D \Rightarrow D = C - 1 \quad 1 = -1 + C - 1 + C + C - 2$$

$$\text{N2 } x^3: 1 = B + C - D + E \quad D = C - 1$$

$$\text{N2 } x^2: 2 = C - E \Rightarrow E = C - 2 \quad 5 = 3C \Rightarrow C = \frac{5}{3} \checkmark$$

$$\text{N2 } x: 0 = -A \Rightarrow A = 0 \checkmark \quad D = 1 - \frac{5}{3} = -\frac{2}{3} \checkmark$$

$$\text{N2 } 1: 1 = B \Rightarrow B = 1 \checkmark \quad E = \frac{5}{3} - 2 = -\frac{1}{3} \checkmark$$

$$E = \frac{5-6}{3} = -\frac{1}{3} \checkmark$$

$$X(x) = \frac{-1}{x^2} + \frac{\frac{5}{3}}{x-1} + \frac{\frac{2}{3}x - \frac{1}{3}}{x^2+x+1} = \frac{-1}{x^2} + \frac{5}{3} \frac{1}{x-1} - \frac{2}{3} \cdot \frac{(x+\frac{1}{2})}{(x+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} \checkmark$$

$$\begin{aligned} x^2+x+1 &= (x+\frac{1}{2})^2 \\ &= x^2+x+\frac{1}{4}-\frac{1}{4}+1 \\ &= (x+\frac{1}{2})^2 + \frac{3}{4} \end{aligned}$$

$$X(x) = -x \checkmark + \frac{5}{3} e^x \checkmark - \frac{2}{3} \cdot e^{-\frac{1}{2}x} \cos\left(\frac{\sqrt{3}}{2}x\right) \checkmark$$

MATEMATIKA 3 - KOLOKVIJ #3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: LUKA SJAUŠ

BROJ INDEKSA: 57680

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu:

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1, \quad x'(0) = 0.$$

2. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu:

$$y'''(t) - y(t) = t, \quad y(0) = 1, \quad y''(0) = 2, \quad y'(0) = 1.$$

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Ukupno:

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$$\textcircled{1} \quad x'''(t) + x'(t) = 0; \quad x(0) = x''(0) = 1, \quad x'(0) = 0.$$

$$\underset{=1}{\sigma^3} X(\sigma) - \underset{=1}{\sigma^2} x(0) - \underset{=0}{\sigma} x'(0) - \underset{=1}{x''(0)} + \sigma X(\sigma) - \underset{=1}{x(0)} = 0$$

$$\sigma^3 X(\sigma) - \sigma^2 - 1 + \sigma X(\sigma) - 1 = 0$$

$$\sigma^3 X(\sigma) + \sigma X(\sigma) = \sigma^2 + 2$$

$$X(\sigma) (\sigma^3 + \sigma) = \sigma^2 + 2$$

$$X(\sigma) = \frac{\sigma^2 + 2}{\sigma^3 + \sigma} = \frac{\sigma^2 + 2}{\sigma(\sigma^2 + 1)} = \frac{A}{\sigma} + \frac{B\sigma + C}{\sigma^2 + 1}$$

$$\sigma^2 + 2 = A(\sigma^2 + 1) + (B\sigma + C)\sigma$$

$$\sigma^2 + 2 = \underline{A\sigma^2 + A} + \underline{B\sigma^2 + C\sigma}$$

$$A + B = 1 \Rightarrow B = 1 - 2 \quad \boxed{B = -1}$$

$$\boxed{C = 0}$$

$$\boxed{A = 2}$$

$$X(\sigma) = \frac{2}{\sigma} + \frac{-1}{\sigma^2 + 1} = 2 \cdot \frac{1}{\sigma} - \frac{1}{\sigma^2 + 1^2}$$

$$x(t) = \underline{2 - \cos t}$$

$$(\cos t)' = -\sin$$

$$x=0 \quad 2 - \cos 0 = 2 - 1 = 1 \quad \checkmark$$

$$x'(t) = \sin t \quad x=0 \quad \sin 0 = 0 \quad \checkmark$$

$$x''(t) = \cos t \quad x=0 \quad \cos 0 = 1 \quad \checkmark$$

$$x'''(t) = -\sin t$$

$$-\sin t + \sin t = 0 \quad \checkmark \checkmark$$

