MATEMATIKA 3 - KOLOKVIJ #3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: LOVRE LOVRIĆ **BROJ INDEKSA:** 58080

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1, \; x'(0) = 0.$$

10

2. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t) - y(t) = t, \quad y(0) = 1, \; y''(0) = 2, \; y'(0) = 1.$$

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Ukupno: 20

1.) $x'''(t) + x'(t) = 0$ $\quad x(0) = 1$, $x''(0) = 1$, $x'(0) = 0$

$$s^3 X(s) - s^2 \underbrace{x(0)}_{1} - s \underbrace{x'(0)}_{0} - \underbrace{x''(0)}_{1} + s X(s) - \underbrace{x(0)}_{1} = 0$$

$$s^3 X(s) - s^2 - 1 + s X(s) - 1 = 0$$

$$s^3 X(s) + s X(s) = s^2 + 2$$

$$X(s)(s^2+1)\cdot s = s^2 + 2$$

$$X(s) = \frac{s^2+2}{(s^2+1)\cdot s}$$

$$\frac{s^2+2}{(s^2+1)\cdot s} = \frac{As+B}{s^2+1} + \frac{C}{s} = \frac{As^2+Bs+Cs^2+C}{(s^2+1)\cdot s} \Rightarrow \frac{-s}{s^2+1} + \frac{2}{s}$$

Nz s^2: $1 = A + C \Rightarrow 1 = A + 2 \Rightarrow -A = 2 - 1$

Nz s: $0 = B \Rightarrow B = 0$ $\quad -A = 1$

Sl. z: $2 = C \Rightarrow C = 2$ $\quad A = -1$

$$X(s) = \frac{-s}{s^2+1} + \frac{2}{s} = (-1)\cdot\frac{s}{s^2+1^2} + 2\cdot\frac{1}{s} = -\cos(t) + 2$$

2.) $y'''(t) - y(t) = t$ $\quad y(0)=1,\ y''(0)=2,\ y'(0)=1$

$$s^3 Y(s) - s^2 \overset{1}{y(0)} - s \overset{1}{y'(0)} - \overset{2}{y''(0)} - Y(s) = \frac{1}{s^2}$$ ✓

$$s^3 Y(s) - s^2 - s - 2 - Y(s) = \frac{1}{s^2}$$

$$s^3 Y(s) - Y(s) = \frac{1}{s^2} + s^2 + s + 2$$

$$Y(s)(s^3 - 1) = \frac{1 + s^4 + s^3 + 2s^2}{s^2}$$

$$Y(s) = \frac{s^4 + s^3 + 2s^2 + 1}{(s^3 - 1)\cdot s^2} = \frac{s^4 + s^3 + 2s^2 + 1}{s^2 \cdot (s-1)\cdot (s^2 + s + 1)}$$ ✓

$a^3 \pm b^3 = (a \pm b)\cdot (a^2 \mp ab + b^2)$

$$\frac{s^4 + s^3 + 2s^2 + 1}{s^2 (s-1)\cdot (s^2+s+1)} = \frac{As + B}{s^2} + \frac{C}{s-1} + \frac{Ds + E}{s^2 + s + 1} = \frac{\cancel{As^4} - \cancel{As} + \cancel{Bs^3} - \cancel{B} + \cancel{Cs^4} + \cancel{Cs^3} + \cancel{Cs^2}}{s^2 \cdot (s-1)\cdot (s^2+s+1)}$$ ✓

$$= \frac{\cancel{Ds^4} - \cancel{Ds^3} + \cancel{Es^3} - \cancel{Es^2}}{s^2 \cdot (s-1)\cdot (s^2+s+1)}$$

Vz s^4: $1 = A + C + D \Rightarrow 1 = C + D \Rightarrow -D = C - 1$, $D = 1 - C$

Vz s^3: $1 = B + C - D + E$

Vz s^2: $2 = C - E \Rightarrow E = C - 2$

Vz s: $0 = -A \Rightarrow A = 0$ ✓

Kz s: $1 = -B \Rightarrow B = -1$ ✓

$1 = -1 + C - 1 + C + C - 2$

$5 = 3C \Rightarrow C = \frac{5}{3}$ ✓

$D = 1 - \frac{5}{3}$

$D = \frac{3-5}{3} = -\frac{2}{3}$ ✓

$E = \frac{5}{3} - 2$

$E = \frac{5-6}{3} = -\frac{1}{3}$ ✓

$$X(s) = -\frac{1}{s^2} + \frac{\frac{5}{3}}{s-1} - \frac{\frac{2}{3}s + \frac{1}{3}}{s^2+s+1} = -\frac{1}{s^2} + \frac{5}{3}\frac{1}{s-1} - \frac{2}{3}\cdot\frac{\left(s + \frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$ ✓ ✓ ✓

$s^2 + s + 1 = \left(s + \frac{1}{2}\right)^2$

$= s^2 + s + \frac{1}{4} - \frac{1}{4} + 1$

$= \left(s + \frac{1}{2}\right)^2 + \frac{3}{4}$

$$x(t) = -t + \frac{5}{3}e^t - \frac{2}{3}\cdot e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)$$ ✓

MATEMATIKA 3 - KOLOKVIJ #3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: LUKA SJAUŠ **BROJ INDEKSA:** 57680

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1, \; x'(0) = 0.$$

10

2. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t) - y(t) = t, \quad y(0) = 1, \; y''(0) = 2, \; y'(0) = 1.$$

~~10~~ 8

Ukupno: 18

(1) $x'''(t) + x'(t) = 0$; $x(0) = x''(0) = 1$, $x'(0) = 0$.

$s^3X(s) - s^2 x(0) - s x'(0) - x''(0) + sX(s) - x(0) = 0$

($x(0) = 1$, $x'(0) = 0$, $x''(0) = 1$, $x(0) = 1$)

$s^3X(s) - s^2 - 1 + sX(s) - 1 = 0$

$s^3X(s) + sX(s) = s^2 + 2$

$X(s)(s^3 + s) = s^2 + 2$

$X(s) = \frac{s^2+2}{s^3+s} = \frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad / \cdot s(s^2+1)$

$s^2 + 2 = A(s^2+1) + (Bs+C)s$

$s^2 + 2 = As^2 + A + Bs^2 + Cs$

$A + B = 1 \Rightarrow B = 1 - 2$ $\boxed{B = -1}$

$\boxed{C = 0}$

$\boxed{A = 2}$

$X(s) = \frac{2}{s} + \frac{-s}{s^2+1} = 2 \cdot \frac{1}{s} - \frac{s}{s^2+1^2}$

$x(t) = 2 - \cos t$ ✓

$(\cos t)' = -\sin$

$x = 0 \quad 2 - \cos 0 = 2 - 1 = 1$ ✓

$x'(t) = \sin t \quad x = 0 \quad \sin 0 = 0$ ✓

$x''(t) = \cos t \quad x = 0 \quad \cos 0 = 1$ ✓

$x'''(t) = -\sin t$

$-\sin t + \sin t = 0$ ✓✓

② $y'''(t) - y(t) = t$; $y(0)=1$, $y''(0)=2$, $y'(0)=1$.

$s^3 Y(s) - s^2 \underbrace{y(0)}_{=1} - s \underbrace{y'(0)}_{=1} - \underbrace{y''(0)}_{=2} - Y(s) = \frac{1}{s^2}$

$s^3 Y(s) - s^2 - s - 2 - Y(s) = \frac{1}{s^2}$ ✓

$s^2 \cdot (s-1) = s^3 - s^2$

$s^3 Y(s) - Y(s) = \frac{1}{s^2} + s^2 + s + 2$

$Y(s)(s^3 - 1) = \frac{1 + s^4 + s^3 + 2s^2}{s^2}$

$Y(s) = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s^3-1)} = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s-1)(s^2+s+1)}$ ✓

$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{Ds+E}{s^2+s+1}$ ✓ / $s^2(s-1)(s^2+s+1)$

$s^4 + s^3 + 2s^2 + 1 = As(s-1)(s^2+s+1) + B(s-1)(s^2+s+1) + Cs^2(s^2+s+1) + (Ds+E)(s^3 - s^2)$

UVRSTITI $s = 0$ $0 + 0 + 0 + 1 = B(0-1)(0+0+1)$ $1 = -B$ $\boxed{B = -1}$

UVRSTITI $s = 1$ $1 + 1 + 2 + 1 = C(1+1+1)$ $5 = 3C$ $3C = 5 / \cdot \frac{1}{3}$ $\boxed{C = \frac{5}{3}}$ ✓

$s^4 + s^3 + 2s^2 + 1 = As(s^3-1) + B(s^3-1) + Cs^4 + Cs^3 + Cs^2 + Ds^4 - Ds^3 + Es^3 - Es^2$

$s^4 + s^3 + 2s^2 + 1 = \underline{As^4} \underline{- As} + \underline{Bs^3} - B + \underline{Cs^4} + \underline{Cs^3} + \underline{Cs^2} + \underline{Ds^4} - \underline{Ds^3} + \underline{Es^3} - \underline{Es^2}$

$s^4 + s^3 + 2s^2 + 1 = s^4(A + C + D) + s^3(B + C - D + E) + s^2(C - E) + s(-A) - B$

$A + C + D = 1$

$B + C - D + E = 1$

$C - E = 2 \Rightarrow \frac{5}{3} - E = 2$ $-E = \frac{6}{3} - \frac{5}{3}$ $-E = \frac{1}{3}$ $\boxed{E = -\frac{1}{3}}$ ✓

$-A = 0$ $\boxed{A = 0}$ ✓

$-B = 1 \Rightarrow \boxed{B = -1}$ ✓

$0 + \frac{5}{3} + D = 1$ $D = 1 - \frac{5}{3}$ $D = \frac{3}{3} - \frac{5}{3}$ $\boxed{D = -\frac{2}{3}}$ ✓

$-1 + \frac{5}{3} + \frac{2}{3} - \frac{1}{3} = 1$ ✓

$Y(s) = -\frac{1}{s^2} + \frac{\frac{5}{3}}{s-1} + \frac{-\frac{2}{3}s - \frac{1}{3}}{s^2+s+1}$ ✓

$s^2 + s + 1$

$s^2 + 2 \cdot s \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \frac{3}{4}$

$\left(s + \frac{1}{2}\right)^2 + \frac{3}{4}$

$Y(s) = -\frac{1}{s^2} + \frac{5}{3} \cdot \frac{1}{s-1} - \frac{2}{3} \frac{s - \frac{1}{2} + \frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{3} \frac{1}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$ ✓

$Y(s) = -\frac{1}{s^2} + \frac{5}{3} \cdot \frac{1}{s-1} - \frac{2}{3} \frac{s + \frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{2} \cdot \left(-\frac{2}{3}\right) \frac{1}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{3} \frac{1}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$y(t) = -t + \frac{5}{3}e^t - \frac{2}{3}\left(e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)\right) - \frac{1}{2}\left(\frac{1}{\frac{\sqrt{3}}{2}} e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)\right) - \frac{1}{3}\left(\frac{1}{\frac{\sqrt{3}}{2}} e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)$

ZA $y=0$ $-0 + \frac{5}{3} - \frac{2}{3}(e^0 \cos 0) - \frac{1}{3}(0) - \frac{1}{3}(0)$

$\frac{5}{3} - \frac{2}{3} = \frac{3}{3} = 1$ ✓

MATEMATIKA 3 - KOLOKVIJ #3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ANTE ŠUŠNJARA **BROJ INDEKSA:** 57679

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

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2. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t) - y(t) = t, \quad y(0) = 1, \; y''(0) = 2, \; y'(0) = 1.$$

10

Ukupno: 10

1) $x'''(t) + x'(t) = 0$

$x(0) = x''(0) = 1$

$x'(0) = 0$

$s^3F(s) - s^2 \cancel{x(0)}^{1} - s\cancel{x'(0)}^{0} - \cancel{x''(0)}^{1} = s^3F(s) - s^2 - 1$

$sF(s) - \cancel{x(0)}^{1} = sF(s) - 1$

$0 = 0$

$s^3F(s) - s^2 - 1 + sF(s) - 1 = 0$

$s^3F(s) + sF(s) = s^2 + 2$

$F(s)(s^3 + s) = s^2 + 2 \;/ : (s^3+s)$

$F(s) = \frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{(s^2+1)} \;/ \cdot s(s^2+1)$

$s^2 + 2 = A(s^2+1) + Bs^2 + Cs$

$= As^2 + A + Bs^2 + Cs$

$s^0 \Rightarrow 2 = A$

$s^1 \Rightarrow 0 = C$

$s^2 \Rightarrow 1 = A + B$

$A = 2$

$C = 0$

$2 + B = 1$

$B = 1 - 2$

$B = -1$

$\sqrt{1} = 1$

$F(s) = 2 \cdot \frac{1}{s} + -1 \frac{s}{(s^2+1)} + 0$

$= 2 * \cos t$ ✓

$$y'''(t) - y(t) = t$$

$$y(0) = 1$$
$$y'(0) = 1$$
$$y''(0) = 2$$

$$s^3F(s) - s^2 \cdot 1 - s \cdot 1 - 2 = s^3F(s) - 1 - 1 - 2$$

$$sF(s) - 1 = sF(s) - 1$$

$$t = \frac{1}{s^2}$$

$$s^3F(s) - 4 - sF(s) + 1 = \frac{1}{s^2}$$

$$s^3F(s) - sF(s) = \frac{1}{s^2} + 3 \qquad \frac{1+3s^2}{s^2}$$

$$F(s)(s^3 - s) = \frac{1+3s^2}{s^2} \;/\cdot (s^3 - s)$$

$$F(s) = \frac{1+3s^2}{s^3(s^2-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{(s^2-1)} \;/\cdot s^3(s^2-1)$$

$$1 + 3s^2 = As^2(s^2-1) + Bs(s^2-1) + C(s^2-1) + Ds^4 + Es^3$$
$$= As^4 - As^2 + Bs^3 - Bs + Cs^2 - C + Ds^4 + Es^3$$

$s^0 \Rightarrow 1 = -C \quad C = -1$

$s^1 \Rightarrow 0 = -B \quad B = 0$

$s^2 \Rightarrow 3 = -A + C$

$s^3 \Rightarrow 0 = B + E \quad E = 0$

$s^4 \Rightarrow 0 = A + D$

$-A + C = 3$

$-A - 1 = 3$

$-A = 3 \;/\cdot(-1)$

$A = -3$

$-3 + D$

$A + D = 0$

$D = 3$

$$F(s) = -3 \cdot \frac{1}{s} + 0 + \frac{1}{s^3} + \frac{3s}{s^2-1} + 0$$

$$= -3 \cdot \frac{1}{s} + \frac{1}{s^3} + 3 \cdot \frac{s}{s^2-1}$$

$\sqrt{1} = 1$

$1^2 = 1$

$$F(s) = -3 - \frac{1}{2}t^2 + 3\cos t$$

MATEMATIKA 3 - KOLOKVIJ #3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: LUKA KURILIĆ **BROJ INDEKSA:** 58076

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

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2. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t) - y(t) = t, \quad y(0) = 1,\ y''(0) = 2,\ y'(0) = 1.$$

10

Ukupno: 10

(1)

$x''' \Rightarrow s^3 X(s) - s^2 x(0) - s^1 x'(0) - x''(0)$

$\boxed{x''' \Rightarrow s^3 X(s) - s^2 - 1}$

$x' \Rightarrow s^1 X(s) - x(0)$

$\boxed{x' \Rightarrow s X(s) - 1}$

$s^3 X(s) - s^2 - 1 + s X(s) - 1 = 0$

$s^3 X(s) + s X(s) = s^2 + 1 + 1$

$X(s)(s^3 + s) = s^2 + 2$

$X(s) = \frac{s^2+2}{s^3+s}$

$X(s) = \frac{s^2+2}{s(s^2+1)}$

$s^2 + 2 = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad / \ s(s^2+1)$

$= A(s^2+1) + (Bs+C)s$

$= As^2 + A + Bs^2 + Cs$

$s^2 \quad 1 = A + B$

$s^1 \quad 0 = C$

$s^0 \quad 2 = A$

$\boxed{A = 2}$

$\boxed{C = 0}$

$\boxed{B = -1}$

$A + B = 1$

$2 + B = 1$

$B = 1 - 2$

$\boxed{B = -1}$

$f(s) = \frac{2}{s} + \frac{-1s+0}{s^2+1}$

$= \frac{2}{s} - \frac{s}{s^2+1}$

$\boxed{f(t) = 2 - \cos t}$

② $y'''(t) - y'(t) = t$ POGREŠNA JEDNADŽBA $y(0)=1,\ y''(0)=2,\ y'(0)=1$

$y''' \Rightarrow s^3 y(s) - s^2 y(0) - s^1 y'(0) - y''(0)$

$\boxed{y''' \Rightarrow s^3 y(s) - s^2 - s - 2}$ ✓

$y' \Rightarrow s^1 y(s) - y(0)$

$\boxed{y' \Rightarrow s\,y(s) - 1}$ ✓

$\Rightarrow$ DRUGI PAPIR

$s^3 y(s) - s^2 - s - 2 - s\,y(s) + 1 = \frac{1}{s^2}$

$s^3 y(s) - s\,y(s) = \frac{1}{s^2} + s^2 + s + 2 - 1$

$y(s)(s^3 - s) = \frac{1 + s^4 + s^3 + s^2}{s^2}$

$y(s)(s^3 - s) = \frac{s^4 + s^3 + s^2 + 1}{s^2}$ ✓

$y(s) = \frac{s^4 + s^3 + s^2 + 1}{s^2(s^3 - s)}$

$y(s) = \frac{s^4 + s^3 + s^2 + 1}{s^3(s^2 - 1)}$ ✓

$s^4 + s^3 + s^2 + 1 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2 - 1}$ ✓ $/\ s^3(s^2-1)$

$= A(s^2(s^2-1)) + B(s(s^2-1)) + C(s^2-1) + (Ds+E)s^3$

$= A(s^4 - s^2) + B(s^3 - s) + C(s^2-1) + Ds^4 + Es^3$

$= As^4 - As^2 + Bs^3 - Bs + Cs^2 - C + Ds^4 + Es^3$

$s^4 \quad 1 = A + D$

$s^3 \quad 1 = B + E$

$s^2 \quad 1 = -A + C$

$s^1 \quad 0 = -B$

$s^0 \quad 1 = -C$

$\boxed{C = -1}$

$\boxed{B = 0}$

$1 = -A + C$

$1 = -A - 1$

$-A - 1 = 1$

$-A = 2$

$\boxed{A = -2}$

$1 = B + E$

$1 = 0 + E$

$\boxed{E = 1}$

$1 = A + D$

$1 = -2 + D$

$-2 + D = 1$

$\boxed{D = 3}$

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② *NASTAVAK

$A=-2$
$B=0$
$C=-1$
$D=3$
$E=1$

$$\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s^3}+\frac{Ds+E}{s^2-1}$$

$$f(s)=\frac{-2}{s}+\frac{0}{s^2}-\frac{1}{s^3}+\frac{3s+1}{s^2-1}$$

$$=\frac{-2}{s}-\frac{1}{s^3}+\frac{3s}{s^2-1^2}+\frac{1}{s^2-1^2}$$

$$f(t)=-2-\frac{2}{2}\cdot\frac{1}{s^3}+\cosh 3t+\sinh t$$

$$f(t)=-2-\frac{1}{2}t^2+\cosh 3t+\sinh t$$

PROVJERA ?