MATEMATIKA 3 - KOLOKVIJ #2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: HRVOJE BATUR **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr. $\int_0^1 \int_0^{x+1} x + \cos y \, \mathrm{d}y \mathrm{d}x$). Nije potrebno integral riješavati do kraja.

1. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{7}{2},0)$, $B(7,\frac{2}{2})$ i $C(\frac{7}{2},\frac{6}{2})$. Izračunati dvostruki integral 10

$$\iint_X x^3 \mathrm{d}x \mathrm{d}y$$

2. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z \geq 1$. 10

3. Izračunati 10

$$\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$$

4. Zadana je kružna uzvojnica (spirala) s jednadžbama $x = 2\cos t$, $y = 2\sin t$ i $z = t$. Skiciraj krivulju. Izračunati duljinu 3 namotaja ove krivulje. (pomoć: jedan namotaj odgovara peridu iskorištenih trigonometrijskih funkcija) 10

Ukupno: 40

Tablica integrala (zapravo ti ne treba)

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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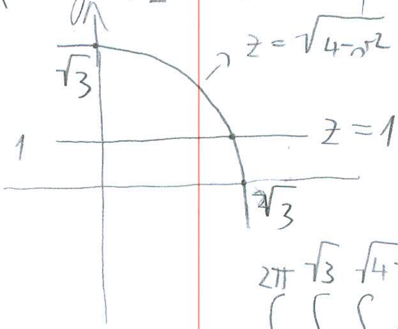
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \coth x \,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

② $x^2+y^2+z^2=4$ $\quad z \geq 1$ $\quad\quad z^2+r^2=4 \Rightarrow z^2=4-r^2$

$z=\sqrt{4-r^2}$



$z\in[1,\sqrt{4-r^2}]$

$r\in[0,\sqrt{3}]$

$\varphi\in[0,2\pi]$

$r^2+1=4$

$r^2=3$

$r=\sqrt{3}$

$$=\int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} 1\cdot r\,dz\,dr\,d\varphi$$

③ $\displaystyle\int_{(3,2)}^{(5,5)} x\,dy+y\,dx=\int_{(3,2)}^{(5,5)} y\,dx+x\,dy$

$\frac{\partial f}{\partial x}=-y \quad / \int dx$

$f(x,y)=-yx+c(y)$

$\frac{\partial f}{\partial y}=-x$

$\cancel{-x}+c(y)=\cancel{-x}$

$c(y)=0$

$f(x,y)=-yx$

$f(3,2)-f(5,5)=-6-(-25)$

$=-6+25$

$=19$

④ $x=2\cos t$

$y=2\sin t$

$z=t$

$t\in[0,2\pi]$

$r=\begin{pmatrix}2\cos t\\ 2\sin t\\ t\end{pmatrix}$ $\quad r'=\begin{pmatrix}-2\sin t\\ 2\cos t\\ 1\end{pmatrix}$

$\|r'(t)\|=\sqrt{(-2\sin t)^2+(2\cos t)^2+1}$

$=\sqrt{4\sin^2 t+4\cos^2 t+1}$

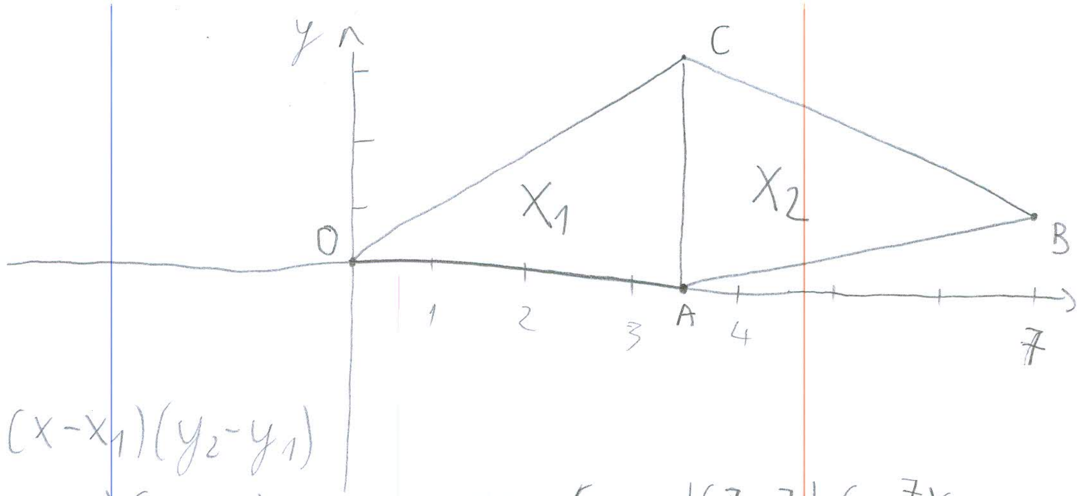
$=\sqrt{4(\sin^2 t+\cos^2 t)+1}$

$=\sqrt{5}$

$$=3\cdot\int_0^{2\pi}\sqrt{5}\,dt$$

HRVOJE BATUR

① $X \ldots$ $O(0,0)$
$A(\frac{7}{2},0)$
$B(7,1)$
$C(\frac{7}{2},3)$



OA: $(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$

$(y-0)(\frac{7}{2}-0)=(x-0)(0-0)$

$\frac{7}{2}y=0$

$\boxed{y=0}$

AB: $(y-0)(7-\frac{7}{2})=(x-\frac{7}{2})(1-0)$

$\frac{7}{2}y=x-\frac{7}{2}$

$\frac{7}{2}y=x-\frac{7}{2} \quad / : \frac{7}{2}$

$\boxed{y=\frac{2}{7}x-1}$ ✓

CB: $(y-3)(7-\frac{7}{2})=(x-\frac{7}{2})(1-3)$

$(y-3)\frac{7}{2}=-2x+7$

$\frac{7}{2}y-\frac{21}{2}=-2x+7$

$\frac{7}{2}y=-2x+\frac{35}{2} \quad / : \frac{7}{2}$

$\boxed{y=-\frac{4}{7}x+5}$ ✓

OC: $(y-0)(\frac{7}{2}-0)=(x-0)(3-0)$

$\frac{7}{2}y=3x \quad / : \frac{7}{2}$

$\boxed{y=\frac{6}{7}x}$ ✓

$$\iint x^3\,dx\,dy=\iint X_1+\iint X_2$$

$$X_1 \ldots \int_0^{\frac{7}{2}}\int_{OA}^{OC} x^3\,dy\,dx=\int_0^{\frac{7}{2}}\int_0^{\frac{6}{7}x} x^3\,dy\,dx$$

$$X_2 \ldots \int_{\frac{7}{2}}^{7}\int_{AB}^{CB} x^3\,dy\,dx=\int_{\frac{7}{2}}^{7}\int_{\frac{2}{7}x-1}^{-\frac{4}{5}x+5} x^3\,dy\,dx$$

$$\iint_X x^3\,dx\,dy=\int_{\frac{7}{2}}^{7}\int_{OA}^{OC} x^3\,dy\,dx+\int_{\frac{7}{2}}^{7}\int_{AB}^{CB} x^3\,dy\,dx$$

MATEMATIKA 3 - KOLOKVIJ #2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK
IME I PREZIME: ANTE ŠUŠNJARA **BROJ INDEKSA:**	Broj ↓ bodova

Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr. $\int_0^1 \int_0^{x+1} x + \cos y \, \mathrm{d}y\mathrm{d}x$). Nije potrebno integral riješavati do kraja.

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2. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 4$ za koji vrijedi $z \geq 1$. 10

3. Izračunati $$\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$$ 10

4. Zadana je kružna uzvojnica (spirala) s jednadžbama $x = 2\cos t$, $y = 2\sin t$ i $z = t$. Skiciraj krivulju. Izračunati duljinu 3 namotaja ove krivulje. (pomoć: jedan namotaj odgovara peridu iskorištenih trigonometrijskih funkcija) 10

Ukupno: 30

3) $w = \begin{bmatrix} y \\ x \end{bmatrix} = -\text{grad} f$

$F_{dx} = -y \,/\!\int$

$F = -y \int dx$

$\boxed{F = -xy + C_y}$

$d_y F = -x$

$d_y(-xy + C_y) = -x$

$-x + d_y C_y = -x$

$d_y C_y = -x + x \,/\!\int dy$

$C_y = 0 \quad \boxed{F = -xy}$

$\int_{3,2}^{5,5} -3\cdot 2 - (-5\cdot 5)$

$= -6 + 25$

$= 19$ ✓

Tablica integrala (zapravo ti ne treba)

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4)

1 namotaj $= 2\pi \quad 3 \cdot 2\pi = 6\pi$

$$r(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ t \end{pmatrix} \quad r'(t) = \begin{pmatrix} -2\sin t \\ 2\cos t \\ 1 \end{pmatrix}$$

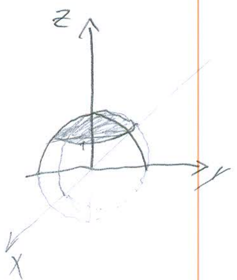
ANTE ŠUŠNJARA

$$|r'(t)| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 1^2}$$

$$= \sqrt{5}$$

$$\int_0^{6\pi} \sqrt{5}\, dt = 6\pi\sqrt{5}$$

2)



$$x^2 + y^2 + z^2 = 4$$

$r^2 = 4 \ /\sqrt{}$

$\boxed{r = 2}$

$R^2 + z^2 = 4$

$R = 4 - z^2 \ /\sqrt{}$

$R = \sqrt{4 - z^2}$

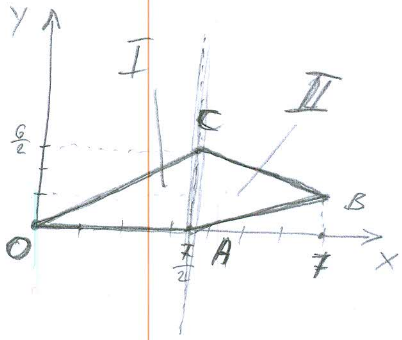
$z \in [1, 2]$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{4 - z^2}]$

$$\int_0^{2\pi} \int_1^2 \int_0^{\sqrt{4-z^2}} r\, dr\, dz\, d\varphi$$

1)



$OA \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{\cancel{0} - 0}{\cancel{\frac{7}{2}} - 0}(x - 0)$

$\boxed{y = 0}$

	x	y
~~O~~	~~0~~	~~0~~
~~A~~	~~$\frac{7}{2}$~~	~~0~~
B	7	1
C	$\frac{7}{2}$	3

$AB \Rightarrow y - 0 = \frac{1 - 0}{7 - \frac{7}{2}}(x - \frac{7}{2})$

$y = \frac{1}{\frac{7}{2}}(x - \frac{7}{2})$

$y = \frac{7}{2}(x - \frac{7}{2})$ ✗

$\boxed{y = \frac{7}{2}x - \frac{49}{4}}$

$3 + \frac{1}{2} = \frac{6+1}{2} =$

$7 - \frac{7}{2} = \frac{14 - 7}{2} =$

$BC \Rightarrow y - 1 = \frac{3 - 1}{\frac{7}{2} - 7}(x - 7)$

$\frac{7}{2} - \frac{7}{1} = \frac{7 - 14}{2}$

$y - 1 = \frac{2}{-\frac{7}{2}}(x - 7)$

$y - 1 = -\frac{7}{4}(x - 7)$

$y = -\frac{7}{4}x + \frac{49}{4} + 1$

$y = -\frac{7}{4}x + \frac{49 + 4}{4}$

$\boxed{y = -\frac{7}{4}x + \frac{53}{4}}$

$OC \Rightarrow y - 0 = \frac{3 - 0}{\frac{7}{2} - 0}(x - 0)$

$y = \frac{\frac{3}{1}}{\frac{7}{2}} x$

$\boxed{y = \frac{7}{6}x}$ ✗

$$\boxed{\int_0^{\frac{7}{2}} \int_0^{\cancel{\frac{7}{6}x}} x^3\, dy\, dx + \int_{\frac{7}{2}}^{7} \int_{\cancel{\frac{7}{2}x - \frac{49}{4}}}^{-\frac{7}{4}x + \frac{53}{4}} x^3\, dy\, dx}$$

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$\int \tan x \, dx = -\ln \|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x \, dx = \ln \|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

1. $O(0,0)$, $A(\underbrace{\tfrac{7}{2}}_{=3.5}, 0)$, $B(7, \underbrace{\tfrac{2}{2}}_{=1})$; $C(\underbrace{\tfrac{7}{2}}_{=3.5}, \underbrace{\tfrac{6}{2}}_{=3})$

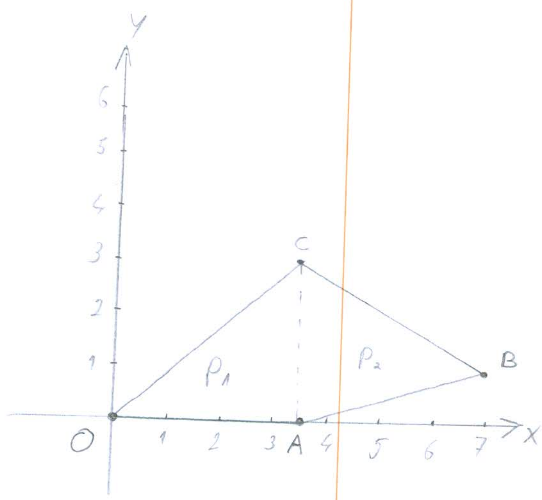
$X =$ ČETVEROKUT $\qquad \iint\limits_X x^3 \, dx\, dy$

$\overline{OC}$ ... $(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$ $\quad O(0,0)$ $\;C(\tfrac{7}{2}, 3)$

$$(y - 0)(\tfrac{7}{2} - 0) = (3 - 0)(x - 0)$$

$$\tfrac{7}{2} y = 3x \;/\cdot \tfrac{7}{2} \quad \cdot \tfrac{2}{7}$$

$$y = \tfrac{21}{2} x$$



$\overline{BC}$ ... $B(7, 1)$ $\;C(\tfrac{7}{2}, 3)$

$$(y - 1)(\tfrac{7}{2} - 7) = (3 - 1)(x - 7)$$

$$-\tfrac{7}{2} y + \tfrac{7}{2} = 2x - 14$$

$$-\tfrac{7}{2} y = 2x - 14 - \tfrac{7}{2}$$

$$-\tfrac{7}{2} y = 2x - \tfrac{35}{2} \;/\cdot(-\tfrac{7}{2})$$

$$y = -7x + \tfrac{245}{4}$$

$\tfrac{7}{2} - 7 = \tfrac{7 - 14}{2} = -\tfrac{7}{2}$

$-14 - \tfrac{7}{2} = \tfrac{-28 - 7}{2} = -\tfrac{35}{2}$

$2 \cdot (-\tfrac{7}{2}) = -\tfrac{14}{2} = -7$

$-\tfrac{35}{2} \cdot (-\tfrac{7}{2}) = \tfrac{245}{4}$

$7 - \tfrac{7}{2} = \tfrac{14 - 7}{2} = \tfrac{7}{2}$

$-\tfrac{7}{2} \cdot \tfrac{7}{2} = -\tfrac{49}{2}$

$\overline{AB}$ ... $(\tfrac{7}{2}, 0)$ $\;B(7, 1)$

$$(y - 0)(7 - \tfrac{7}{2}) = (1 - 0)(x - \tfrac{7}{2})$$

$$\tfrac{7}{2} y = x - \tfrac{7}{2} \;/\cdot \tfrac{7}{2}$$

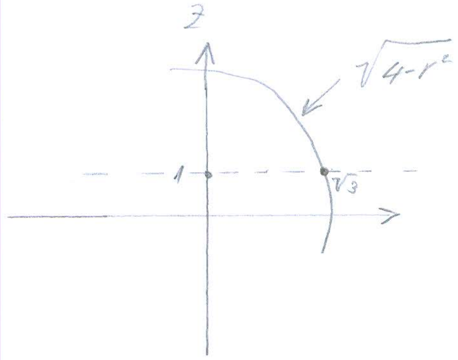
$$y = \tfrac{7}{2} x - \tfrac{49}{2}$$

$$I = \int_0^{3.5} \int_0^{\frac{21}{2}x} x^3 \, dy\, dx + \int_{3.5}^{7} \int_{\frac{7}{2}x - \frac{49}{2}}^{-7x + \frac{245}{4}} x^3 \, dy\, dx$$

(2.) $x = r\cos\varphi$ $\quad x^2+y^2+z^2=4 \quad , \quad z \geq 1$

$y = r\sin\varphi$

$z = z$



$\varphi \in [0, 2\pi]$

$x^2+y^2+z^2=4$

$r^2+z^2=R^2$

$r^2+z^2=2^2$

$z = \pm\sqrt{2^2-r^2}$

PRESJEČIŠTE

$r^2+z^2=4$ UVRSTITI $z=1$

$r^2+1^2=4$

$r^2=4-1$

$r^2=3$

$r=\sqrt{3}$

$z \in [1, \sqrt{4-r^2}]$

$r \in [0, \sqrt{3}]$

$$V = \int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r \, dz\, dr\, d\varphi$$ ✓

(3.) IZRAČUNATI :
$$\int_{(3,2)}^{(5,5)} x\,dy + y\,dx$$

$$w = \begin{pmatrix} x \\ y \end{pmatrix} = -\operatorname{grad} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$\frac{\partial f}{\partial x} = -x \;/\int dx \quad \Rightarrow \int -x\,dx = -\frac{x^2}{2} + C(y)$

$\frac{\partial f}{\partial y} = -y \;/\int dy \quad 0 + \frac{\partial C(y)}{\partial y} = -y \quad C'(y) = -y \;/\int dy = -\frac{y^2}{2} + C_{=0}$

$f = -\frac{x^2}{2} - \frac{y^2}{2}$

$f = f(3,2) - f(5,5) = \left(-\frac{3^2}{2} - \frac{2^2}{2}\right) - \left(-\frac{5^2}{2} - \frac{5^2}{2}\right) = \ldots$

4.) KRUŽNA UZVOJNICA , $t \in [0, 6\pi]$, $x = 2\cos t$, $y = 2\sin t$, $z = t$, $\ell = ?$

$$r(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ t \end{pmatrix} \qquad r'(t) = \begin{pmatrix} -2\sin t \\ 2\cos t \\ 1 \end{pmatrix}$$

$(\cos t)' = -\sin t$

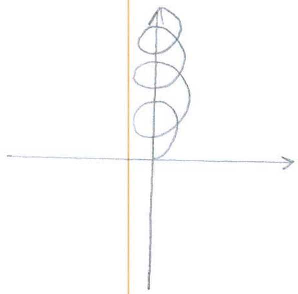
$(\sin t)' = \cos t$

$$\| r'(t) \| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 1^2} = \sqrt{4\sin^2 t + 4\cos^2 t + 1} = \sqrt{4\underbrace{(\sin^2 t + \cos^2 t)}_{=1} + 1}$$

$$= \sqrt{4+1} = \sqrt{5}$$

$$\ell = \int_0^{6\pi} \sqrt{5} \cdot dt$$

SKICA :



3.) $$\int_{(3,2)}^{(5,5)} y\,dx + x\,dy$$

$$\begin{pmatrix} y \\ x \end{pmatrix} = -\operatorname{grad} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = -y \;/\!\int dx \qquad \int -y\,dx = -y\int dx = -yx + C(y)$$

$$\frac{\partial f}{\partial x} = -x \qquad \cancel{-x} + \frac{\partial C(y)}{\partial y} = \cancel{-x}$$

$$f = -yx + \underset{=0}{C} \qquad \underline{f = -yx}$$

$$f = f(3,2) - f(5,5) = -(2\cdot 3) + (5\cdot 5)$$

MATEMATIKA 3 - KOLOKVIJ #2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: LOVRE LOVRIĆ **BROJ INDEKSA:** 58080

POPUNJAVA NASTAVNIK
Broj ↓ bodova

`Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr.` $\int_0^1 \int_0^{x+1} x + \cos y \, dy dx$`). Nije potrebno integral riješavati do kraja.`

1. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{7}{2},0)$, $B(7,\frac{2}{2})$ i $C(\frac{7}{2},\frac{6}{2})$. Izračunati dvostruki integral ~~10~~

$$\iint_X x^3 dx dy$$

2. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z \geq 1$. 10

3. Izračunati ~~10~~

$$\int_{(3,2)}^{(5,5)} x dy + y dx$$

4. Zadana je kružna uzvojnica (spirala) s jednadžbama $x = 2\cos t$, $y = 2\sin t$ i $z = t$. Skiciraj krivulju. Izračunati duljinu 3 namotaja ove krivulje. (pomoć: jedan namotaj odgovara peridu iskorištenih trigonometrijskih funkcija) 10

Ukupno: 20

4.) $x = 2\cos t$
$y = 2\sin t$
$z = t$

$$r = \begin{bmatrix} 2\cos t \\ 2\sin t \\ t \end{bmatrix} \Rightarrow r' = \begin{bmatrix} -2\sin t \\ 2\cos t \\ 1 \end{bmatrix}$$

$t \in [0, 6\pi]$

$$\|r'\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 1^2}$$

$$\|r'\| = \sqrt{4\sin^2 t + 4\cos^2 t + 1}$$

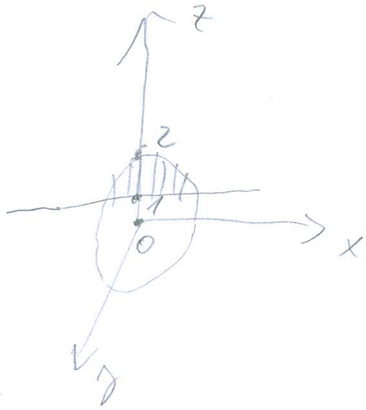
$\Rightarrow$ NASTAVAK

Tablica integrala (zapravo ti ne treba)

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\, dx = -\cos x + C$	$\int \tanh x\, dx = \ln\|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\, dx = \sin x + C$	$\int \coth x\, dx = \ln\|\sinh x\|$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\, dx = \ln\|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

4) $\int_0^{6\pi} \sqrt{4\sin^2 t + 4\cos^2 t + 1}\, dt =$

2.) $x^2+y^2+z^2=4$ $\quad z \geq 1$



$r^2+z^2=4$ $\quad z \in [1,2]$

$r^2 = 4-z^2$ $\quad r \in [0, \sqrt{4-z^2}]$

$r = \sqrt{4-z^2}$ $\quad \varphi \in [0, 2\pi]$

$$\int_0^{2\pi}\int_1^2\int_0^{\sqrt{4-z^2}} r \, dr \, dz \, d\varphi = \checkmark$$

JEDNADŽBA ZA KUGLU

$x^2+y^2+z^2=R^2$

$x^2+y^2+z^2=2^2$ $\Rightarrow$ radijus je 2

$x^2+y^2+z^2=$ stoga je $z \in [1,2]$

3.) $$\int_{(3,2)}^{(5,5)} x\,dy + y\,dx = \int_{(3,2)}^{(5,5)} y\,dx + x\,dy$$

$= f(3,2) - f(5,5)$

$= -6-(-25) = 19$

$W = \begin{bmatrix} y \\ x \end{bmatrix} = -\operatorname{grad} f$

$\frac{\partial f}{\partial x} = -y \quad / \int dx$

$f = -yx + C(y) \Rightarrow f = -yx$ ✓

$\frac{\partial f}{\partial y} = -x$

$\frac{\partial(-yx + C(y))}{\partial y} = -x$

$-x + C(y) = -x$

$C(y) = 0$

$f = f_1 - f_2 = \underset{x_1 y_1}{(3,2)} - \underset{x_2 y_2}{(5,5)} =$

$= -(2\cdot 3) - (5\cdot 5) = -6-25 = \underline{\underline{-31}}$

?

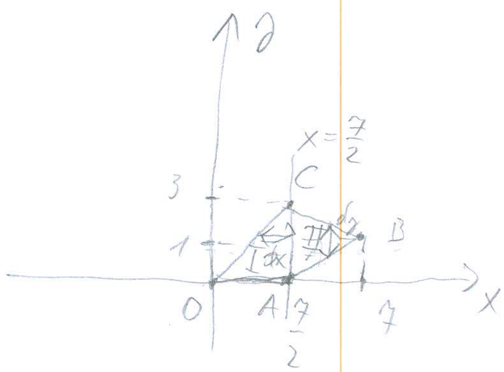
LOVRE LOVRIĆ 58080

1.) $O(0,0)$ $\quad X \to$ četverokut

$A\left(\frac{7}{2},0\right)$

$B(7,1)$ $\quad \iint\limits_X x^3\,dx\,dy$

$C\left(\frac{7}{2},3\right)$



OA:

$(y-0)\cdot\left(\frac{7}{2}-0\right)=(x-0)\cdot(0-0)$

$y=0$

AB:

$(y-0)\cdot\left(7-\frac{7}{2}\right)=\left(x-\frac{7}{2}\right)\cdot(1-0)$

$\frac{7}{2}y=x-\frac{7}{2} \quad /:\frac{7}{2}$

$y=\frac{2}{7}x-1$

OC:

$(y-0)\cdot\left(\frac{7}{2}-0\right)=(x-0)\cdot(3-0)$

$\frac{7}{2}y=3x$

$y=\frac{3x}{\frac{7}{2}}$

$y=\frac{6}{7}x$

$x=\frac{7}{6}y$

BC:

$(y-1)\cdot\left(\frac{7}{2}-7\right)=(x-7)\cdot(3-1)$

$(y-1)\cdot\left(-\frac{7}{2}\right)=2x-14$

$-\frac{7}{2}y+\frac{7}{2}=2x-14$

$-\frac{7}{2}(y-1)=2(x-7)$

$(y-1)=\frac{2(x-7)}{-\frac{7}{2}}$

$y-1=-\frac{4(x-7)}{7}$

$y=-\frac{4}{7}(x-7)+1$

$y=-\frac{4}{7}x+5$

$$\mathrm{I}: \int_0^{3.5}\int_{\frac{7}{6}y}^{\frac{7}{2}} x^3\,dx\,dy$$

$$\mathrm{II}: \int_{\frac{7}{2}}^{7}\int_{\frac{2}{7}x-1}^{-\frac{4}{7}x+5} x^3\,dy\,dx$$

$$\iint\limits_X x^3\,dx\,dy=\mathrm{I}+\mathrm{II}=\int_0^{3.5}\int_{\frac{7}{6}y}^{\frac{7}{2}} x^3\,dx\,dy+\int_{\frac{7}{2}}^{7}\int_{\frac{2}{7}x-1}^{-\frac{4}{7}x+5} x^3\,dy\,dx$$

MATEMATIKA 3 - KOLOKVIJ #2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: LUKA KURILIĆ **BROJ INDEKSA:** 58076

POPUNJAVA NASTAVNIK
Broj ↓ bodova

Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr. $\int_0^1 \int_0^{x+1} x + \cos y \, dy dx$). Nije potrebno integral riješavati do kraja.

1. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{7}{2}, 0)$, $B(7, \frac{2}{2})$ i $C(\frac{7}{2}, \frac{6}{2})$. Izračunati dvostruki integral ~~10~~

$$\iint_X x^3 \, dx dy$$

2. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2 + y^2 + z^2 = 4$ za koji vrijedi $z \geq 1$. ~~10~~
3. Izračunati 10

$$\int_{(3,2)}^{(5,5)} x dy + y dx$$

4. Zadana je kružna uzvojnica (spirala) s jednadžbama $x = 2\cos t$, $y = 2\sin t$ i $z = t$. Skiciraj krivulju. Izračunati duljinu 3 namotaja ove krivulje. (pomoć: jedan namotaj odgovara peridu iskorištenih trigonometrijskih funkcija) 10

Ukupno: 20

4

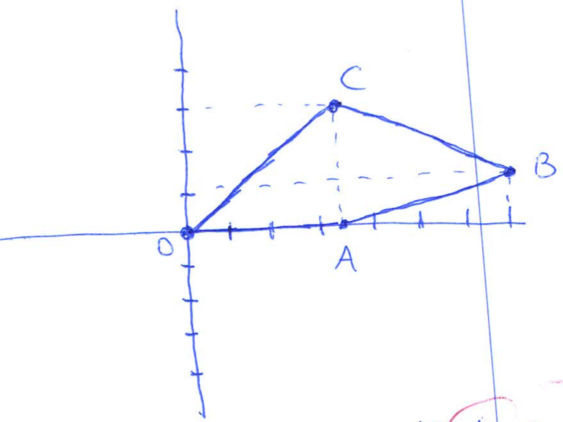
$$r(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ t \end{pmatrix} \qquad r'(t) = \begin{pmatrix} -2\sin t \\ 2\cos t \\ 1 \end{pmatrix}$$

$$|r'(t)| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 1}$$
$$= \sqrt{4\sin^2 t + 4\cos^2 t + 1}$$
$$= \sqrt{4(\underbrace{\sin^2 t + \cos^2 t}_{1}) + 1}$$
$$= \sqrt{5}$$

$$\int_0^{6\pi} \sqrt{5} \, dt$$
$$\sqrt{5} \int_0^{6\pi} dt$$

Tablica integrala (zapravo ti ne treba)

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n \, dx = \frac{x^{n+1}}{n+1}, \; n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x \, dx = \frac{a^x}{\ln a} + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x \, dx = -\cos x + C$	$\int \tanh x \, dx = \ln \lvert \cosh x \rvert$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, dx = \sin x + C$	$\int \coth x \, dx = \ln \lvert \sinh x \rvert$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \tan x \, dx = -\ln \lvert \cos x \rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x \, dx = \ln \lvert \sin x \rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$



O (0,0)
$A\left(\frac{7}{2},0\right)$
$B(7,1)$
$C\left(\frac{7}{2},3\right)$

$$\iint_X x^3\,dx\,dy$$

$$\int\int_0^{\cancel{\frac{6}{7}x-3}} x^3\,dx\,dy + \int_{\frac{7}{2}}^{7}\int_{\frac{2}{7}x}^{\frac{4}{7}x-3} x^3\,dx\,dy$$

$\overline{OA}$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-0=\frac{0-0}{\frac{7}{2}-0}(x-0)$$
$$y=\frac{0}{\frac{7}{2}}(x)$$
$$\boxed{y=0} \Leftarrow OA$$

$$\Rightarrow y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-0=\frac{1-0}{7-\frac{7}{2}}\left(x-\frac{7}{2}\right)$$
$$y=\frac{x-\frac{7}{2}}{\frac{14}{2}-\frac{7}{2}}$$
$$y=\frac{x-\frac{7}{2}}{\frac{7}{2}}$$
$$y=\frac{x}{\frac{7}{2}}-\frac{\frac{7}{2}}{\frac{7}{2}} = \frac{2}{7}x-1$$
$$\boxed{y=\frac{2}{7}x} \Leftarrow AB$$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-0=\frac{3-0}{\frac{7}{2}-0}\left(x-\frac{7}{2}\right)$$
$$y=\frac{3\left(x-\frac{7}{2}\right)}{\frac{7}{2}}$$
$$y=\frac{3x-\frac{21}{2}}{\frac{7}{2}}$$
$$y=\frac{3x}{\frac{7}{2}}-\frac{\frac{21}{2}}{\frac{7}{2}}$$
$$y=\frac{3x}{\frac{7}{2}}-3$$
$$\boxed{y=\frac{6}{7}x-3} \Rightarrow \overline{OC}$$

$$\overline{BC} \to y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-1=\frac{3-1}{\frac{7}{2}-7}(x-7)$$
$$y-1=\frac{2}{\frac{7}{2}-\frac{14}{2}}(x-7)$$
$$y-1=\frac{2x-14}{\frac{7}{2}}$$
$$y-1=\frac{2x}{\frac{7}{2}}-\frac{14}{\frac{7}{2}}$$
$$y-1=\frac{4}{7}x-\frac{28}{7}$$
$$y-1=\frac{4}{7}x-4+1$$
$$\boxed{y=\frac{4}{7}x-3} \Rightarrow BC$$

$$\int_{(3,2)}^{(5,5)} x\,dy + y\,dx$$

$$\begin{bmatrix} y \\ x \end{bmatrix} = -\operatorname{grad} f$$

$$dx\, f = -y \,/ \int$$

$$f = \int -y\,dx$$

$$f = -y \int dx$$

$$\boxed{f = -yx + c(y)}$$

$$dy\, f = -x$$

$$dy\,(-yx + c(y)) = -x$$

$$dy\,(-x) + c(y) = -x$$

$$\cancel{c(y) = \int (-x + x)\,dy}$$

$$\cancel{c(y) = \int 0\,dy}$$

$$\cancel{dy\, c(y) = 0}$$

$$\cancel{\int c(y) = 0 \int dy}$$

$$\cancel{\int c(y) = 0}$$

$$c(y) = 0 \int dy$$

$$c(y) = 0$$

$$f = -yx + 0$$

$$\boxed{f = -yx}$$

$$= (-2 \cdot 3) - (-5 \cdot 5)$$

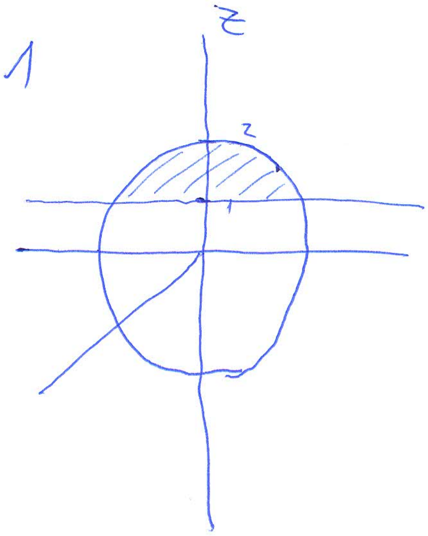
$$= -6 - (-25)$$

$$= -6 + 25$$

$$= \boxed{19}$$

✓

② $x^2+y^2+z^2=4$ $\quad z \geq 1$



$r^2+z^2=16$ ×

$r^2=16-z^2$

$\boxed{r=\sqrt{16-z^2}}$

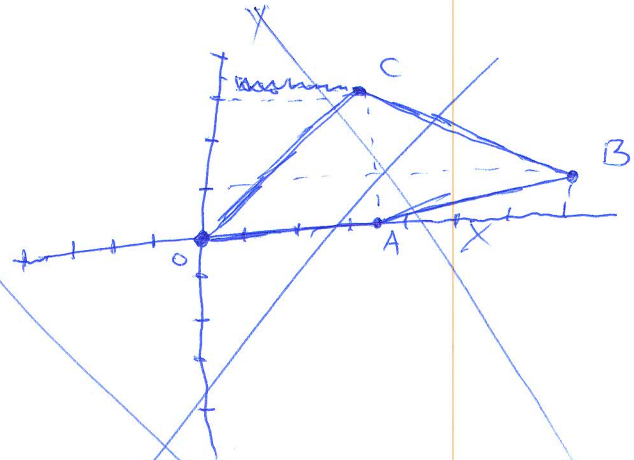
$x^2+y^2=4$

$r^2=4$

$\boxed{r=2}$

$\int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{16-z^2}} r\,dr\,d\varphi\,dz$ ×

① $\iint_X x^3\,dx\,dy$



$F=\begin{pmatrix}0\\0\\x^3\end{pmatrix}$

$F'=\begin{pmatrix}0\\0\\0\end{pmatrix}$

$\operatorname{div} F = 0+0+0=0$

$\iiint 0\,dx\,dy\,dz = 0$

Nevalja

$O(0,0)$

$A(\frac{7}{2},0)$

$B(7,1)$

$C(\frac{7}{2},3)$

MATEMATIKA 3 - KOLOKVIJ #2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: STIPE JURLINA (Stipe Jurlina) **BROJ INDEKSA:**	

Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr. $\int_0^1\int_0^{x+1} x+\cos y\,\mathrm{d}y\mathrm{d}x$). Nije potrebno integral riješavati do kraja.

1. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{7}{2},0)$, $B(7,\frac{2}{2})$ i $C(\frac{7}{2},\frac{6}{2})$. Izračunati dvostruki integral ~~10~~

$$\iint_X x^3\mathrm{d}x\mathrm{d}y$$

2. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z\geq 1$. ~~10~~

3. Izračunati ~~10~~

$$\int_{(3,2)}^{(5,5)} x\mathrm{d}y+y\mathrm{d}x$$

4. Zadana je kružna uzvojnica (spirala) s jednadžbama $x=2\cos t$, $y=2\sin t$ i $z=t$. Skiciraj krivulju. Izračunati duljinu 3 namotaja ove krivulje. (pomoć: jedan namotaj odgovara peridu iskorištenih trigonometrijskih funkcija) ~~10~~

Ukupno: $\cancel{0}$

$(y-3)\cdot(\frac{7}{2})=-2(x-\frac{7}{2})$

$\frac{7}{2}y-\frac{21}{2}=-2x$

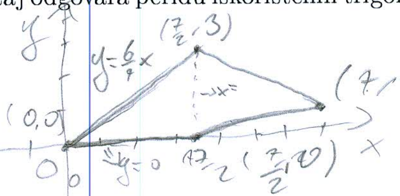
$(y-3)\cdot(7-\frac{7}{2})=(1-3)(x-\frac{7}{2})$

1) $O(0,0)$
$A(\frac{7}{2},0)$
$B(7,1)$
$C(\frac{7}{2},3)$

$\frac{7}{2}y = x-\frac{7}{2} \;/:\frac{7}{2}$

$y=\frac{x}{\frac{7}{2}}-\frac{\frac{7}{2}}{\frac{7}{2}}$

$y=\frac{2}{7}x-1$



$$\int_0^{\frac{7}{2}}\int_0^{\frac{6}{7}x} x^3\,dy\,dx+\int_{\frac{7}{2}}^{7}\int_{\frac{2}{7}x-1}^{(-\frac{4}{7}x+1)} x^3\,dy\,dx$$

$7-\frac{7}{2}=\frac{14-7}{2}=\frac{7}{2}$

$(y-y_1)(x_2-x_1)=(y_2-y_1)(x-x_1)$

$(y-0)(\frac{7}{2}-0)=(3-0)(x-0)$

$y\cdot\frac{7}{2}=3\cdot x$

$\frac{7}{2}y=3x\;/:\frac{7}{2}$

$y=\frac{3x}{\frac{7}{2}}=\frac{6}{7}x$

$(y-0)\cdot(7-\frac{7}{2})=(1-0)\cdot(x-\frac{7}{2})$

$y\cdot\frac{7}{2}=1\cdot(x-\frac{7}{2})$

Tablica integrala (zapravo ti ne treba)

$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x}=\tan x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,\mathrm{d}x=\frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x}=-\cot x+C$	$\int \frac{\mathrm{d}x}{a^2-x^2}=\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+C$
$\int \frac{\mathrm{d}x}{x}=\ln\|x\|+C$	$\int \sinh x\,\mathrm{d}x=\cosh x+C$	$\int \frac{\mathrm{d}x}{x^2-a^2}=\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+C$
$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C$	$\int \cosh x\,\mathrm{d}x=\sinh x+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}}=\ln\left\|x+\sqrt{x^2\pm a^2}\right\|+C$
$\int \sin x\,\mathrm{d}x=-\cos x+C$	$\int \tanh x\,\mathrm{d}x=\ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int \cos x\,\mathrm{d}x=\sin x+C$	$\int \coth x\,\mathrm{d}x=\ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int \tan x\,\mathrm{d}x=-\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x}=\tanh x+C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x=\ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x}=-\coth x+C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$

(2.) $x^2+y^2+z^2=4$ $\quad z \geq 1$

$r<2$

$x = r\cos\varphi$

$y = r\sin\varphi$

$r \in [0, 2]$

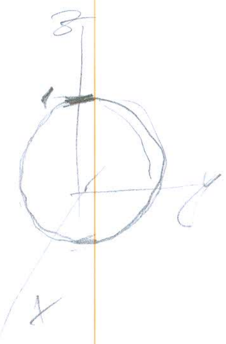
$\varphi \in [0, 2\pi]$

$z \in [\sqrt{4-x^2-y^2}, 1]$

$x^2+y^2+z^2=4$

$z^2 = 4-x^2-y^2$

$z = \sqrt{4-x^2-y^2}$



$$\int_0^{2\pi}\int_0^{2}\int_{\sqrt{4-x^2-y^2}}^{1} 1\, r\, dz\, dr\, d\varphi$$

$\sqrt{4-x^2-y^2} \to$ zamijeniti sa $\boxed{\begin{array}{l} x \to r\cos\varphi \\ y = r\sin\varphi \end{array}}$

$$\boxed{\int_0^{2\pi}\int_0^{2}\int_{\sqrt{4-r^2\cos^2\varphi - r^2\sin^2\varphi}}^{1} 1 \quad r\, dz\, dr\, d\varphi}$$ $\Rightarrow$ RJEŠENJE

(3.) $$\int_{(3,2)}^{(5,5)} x\,dy + y\,dx$$ Potencijalno polje:

$\begin{pmatrix} x \\ y \end{pmatrix} = -\operatorname{grad} f$

$-x \quad \int dx$ $\quad \boxed{-\frac{x^2}{2} - \frac{y^2}{2}}$ $\Rightarrow$ funkcija

$-\frac{x^2}{2} + Cy \Rightarrow$

$0 + \left(\frac{\partial Cy}{\partial y}\right) = -y$

$0 + Cy = -\frac{y^2}{2}$

$\left[-\frac{3^2}{2} - \frac{2^2}{2}\right] - \left[-\frac{5^2}{2} - \frac{5^2}{2}\right]$

$\left[-\frac{9}{2} - 2\right] + \frac{25}{2} + \frac{25}{2}$

$\frac{-9-4}{2} + \frac{50}{2} = \boxed{\frac{37}{2}}$ $\Rightarrow$ Rješenje

$\frac{7}{2}y - \frac{21}{2} = -2x - 7$

$\frac{7}{2}y = -2x - 7 + \frac{21}{2}$

$\frac{7}{2}y = -2x - \frac{14+21}{2}$

$\frac{7}{2}y = -2x + \frac{7}{2} \quad / : \frac{7}{2}$

$y = -\frac{2x}{\frac{7}{2}} + 1$

$\boxed{y = -\frac{4}{7}x + 1}$ → Pri toolotok

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4. $r \begin{pmatrix} x = 2\cos t \\ y = 2\sin t \\ z = t \end{pmatrix}$ Parametrizacija

$t \in [0, 2\pi]$

$\boxed{3 \int_0^{2\pi} 1 \cdot \begin{pmatrix} -2\sin t \\ 2\cos t \\ 1 \end{pmatrix} dt}$

$r' \begin{pmatrix} -2\sin t \\ 2\cos t \\ 1 \end{pmatrix}$

$\Downarrow$

RJEŠENJE