MATEMATIKA 3 - KOLOKVIJ #2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: HRVOJE BATUR **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr. $\int_0^1 \int_0^{x+1} x + \cos y \, dy dx$). Nije potrebno integral riješavati do kraja.

1. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{7}{2},0)$, $B(7,\frac{2}{2})$ i $C(\frac{7}{2},\frac{6}{2})$. Izračunati dvostruki integral 10

$$\iint_X x^3 \mathrm{d}x\mathrm{d}y$$

2. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=4$ za koji vrijedi $z \geq 1$. 10

3. Izračunati 10

$$\int_{(3,2)}^{(5,5)} x\mathrm{d}y + y\mathrm{d}x$$

4. Zadana je kružna uzvojnica (spirala) s jednadžbama $x = 2\cos t$, $y = 2\sin t$ i $z = t$. Skiciraj krivulju. Izračunati duljinu 3 namotaja ove krivulje. (pomoć: jedan namotaj odgovara peridu iskorištenih trigonometrijskih funkcija) 10

Ukupno: 40

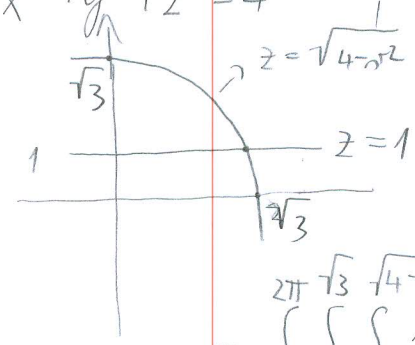
Tablica integrala (zapravo ti ne treba)

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x \,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \tanh x \,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \coth x \,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

② $x^2+y^2+z^2=4 \quad z \geq 1$

$z^2+r^2=4 \Rightarrow z^2=4-r^2$

$z=\sqrt{4-r^2}$



$z \in [1, \sqrt{4-r^2}]$

$r \in [0, \sqrt{3}]$

$\varphi \in [0, 2\pi]$

$r^2+1=4$

$r^2=3$

$r=\sqrt{3}$

$$= \int_0^{2\pi} \int_0^{\sqrt{3}} \int_1^{\sqrt{4-r^2}} 1 \cdot r \, dz \, dr \, d\varphi$$ ✓

③ $$\int_{(3,2)}^{(5,5)} x\,dy + y\,dx = \int_{(3,2)}^{(5,5)} y\,dx + x\,dy$$

$\frac{\partial A}{\partial x} = -y \quad / \int dx$

$A(x,y) = -yx + C(y)$

$\frac{\partial A}{\partial y} = -x$

$-x + C(y) = -x$

$C(y) = 0$

$A(x,y) = -yx$

$A(3,2) - A(5,5) = -6 - (-25)$

$= -6 + 25$

$= 19$ ✓

④ $x = 2\cos t$

$y = 2\sin t$

$z = t$

$t \in [0, 2\pi]$

$$r = \begin{pmatrix} 2\cos t \\ 2\sin t \\ t \end{pmatrix} \qquad r' = \begin{pmatrix} -2\sin t \\ 2\cos t \\ 1 \end{pmatrix}$$

$$\|r'(t)\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + 1}$$

$$= \sqrt{4\sin^2 t + 4\cos^2 t + 1}$$

$$= \sqrt{4(\sin^2 t + \cos^2 t) + 1}$$

$$= \sqrt{5}$$

$$= 3 \cdot \int_0^{2\pi} \sqrt{5}\, dt$$ ✓