MATEMATIKA 3 - KOLOKVIJ #1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ANTE ŠUŠNJARA **BROJ INDEKSA:** 57679

POPUNJAVA NASTAVNIK
Broj ↓ bodova

Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr. $\int_0^1 \int_0^{x+1} x + \cos y \, dy dx$). Nije potrebno integral riješavati do kraja.

1. Neka je C plašt cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z–osi, visine $v = 2$ s centrom u ishodištu ($z \in [-1, 1]$). Podrazumijeva se orijentacija plašta cilindra prema van. Kako treba računati $\iint_C 2x + 3 dy dz$? 10

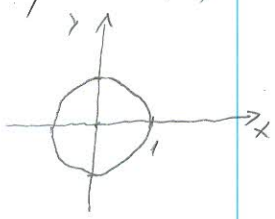
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Kako izračunati $\iint_K (2x + 3)\, dx dy$? 10

3. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Kako se može izračunati $\iint_{\partial \widehat{K}} (2x + 3)\, dx dy$? 10

4. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) orijentirana prema van. Kako se može izračunati $\iint_S 3 dx dy$? (pomoć: rot $(3x\mathbf{j}) = 3\mathbf{k}$) 10

Ukupno: 20

2) $T(2, 1)$ $r = 1$



$x = r\cos\varphi + 2$
$y = r\sin\varphi + 1$
$dx\,dy = r\,dr\,d\varphi$
$\varphi \in [0, 2\pi]$
$r \in [0, 1]$

$$\int_0^{2\pi}\int_0^1 (3 + 2(r\cos\varphi + 2))\, r\, dr\, d\varphi$$

$$\int_0^{2\pi}\int_0^1 (3r + 2r^2\cos\varphi + 4r)\, dr\, d\varphi$$ ✓

3) $a = 2$ $T(0, 0)$ $\iint_{\partial K} (2x + 3)\, dx\, dy$

$$\omega = \begin{bmatrix} 0 \\ 0 \\ 2x + 3 \end{bmatrix}$$

$$\operatorname{div} \omega = dx + dy + dz = 0 + 0 + 0 = 0$$

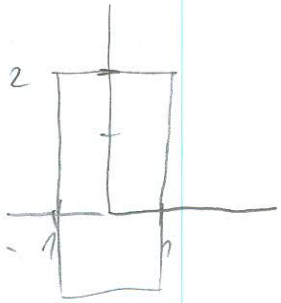
$$\iiint_K 0\, dx\, dy\, dz = 0$$

Tablica integrala (zapravo ti ne treba)

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x\, dx = -\cos x + C$	$\int \tanh x\, dx = \ln \lvert \cosh x \rvert$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\, dx = \sin x + C$	$\int \coth x\, dx = \ln \lvert \sinh x \rvert$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\, dx = -\ln \lvert \cos x \rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\, dx = \ln \lvert \sin x \rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

1) $r = 1$

$V = 2$



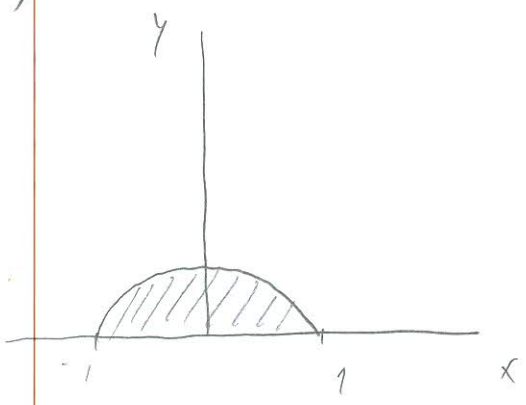
$z \in [-1, 1]$ Girovu formulu

$r \in [0, 1]$

$$\iint_C 2x + 3 \, dy \, dz$$

Ø

4)



$x = \cos t$

$y = \sin t$

$dx\,dy = dt$

$$r = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$

$$r' = \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix}$$

$r = 1$

$T(0,0)$

$|r(t)| = \sqrt{\sin^2 t + \cos^2 t}$

$|r(t)| = 1$

$$\int_0^{2\pi} 3 \cdot 1 \, dt$$

$$3 \int_0^{2\pi} dt$$

$3 \cdot 2\pi$

6π //

Ø

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IME I PREZIME: LOVRE LOVRIĆ **BROJ INDEKSA:** 58080

POPUNJAVA NASTAVNIK
Broj ↓ bodova

Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr. $\int_0^1 \int_0^{x+1} x + \cos y \, dy dx$). Nije potrebno integral riješavati do kraja.

1. Neka je C plašt cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z–osi, visine $v = 2$ s centrom u ishodištu ($z \in [-1, 1]$). Podrazumijeva se orijentacija plašta cilindra prema van. Kako treba računati $\iint_C 2x + 3 \mathrm{d}y\mathrm{d}z$? 10

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Kako izračunati $\iint_K (2x + 3) \, \mathrm{d}x\mathrm{d}y$? 10

3. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Kako se može izračunati $\iint_{\widehat{\partial K}} (2x + 3) \, \mathrm{d}x\mathrm{d}y$? 10

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Ukupno: 20

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$\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1}, \; n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
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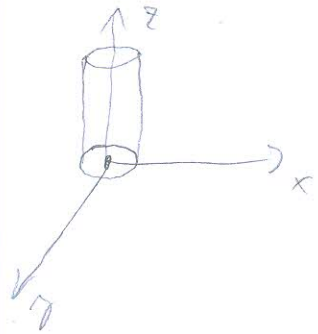
LOVRE LOVRIĆ 58080

1.) $r = 1$ $\quad z \in [-1, 1]$

$N = 2$ $\quad r \in [0, 1]$

$\varphi \in [0, 2\pi]$



$$\iint_C 2x + 3 \, dy \, dz$$

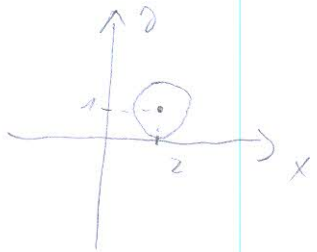
$$W = \begin{bmatrix} 0 \\ 0 \\ 2x+3 \end{bmatrix} \Rightarrow div = 0 + 0 + 0 = 0$$

NIJE ZATVOREN PA NE MOŽE PREKO DIVERGENCIJE

$$\int_0^{2\pi} \int_{-1}^{1} \int_0^1 r \, dr \, dz \, d\varphi =$$

2.) $r = 1$

$T(2,1)$



$$\iint_K (2x+3) \, dx \, dy$$

$x = r\cos\varphi + 2$

$y = r\sin\varphi + 1$

$(x-a)^2 + (y-b)^2 = r^2$

$(r\cos\varphi + 2 - 2)^2 + (r\sin\varphi + 1 - 1)^2 = r^2$

$r^2\cos^2\varphi + r^2\sin^2\varphi = 1$

$r^2 = 1$

$r = 1$

$$\int_0^{2\pi} \int_0^1 (2(r\cos\varphi + 2) + 3) r \, dr \, d\varphi =$$ ✓

$r \in [0, 1]$

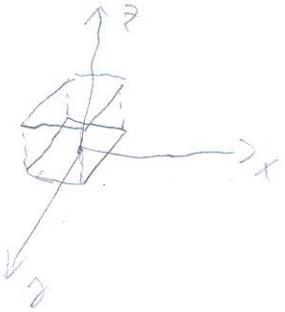
$\varphi \in [0, 2\pi]$

10

$$= \int_0^{2\pi} \int_0^1 (2r\cos\varphi + 7) r \, dr \, d\varphi = \int_0^{2\pi} \int_0^1 2r^2\cos\varphi + 7r \, dr \, d\varphi =$$

3.) $a = 2$ kocka

$x \in [-1,1]$
$y \in [-1,1]$
$z \in [-1,1]$



$$\iint_{\partial K} (2x+3)\,dx\,dy$$

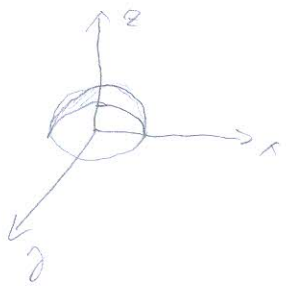
$$N = \begin{bmatrix} 2x+3 \\ 0 \\ 0 \end{bmatrix} \Rightarrow div = 2 + 0 + 0 = 2$$

✓ 10

$$2\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} dx\,dy\,dz =$$

4.) $r = 1$
$z \geq 0$

$r \in [0,1]$
$z \in [0,1]$
$\varphi \in [0, 2\pi]$



$$\iint_S 3\,dx\,dy$$

NIJE ZATVOREMA PLOHA

$$N = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \Rightarrow div = 0 + 0 + 0 = 0$$

∅

$$\int_0^{2\pi}\int_0^1\int_0^1 r\,dr\,dz\,d\varphi =$$

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IME I PREZIME: SURLINA **BROJ INDEKSA:**

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Broj ↓ bodova

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Ukupno: 0

(1)

$\iint 2x+3\,dy\,dz$

$W\begin{pmatrix} 2x+3 \\ 0 \\ 0 \end{pmatrix}$

$r=1 \quad t\in[-1,1]$

$x = r\cos v$
$y = r\sin v$

PARAMETRIZACIJA
$r\begin{pmatrix} r\cos v \\ r\sin v \\ u \end{pmatrix}$

$r\begin{pmatrix} \cos v \\ \sin v \\ u \end{pmatrix}$

$\int_0^{2\pi}\int_0^1 \begin{pmatrix} 2x+3 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \cos v \\ \sin v \\ 0 \end{pmatrix} dr\,dv$

$\int_0^{2\pi}\int_0^1 (2x+3)\cdot\cos v \cdot r\,dr\,dv$

$\frac{\partial r}{\partial v} \times \frac{\partial r}{\partial u}$

orijentacija prema van

$\begin{pmatrix} -\sin v \\ \cos v \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos v \\ \sin v \\ 0 \end{pmatrix}$

$-\sin v \quad 0$
$\cos v \quad 0$

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$\int \cot x\, dx = \ln \lvert \sin x \rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

2. $K \Rightarrow$ krug $\quad T(2,1)$

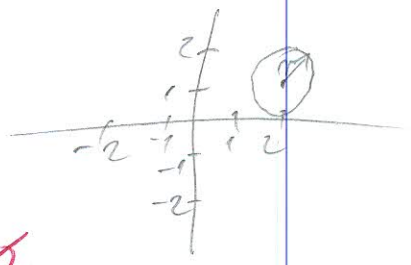
$r = 1$

$x = r\cos\varphi \Rightarrow r = 1 \quad x = \cos\varphi + 2$

$y = r\sin\varphi \quad y = \sin\varphi + 1$

$$\iint (2x+3)\,dx\,dy$$

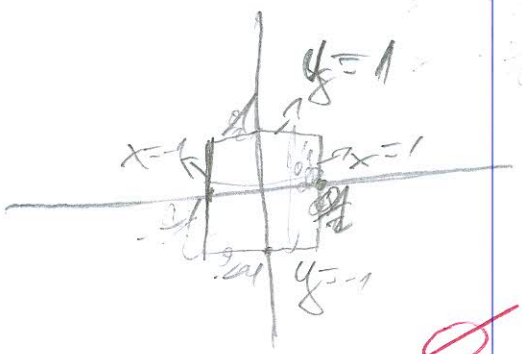
$$\int_0^{2\pi}\int_0^1 2\cdot(\cos\varphi+2)+3\cdot r\,dr\,d\varphi \quad \times$$



$$\int_0^{2\pi}\int_0^1 2\cos\varphi + 4 + 3\,r\,dr\,d\varphi \quad \emptyset$$

3. Kako je $K \Rightarrow$ kocka $a = 2$ centar u ishodištu

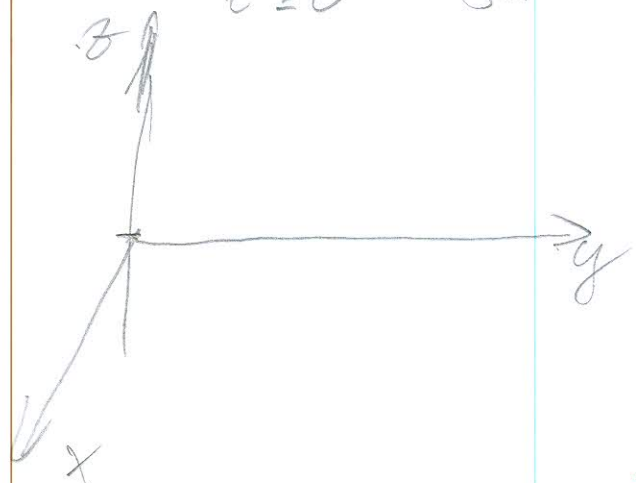
$$\iint (2x+3)\,dx\,dy$$



$$\int_{-1}^{1}\int_{-1}^{1} (2x+3)\,dx\,dy \quad \times$$

4. Sporno polustožac $r = 1$ centar u ishodištu

$z \geq 0$ Izračunati $\iint_S 3\,dx\,dy$



$$\vec{N} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

$$\begin{matrix} \partial x & 0 \\ \partial y & 0 \\ \partial z & 3 \\ \partial x & 0 \\ \partial y & 0 \end{matrix} \Rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\iint_S 0\,dx\,dy \Rightarrow 0$$