

o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME:

LUKA KURILIĆ

BROJ INDEKSA:

58076

Svaki sljedeći zadatak svesti na rješavanje jednog ili serije jednostrukih određenih integrala (npr. $\int_0^1 \int_0^{x+1} x + \cos y \, dy \, dx$). Nije potrebno integral rješavati do kraja.

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Kako se može računati $\int_{\partial K} (2x + 3) \, ds$? 10
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0, -1)$, a ∂K kružnica orijentirana suprotno od kazaljke na satu. Kako se može izračunati $\int_{\partial K} (2x + 3) \, dy$? 10
3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Kako se može računati $\iiint_K (2x + 3) \, dx \, dy \, dz$? 10
4. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint_{\partial K} 3 \, dS$? 10

Ukupno:

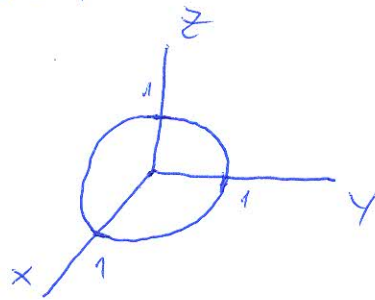
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Tablica integrala (zapravo ti ne treba)

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int \sin x \, dx = -\cos x + C$	$\int \tanh x \, dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, dx = \sin x + C$	$\int \coth x \, dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \tan x \, dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x \, dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

④ $r=1$

$$\iint_{\partial K} 3 \, ds$$



$$r(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$$

$$r'(t) = \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix} \quad \times$$

$$|r'(t)| = \sqrt{\sin^2 t + \cos^2 t}$$

$$|r'(t)| = 1$$

$$\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{1-z^2}} 3r \, dr \, dt \, dz$$

$$x = \cos t$$

$$y = \sin t$$

$$x^2 + y^2 + z^2 = R \quad \rightarrow r^2$$

$$x^2 + y^2 + z^2 = 1$$

$$r^2 + z^2 = 1$$

$$r^2 = 1 - z^2$$

$$r = \sqrt{1 - z^2}$$

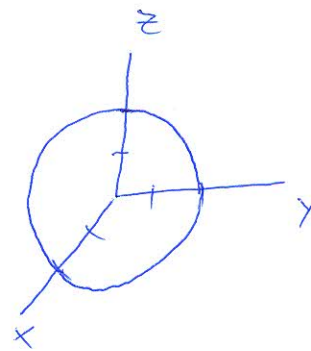
3) Kugla

$r = 2$

$\iiint (2x+3) dx dy dz$

$\int_0^{2\pi} \int_0^{\sqrt{4-z^2}} \int_0^{\sqrt{4-z^2}} (2x+3) r dr dt dz$

$\int_0^{2\pi} \int_0^{\sqrt{4-z^2}} \int_0^{\sqrt{4-z^2}} (2r \cos t + 3) r dr dt dz$ ✗



$x = r \cos t$
 $y = r \sin t$
 $x \in [0, 2\pi]$
 $y \in [0, 2]$
 $z \in [0, \sqrt{4-z^2}]$

$x^2 + y^2 = r^2$
 $x^2 + y^2 + z^2 = R^2$
 $r^2 + z^2 = 2$
 $r^2 + z^2 = 4$
 $r^2 = 4 - z^2$
 $r = \sqrt{4 - z^2}$

4) ~~$r = 1$ $x = \cos t$~~

~~$\iint_{\partial K} 3 ds$ $y = \sin t$~~

~~$r(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$~~

~~$r'(t) = \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix}$~~

~~$|r'(t)| = \sqrt{\sin^2 t + \cos^2 t}$~~

~~$|r'(t)| = 1$~~

~~$\int_0^{2\pi} 3 \cdot 1 dt$~~

~~$3 \int_0^{2\pi} dt \Rightarrow 3 \cdot 2\pi = 6\pi$~~

Nevalja

1) $r = 2$ $T(0,0)$

$\int_{\partial K} (2x+3) ds$

$x = \cos t$
 $y = \sin t$

$r(t) = \begin{bmatrix} 2 \cos t \\ 2 \sin t \end{bmatrix}$

$r'(t) = \begin{bmatrix} -2 \sin t \\ 2 \cos t \end{bmatrix}$

$|r'(t)| = \sqrt{2 \sin^2 t + 2 \cos^2 t}$
 $= \sqrt{2(\sin^2 t + \cos^2 t)}$
 $= \sqrt{2 \cdot 1} = \sqrt{2}$

$\int (2x+3) \sqrt{2} dt$
 $\sqrt{2} \int_0^{2\pi} (2 \cdot \cos t + 3) dt$ ✓

10

$$\textcircled{2} \quad r=1 \quad T(0, -1)$$

$$\int \frac{\partial Q}{\partial x} (2x+3) dy$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2 - 0 = 2$$

$$\int_0^{2\pi} \int_0^1 2 r dr d\theta \quad \checkmark \quad \underline{10}$$

$$\int_0^{2\pi} d\theta \int_0^1 r dr$$

$$\int_0^{2\pi} d\theta \left(\frac{r^2}{2} \Big|_0^1 \right)$$

$$\int_0^{2\pi} d\theta \frac{1}{2}$$

$$\frac{1}{2} \int_0^{2\pi} d\theta$$

$$\frac{1}{2} \cdot 2\pi = \pi$$

