MATEMATIKA 3 - KOLOKVIJ #1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: LUKA KURILIĆ **BROJ INDEKSA:** 58076

POPUNJAVA NASTAVNIK
Broj ↓ bodova

`Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr.` $\int_0^1 \int_0^{x+1} x + \cos y \, dy dx$`). Nije potrebno integral riješavati do kraja.`

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Kako se može računati $\int\limits_{\partial K} (2x+3)\,ds$? 10

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\partial\widehat{K}$ kružnica orjentirana suprotno od kazaljke na satu. Kako se može izračunati $\int\limits_{\partial\widehat{K}} (2x+3)\,dy$? ~~10~~ 10

3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Kako se može računati $\iiint\limits_K (2x+3)\,dxdydz$? ~~10~~

4. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 3dS$? ~~10~~

Ukupno: ~~31~~

Tablica integrala (zapravo ti ne treba)

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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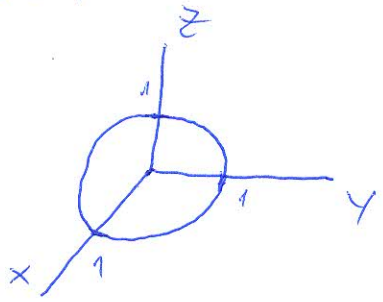
④ $r = 1$

$\iint_{\partial K} 3\,ds$



$x = \cos t$

$y = \sin t$

$x^2 + y^2 + z^2 = R^2 \nearrow 1$

$\underbrace{x^2 + y^2}_{r^2} + z^2 = 1$

$r^2 + z^2 = 1$

$r^2 = 1 - z^2$

$\boxed{r = \sqrt{1 - z^2}}$

$r(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$

$r'(t) = \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix}$ ×

$|r'(t)| = \sqrt{\sin^2 t + \cos^2 t}$

$|r'(t)| = 1$

$$\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{1-z^2}} 3r\,dr\,dt\,dz$$

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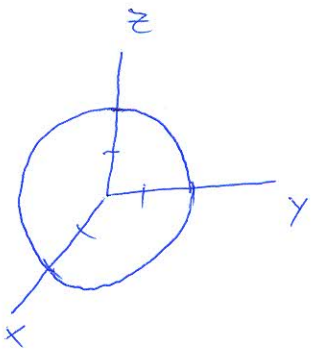
(3) kugla

$r = 2$

$\iiint (2x+3)\,dx\,dy\,dz$

$\int_0^{2\pi}\int_0^{K}\int_0^{\sqrt{4-z^2}} (2x+3)\,r\,dr\,d\varphi\,dz$

$\int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{4-z^2}} (2r\cos\varphi+3)\,r\,dr\,d\varphi\,dz$ ✗



$x = r\cos\varphi$

$y = r\sin\varphi$

$x \in [0, 2\pi]$

$y \in [0, 2]$

$z \in [0, \sqrt{4-z^2}]$

$x^2 + y^2 = r^2$

$x^2 + y^2 + z^2 = R^2$

r^2

$r^2 + z^2 = 2^2$

$r^2 + z^2 = 4$

$r^2 = 4 - z^2$

$r = \sqrt{4-z^2}$

(4) $r = 1$ $\quad x = \cos t$ $\quad y = \sin t$

$\int_{\partial K} 3\,ds$

$r(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$

$r'(t) = \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix}$

$\|r'(t)\| = \sqrt{\sin^2 t + \cos^2 t}$

$\|r'(t)\| = 1$

$\int_0^{2\pi} 3 \cdot 1\,dt$

$3\int_0^{2\pi} dt \Rightarrow 3 \cdot 2\pi \neq 6\pi$

Nevalja

(1) $r = 2$ $\quad T(0,0)$ $\quad \int_{\partial K} (2x+3)\,ds$ $\quad x = \cos t$ $\quad y = \sin t$

$r(t) = \begin{bmatrix} 2\cos t \\ 2\sin t \end{bmatrix}$

$r'(t) = \begin{bmatrix} 2\sin t \\ -2\cos t \end{bmatrix}$

$\|r'(t)\| = \sqrt{2\sin^2 t + 2\cos^2 t}$

$= \sqrt{2(\sin^2 t + \cos^2 t)}$

$= \sqrt{2 \cdot 1} = \sqrt{2}$

$\int (2x+3)\sqrt{2}\,dt$

$\sqrt{2}\int_0^{2\pi} (2\cdot\cos t + 3)\,dt$ ✓

10

(2) $r=1 \quad T(0,-1)$

$$\oint_{\partial K} (2x+3)\,dy$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2 - 0 = 2$$

$$\int_0^{2\pi}\int_0^1 2\,r\,dr\,d\varphi \quad \checkmark \quad 10$$

$$\int_0^{2\pi} d\varphi \int_0^1 r\,dr$$

$$\int_0^{2\pi} d\varphi \left(\frac{r^2}{2}\Big|_0^1\right)$$

$$\int_0^{2\pi} d\varphi \; \frac{1}{2}$$

$$\frac{1}{2}\int_0^{2\pi} d\varphi$$

$$\frac{1}{2}\cdot 2\pi = \boxed{\pi}$$

MATEMATIKA 3 - KOLOKVIJ #1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: HRVOJE BATUR **BROJ INDEKSA:** 17-2-0006

POPUNJAVA NASTAVNIK
Broj ↓ bodova

Svaki sljedeći zadatak svesti na riješavanje jednog ili serije jednostrukih određenih integrala (npr. $\int_0^1 \int_0^{x+1} x + \cos y \, \mathrm{d}y\mathrm{d}x$). Nije potrebno integral riješavati do kraja.

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,0)$. Kako se može računati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$? 10

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3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Kako se može računati $\iiint\limits_K (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$? 10

4. Neka je K kugla radijusa $r = 1$ sa centrom u ishodištu. Kako preko definicije izračunati $\iint\limits_{\partial K} 3\mathrm{d}S$? 10

Ukupno: 10

Tablica integrala (zapravo ti ne treba)

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
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HRVOJE BATUR 17-2-0006

③ K... kugla $r=2$ $T(0,0)$ $z^2+r^2=2^2$ $z^2=2^2-r^2$ $z=\sqrt{4-r^2}$

$\varphi \in [0,2\pi]$

$z \in [0, \sqrt{4-r^2}]$

$r \in [0, \sqrt{2}]$

$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_0^{\sqrt{4-r^2}} (2r\cos\varphi+3)\, r\, dz\, dr\, d\varphi \quad \times$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}}\int_0^{\sqrt{4-r^2}} (2r^2\cos\varphi+3r)\, dz\, dr\, d\varphi$$

$x = r\cos\varphi$

$y = r\sin\varphi$

⊘

① K... krug $r=2$ $T(0,0)$ $\int_{\partial K}(2x+3)\,ds$

$x = r\cos\varphi$

$y = r\sin\varphi$

$r \in [0,2]$

$\varphi \in [0, 2\pi]$

$$\int_0^{2\pi}\int_0^{2} (2r\cos\varphi+3)\, r\, dr\, d\varphi$$

$$= \int_0^{2\pi}\int_0^{2} (2r^2\cos\varphi+3r)\, dr\, d\varphi \quad \times \quad ⊘$$

④ K... kugla $r=1$ $T(0,0)$ $\iint_{\partial K} 3\,ds$

$z^2+r^2=1$

$z=\sqrt{1-r^2}$

$$\int_0^{2\pi}\int_0^{1}\int_0^{\sqrt{1-r^2}} 3r\, dz\, dr\, d\varphi \quad \times$$

$z \in [0, \sqrt{1-r^2}]$

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$

② K ... krug $r=1$ $T(0,-1)$, ∂K suprotno od kazaljke orijentirana

$\int_{\partial K}(2x+3)\,dy$

$x = r\cos\varphi$ $r \in [0,1]$

$y = r\sin\varphi$ $\varphi \in [0,2]$

$P=0$

$Q=2x+3$

$$\iint\left(\frac{Q}{dx}-\frac{P}{dy}\right) = \iint 2\, dx\, dy$$

↙ Greenova formula

$$= \int_0^{2\pi}\int_0^{1} 2\cdot r\, dr\, d\varphi \quad ✓ \quad 10$$

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IME I PREZIME: LUKA SJAUŠ **BROJ INDEKSA:** 57680

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 10

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LUKA SJAUŠ

③ KUGLA

K, $r = 2$

$\iiint_K (2x+3)\,dx\,dy\,dz = ?$

$x^2 + y^2 + z^2 = R^2$

$r^2 + z^2 = R^2$

$r = \sqrt{R^2 - z^2}$

$r = \sqrt{2^2 - z^2}$

$d\varphi \in [0, 2\pi]$ ✓

$dr \in [0, \sqrt{4-z^2}]$ ✓

$dz \in [-2, 2]$ ✓

$$\iiint_K (2x+3)\,dx\,dy\,dz = \int_0^{2\pi}\int_{-2}^{2}\int_{0}^{\sqrt{4-z^2}} ? \cdot r\,dr\,dz\,d\varphi$$ ✗

① k krog, $r = 2$, $S(0,0)$.

$\int_{\partial K} (2x+3)\,ds = ?$

(2.) K kreis, r=1 r=(0,-1)

$$\oint_{\partial K} (2x+3)\,dy$$

$$W = \begin{pmatrix} 0 \\ 2x+3 \\ 0 \end{pmatrix}$$

$$\text{rot} = \begin{pmatrix} \partial x \\ \partial y \\ \partial z \end{pmatrix} \times \begin{pmatrix} 0 \\ 2x+3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0-0 \\ 0-0 \\ 2-0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

∂x · 0, ∂y · 0, ∂x · $2x+3$, ∂y · $2x+3$

$$\begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$$

$$\frac{\partial}{\partial u}\begin{pmatrix} u \\ v \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{\partial}{\partial v}\begin{pmatrix} u \\ v \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{\partial}{\partial u} \times \frac{\partial}{\partial v} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\oint_{\partial K} (2x+3)\,dy = \int_0^{2\pi}\int_0^1 \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r\,dr\,d\varphi = \int_0^{2\pi}\int_0^1 2r\,dr\,d\varphi$$ ✓

10