Popuniti odmah!

Ime i prezime: DUJE KRALJEV **Broj indeksa:** 17-2-0015-2010 26

Datum: Vrijeme: od do .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Izračunati $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{4x^2+6x+3}}$. ~~15~~
2. Izračunati $\displaystyle\int_{-1}^{0} 3x\,\mathrm{e}^{x+1}\,\mathrm{d}x$. ~~15~~ 8
3. Grafički prikazati funkciju $f(x,y)=\dfrac{x^2}{y}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$? 15
4. Istražiti domenu i ekstreme funkcije $f(x,y)=x^3-3xy+y^2$. ~~20~~ 10
5. Pronaći opće rješenje problema: $y'+xy^2+x=0$. ~~20~~
6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x)=e^{x^2}$ oko točke $x_0=0$. ~~15~~ 8

IME I PREZIME: DUJO KRALJEV BROJ INDEKSA: 17-1-0015-2010

1) $\int \frac{dx}{\sqrt{4x^2+6x+3}} =$

5) $y' + xy^2 + x = 0$

separacija

$\frac{dy}{dx} = -xy^2 - x$

$\frac{dy}{dx} = -x(y^2+1) \,/\cdot dx$

$\frac{dy}{y^2+1} = -x\,dx \quad \cdot \int$

$\int \frac{dy}{y^2+1} = -\int x\,dx$

$\ln|y^2+1| = -\frac{x^2}{2} + C$

$y^2+1 = e^{-\frac{x^2}{2}+C}$

$y^2 = e^{-\frac{x^2}{2}+C} - 1$

$\emptyset$

Ime i prezime: DUJE KRALJEV **Broj indeksa:** 17-2-0015-2do

$$2)\ \int_{-1}^{0} 3x e^{x+1} dx = \left(x \cdot 3e^{x+1} - 3e^{x+1} \right) \Big|_{-1}^{0}$$

$$= \left(\underbrace{-1 \cdot 3e^{-1+1} - 3e^{-1+1}} - \left(\underbrace{0 \cdot 3e^{0+1} - 3e^{0+1}} \right) \right) = -3 - 3 - 0 + 3 = -3 \ //$$

OBRNUTO

$$e^{0+1} = e^1 = e \approx 2.71$$

$$\int 3x e^{x+1} dx = \begin{bmatrix} x = u & du = 3e^{x+1} dx \\ dx = du & v = \int 3e^{x+1} dx \ \begin{bmatrix} x+1 = t \\ 1 dx = dt \end{bmatrix} \\ & v = 3\int e^t dt \\ & v = 3e^t \\ & v = 3e^{x+1} \end{bmatrix}$$

$$= u \cdot v - \int v \cdot du = x \cdot 3e^{x+1} - \int 3e^{x+1} \cdot dx$$

$$= x \cdot 3e^{x+1} - 3\int e^{x+1} dx = x \cdot 3e^{x+1} - 3e^{x+1} \ ✓$$

~~ODREĐENI~~ INTEGRAL ? $\underline{\underline{8}}$

$$\left(3xe^{x+1} - 3e^{x+1} \right)\Big|_{-1}^{0} = \left(\underbrace{3 \cdot 0 \cdot e^{0+1}}_{=0} - 3\underbrace{e^{0+1}}_{e^1} \right) - \left(3(-1)e^{-1+1} - 3e^{-1+1} \right)$$

$$= -3e + 3\underbrace{e^0}_{=1} + 3\underbrace{e^0}_{=1}$$

$$= -3e + 3 + 3$$

$$= 6 - 3e$$

$$\approx -2.13$$

Ime i prezime: DUJE KRALJEV **Broj indeksa:** 17-2-0015-2010

4) $f(x,y) = x^3 - 3xy + y^2$

$Df(x,y) = \mathbb{R}^2$

$\partial_x f = 3x^2 - 3y$

$\partial_{xx} f = 6x$

$\partial_{xy} f = -3$

$\partial_y f = -3x + 2y$

$\partial_{yy} f = 2$

$$\begin{aligned} \partial_x f &= 0 \\ \partial_y f &= 0 \end{aligned}$$

$3x^2 - 3y = 0$

$-3x + 2y = 0 \Rightarrow 2y = -3x \;/ :2$

$y = -\frac{3}{2}x$

$3x^2 - 3\left(-\frac{3}{2}x\right) = 0$

$3x^2 + \frac{9}{2}x = 0 \;/ :2$

$6x^2 + 9x = 0$

$x(6x+9) = 0$

$x_1 = 0$

$6x + 9 = 0$

$6x = -9 \;/ :6$

$x_2 = -\frac{3}{2}$

$y_1 = -\frac{3}{2} \cdot 0$

$y_1 = 0$

$y_2 = -\frac{3}{2} \cdot \left(-\frac{3}{2}\right)$

$y_2 = \frac{9}{4}$

$T_1(0,0)$

$T_2\left(-\frac{3}{2}, \frac{9}{4}\right)$

$T_1(0,0)$

10

$A = \partial_{xx} f = 6x = 6 \cdot 0 = 0$

$$\Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} 0 & -3 \\ -3 & 2 \end{vmatrix} = 0 - 9 = -9$$

$A = 0$
$\Delta < 0$ sedlasta točka

Ime i prezime: DUJE KRALJEV **Broj indeksa:** 17-2-0025-2010

$T_2\left(-\frac{3}{2}, \frac{9}{4}\right)$

$A = \partial_{xx} f = 6x = 6\cdot\left(-\frac{3}{2}\right) = -9$

$\Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} -9 & -3 \\ -3 & 2 \end{vmatrix} = -18 - 9 = -27$

$A < 0$
$\Delta < 0$ funkcija nema ekstrema

6) $f(x) = e^{x^2}$ $x_0 = 0$

$f(x_0) = e^{x^2} = e^{0^2} = 1$ ✓

$f'(x) = e^{x^2}\cdot 2x, f'(x_0) = e^{0^2}\cdot 2\cdot 0 = 0$ ✓

$f''(x) = e^{x^2}\cdot 2x\cdot 2x + e^{x^2}\cdot 2 = e^{x^2}\cdot 4x + 2e^{x^2}; f''(x_0) = e^{0^2}\cdot 4\cdot 0 + 2e^{0^2} = 2$ ×

$f'''(x) = e^{x^2}\cdot 2x\cdot 4x + e^{x^2}\cdot 4 + 2e^{x^2}\cdot 2x = e^{x^2}\cdot 8x^2 + 4e^{x^2} + 2e^{x^2}\cdot 2x,$

$f'''(x_0) = e^{0^2}\cdot 8\cdot 0^2 + 4e^{0^2} + 2\cdot e^{0^2}\cdot 2\cdot 0 = 4$

$f(x) = f(x_0) + (x - x_0)\cdot f'(x_0) + \frac{(x-x_0)^2}{2!} f''(x_0) + \frac{(x-x_0)^3}{3!}\cdot f'''(x_0) + \dots$

$f(x) = 1 + (x-0)\cdot 0 + \frac{(x-0)^2}{2}\cdot 2 + \frac{(x-0)^3}{6}\cdot 4 + \dots$

$f(x) = 1 + \frac{(x-0)^2}{\cancel{2}_1}\cdot\cancel{2}^1 + \frac{(x-0)^3}{\cancel{6}_3}\cdot\cancel{4}^2 + \dots$

$f(x) = 1 + x^2 + \frac{x^3}{3}\cdot 2 + \dots$

8