Popuniti odmah!

IME I PREZIME: Domagoj Nehić **BROJ INDEKSA:** 17-2-0028-2010

80

DATUM: VRIJEME: OD 09:10 DO

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Riješiti: $\int_0^1 x^2 \sin(2x^3 - 3)\,\mathrm{d}x$ 10
2. Izračunati površinu lika između parabole $y = x^2 - 3$ i pravca $y = 2x$. 15
3. Odrediti Taylorov razvoj funkcije $f(x) = x^3 + 3x - 4$ oko točke $x_0 = 1$. 15
4. Ispitati ekstreme funkcije $f(x, y) = x^2 + y^2 - xy - 2y + 1$. 20
5. Riješiti: $\int \frac{x^3+1}{x^3+x}\,\mathrm{d}x$. 20
6. Riješiti diferencijalnu jednadžbu: $y'' + 2y' = 2x$. ~~20~~

1) $\int_0^1 x^2 \sin(2x^3-3)\, dx$

$$\begin{cases} 2x^3-3=t \\ 6x^2 dx = dt /:6 \end{cases}$$
$$x^2 dx = \frac{dt}{6}$$

$\int_0^1 \sin t \cdot \frac{dt}{6}$ ✓

$\frac{1}{6}\int_0^1 \sin t \cdot dt$ ✓

$\frac{1}{6}\cdot(-\cos t)\Big|_0^1 = -\frac{1}{6}\cos(2x^3-3)\Big|_0^1$ ✓

$= -\frac{1}{6}\cdot\left[\cos(2\cdot 1^3-3) - \cos(2\cdot 0^3-3)\right]$

$= -\frac{1}{6}\left[\cos(-1) - \cos(-3)\right] = -\frac{1}{6}\cdot(0.54+0.99)$ ✓

$= -\frac{1}{6}\cdot 1.53 = -0.255$ // ✓

10

IME I PREZIME: Domagoj Nehić   BROJ INDEKSA: 17-2-0028-2010

$a=1,\ b=0,\ c=-3$

2) $P \ldots y = x^2 - 3$

$p \ldots y = 2x$

$y = x^2 - 3$

$x^2 - 3 = 0$

$x^2 = 3 \,/\sqrt{\ }$

$x = \pm\sqrt{3}$

$x_1 = -1.73$

$x_2 = 1.73$

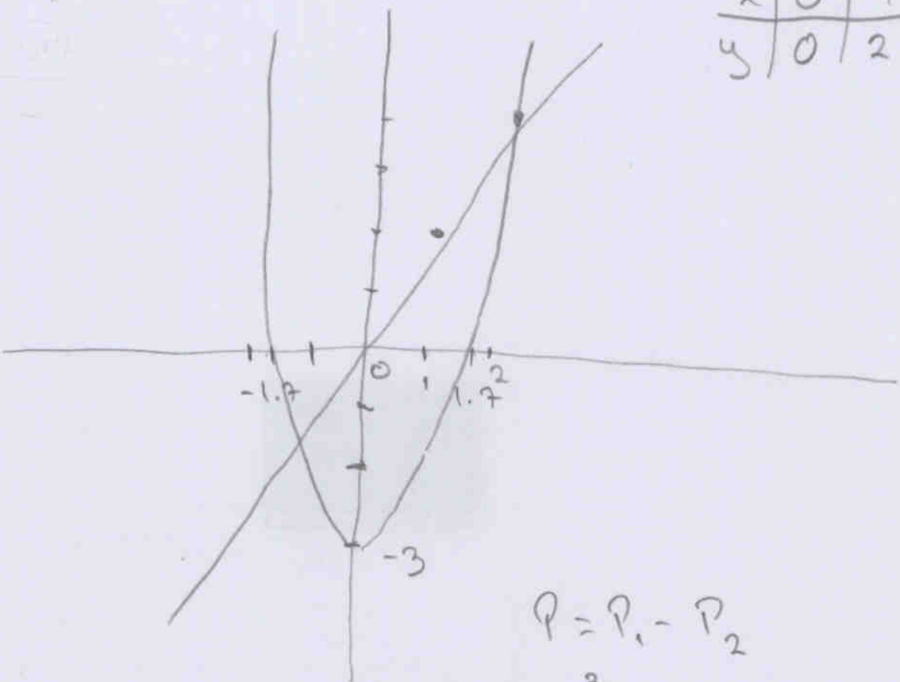
$T\left(-\frac{b}{2a},\ \frac{4ac-b^2}{4a}\right)$

$T\left(-\frac{0}{2},\ \frac{-12-0}{4}\right)$

$T(0,\ -3)$

$y = 2x$

x	0	1	2
y	0	2	4



$P \ldots y = x^2 - 3$

$p \ldots y = 2x$

$y = y$

$x^2 - 3 = 2x$

$x^2 - 2x - 3 = 0$

$a = 1$
$b = -2$
$c = -3$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2}$

$x_1 = \frac{2-4}{2} = -1$

$x_2 = \frac{2+4}{2} = 3$

$P = P_1 - P_2$

$P_1 = \int_{-1}^{3} 2x\,dx = 2\int_{-1}^{3} x\,dx$

$= 2\ \frac{x^2}{2}\Big|_{-1}^{3} = 2\cdot\left[\frac{3^2}{2} - \frac{(-1)^2}{2}\right]$

$= 2\cdot\left[\frac{9}{2} - \frac{1}{2}\right] = 2\cdot 4 = 8$

$P_2 = \int_{-1}^{3}(x^2-3)\,dx = \int_{-1}^{3} x^2 dx - 3\int_{-1}^{3} dx$

$= \frac{x^3}{3}\Big|_{-1}^{3} - 3\cdot x\Big|_{-1}^{3} = \left[\frac{3^3}{3} - \frac{(-1)^3}{3}\right] - 3\cdot\left[3+1\right]$

$= \left[9 + \frac{1}{3}\right] - 3\cdot 4 = \frac{28}{3} - 12 = -\frac{8}{3} = -2.67$

$P = P_1 - P_2 = 8 + 2.67 = 10.67$

15

IME I PREZIME: Domagoj Hehić   BROJ INDEKSA: 17-2-0028-2010

3. $f(x) = x^3 + 3x - 4$ $x_0 = 1$

$f(x) = x^3 + 3x - 4$ $f(0) = 1^3 + 3 \cdot 1 - 4 = 0$

$f'(x) = 3x^2 + 3$ $f'(0) = 3 \cdot 1^2 + 3 = 6$

$f''(x) = 6x$ $f''(0) = 6 \cdot 1 = 6$

$f'''(x) = 6$ $f'''(0) = 6$

$$f(x) = f(0) + \frac{f'(0)}{1!}(x - x_0) + \frac{f''(0)}{2!}(x - x_0)^2 + \frac{f'''(0)}{3!}(x - x_0)^3$$

$$f(x) = 0 + \frac{6}{1!}(x - 1) + \frac{6}{2!}(x - 1)^2 + \frac{6}{3!}(x - 1)^3$$

$$f(x) = 0 + 6(x - 1) + 3(x - 1)^2 + 1(x - 1)^3$$

✓ 15

IME I PREZIME: Domagoj Nehić   BROJ INDEKSA: 17-2-0028-2010

4) $f(x,y) = x^2 + y^2 - xy - 2y + 1$

$z_x = 2x - y$

$z_y = 2y - x - 2$

$2x - y = 0$

$2y - x - 2 = 0$

$2x - y = 0$

$2x = y \,/ : 2$

$x = \frac{y}{2}$

$x = \frac{\frac{4}{3}}{\frac{2}{1}} = \frac{2}{3}$

$2y - x - 2 = 0$

$2y - \frac{1}{2}y - 2 = 0$

$\frac{3}{2}y - 2 = 0$

$\frac{3}{2}y = 2 \,/ : \frac{3}{2}$

$y = \frac{4}{3}$

$T\left(\frac{2}{3}, \frac{4}{3}\right)$

$z_{xx} = 2$ $r_0 = 2 > 0$ min

$z_{xy} = -1$ $s_0 = -1$

$z_{yy} = 2$ $t_0 = 2$

$\Delta = r_0 \cdot t_0 - (s_0)^2 = 2 \cdot 2 - (-1)^2 = 4 - 1 = 3$ ima ekstrem

$f(x,y) = x^2 + y^2 - xy - 2y + 1 = \left(\frac{2}{3}\right)^2 + \left(\frac{4}{3}\right)^2 - \frac{2}{3} \cdot \frac{4}{3} - 2 \cdot \frac{4}{3} + 1$

$= \frac{4}{9} + \frac{16}{9} - \frac{8}{9} - \frac{8}{3} + 1 = -\frac{1}{3}$ //

20

IME I PREZIME: Domagoj Wehić BROJ INDEKSA: 17-2-0028-2010

5) $\int \frac{x^3+1}{x^3+x}\,dx$

$x^3+1 : x^3+x = 1 + \left(\frac{-x+1}{x^3+x}\right)$ ✓

$\cancel{x^3}+1$
$-\cancel{x^3} \pm x$
$-x+1$

$\int\left[1 + \frac{-x+1}{x^3+x}\right] dx$

$\underbrace{\int dx}_{I_1} + \underbrace{\int \frac{-x+1}{x^3+x}\,dx}_{I_2}$ ✓ $= x + \ln x - \frac{1}{2}\ln|x^2+1| - \operatorname{arc\,tg} x$ // ✓ 20

$x^2+1=0$
$x^2=-1$
$x=\sqrt{-1}$

$I_1 = \int dx = x$ // ✓

$I_2 = \int \frac{-x+1}{x^3+x} = \int \frac{-x+1}{x(x^2+1)} = \int \frac{A}{x}dx + \int \frac{Bx+C}{x^2+1} \Big/ \cdot (x)\cdot(x^2+1)$ ✓

$-x+1 \equiv A(x^2+1) + (Bx+C)(x)$
$-x+1 \equiv Ax^2+A + Bx^2+Cx$
$-x+1 \equiv (A+B)x^2 + Cx + A$

I $A+B=0$ $\Rightarrow$ $A+B=0$
II $C=-1$ $\quad 1+B=0$
III $A=1$ $\quad B=-1$ //

$\left\{\begin{array}{l} x^2+1=t \\ 2x\,dx=dt \\ x\,dx=\frac{dt}{2}\end{array}\right.$

$\int \frac{-x+1}{x(x^2+1)} = \int \frac{1}{x}dx + \int \frac{-1x+(-1)}{x^2+1}$

$= \ln|x| + \int \frac{-x\cdot(-1)}{x^2+1} = \ln|x| - \int \frac{x}{x^2+1}dx - \int \frac{dx}{x^2+1}$

$= \ln|x| - \frac{1}{2}\int \frac{dt}{t} - \int \frac{dx}{x^2+1} = \ln|x| - \frac{1}{2}\ln|x^2+1| - \frac{1}{1}\operatorname{arc\,tg}\frac{x}{1}$

$= \ln|x| - \frac{1}{2}\ln|x^2+1| - \operatorname{arc\,tg} x$ //

IME I PREZIME: Damagoj Lukić **BROJ INDEKSA:** 17-2-0028-2010

6) $y'' + 2y = 2x$

POGREŠNO PREPISANO

$y'' + 2y' = 2x$

$r^2 + 2 = 0$

$r^2 = -2$

$r = \sqrt{-2}$ → NE POSTOJI U $\mathbb{R}$

$r_1 = -1.4i$

$r_2 = 1.4i$

$y_0 = C_1 e^{-1.4xi} + C_2 x e^{1.4xi}$

$y = \dfrac{2x}{P'(b)}$

$P(b) = b^2 + 2b + 0$

$P'(b) = 2b + 2$

$P'(1) = 2 \cdot 1 + 2 = 4$

$y = \dfrac{2x}{4}$

$y = y_0 + y = C_1 e^{-1.4xi} + C_2 x e^{1.4xi} + \dfrac{2x}{4}$

Popuniti odmah!

IME I PREZIME: JURE PORTADA **BROJ INDEKSA:**

DATUM: VRIJEME: OD DO

78

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Riješiti: $\int_0^1 x^2 \sin(2x^3 - 3)\,dx$ — ~~10~~ 5
2. Izračunati površinu lika između parabole $y = x^2 - 3$ i pravca $y = 2x$. — 15
3. Odrediti Taylorov razvoj funkcije $f(x) = x^3 + 3x - 4$ oko točke $x_0 = 1$. — ~~15~~ 10
4. Ispitati ekstreme funkcije $f(x, y) = x^2 + y^2 - xy - 2y + 1$. — 20
5. Riješiti: $\int \frac{x^3+1}{x^3+x}\,dx$. — ~~20~~ 18
6. Riješiti diferencijalnu jednadžbu: $y'' + 2y' = 2x$. — ~~20~~ 10

1) $\int_0^1 x^2 \sin(2x^3-3)\,dx =$ $\begin{cases} 2x^3 - 3 = t \\ 6x^2\,dx = dt \\ x^2\,dx = \frac{dt}{6} \end{cases}$

$= \int_0^1 \sin t \frac{dt}{6} = \frac{1}{6}\int_0^1 \sin t = -\frac{1}{6} \cos(2x^3-3) + C \Big|_0^1$ ✓ 5

$= -\frac{1}{6}\cos(2\cdot(1^3 - 0^3) - 3) + C = -\frac{1}{6}\cos(-1) = -0{,}167$

2) $y = x^2 - 3$ $y = 2x$

x	0	1
y	0	2

$x^2 - 3 = 0$

$x^2 = 3$

$x_{12} = \pm\sqrt{3}$

$x_1 = -\sqrt{3}$

$x_2 = \sqrt{3}$

Sjecišta $y = y$

$x^2 - 2x - 3 = 0$

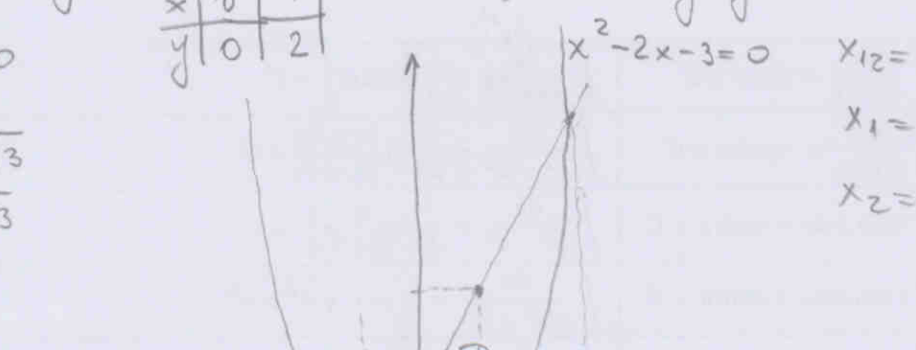
$x_{12} = \frac{2 \pm \sqrt{4+12}}{2}$

$x_1 = \frac{2-4}{2} = -1$

$x_2 = \frac{2+4}{2} = 3$

$T\left(-\frac{0}{2a}; \frac{-12}{4}\right)$

$(0; -3)$



$P = \int_{-1}^{3} 2x - (x^2-3)\,dx = \int_{-1}^{3} -x^2 + 2x + 3\,dx = -\int_{-1}^{3} x^2 dx \; 2\int_{-1}^{3} x\,dx + 3\int_{-1}^{3} dx$

$= -\left(\frac{x^3}{3}\right)\Big|_{-1}^{3} + 2\left(\frac{x^2}{2}\right)\Big|_{-1}^{3} + 3x\Big|_{-1}^{3} = -\left(\frac{27}{3} + \frac{1}{3}\right) + (9-1) + 3(3+1)$

$= -9{,}34 + 8 + 12 = 10{,}66$ ✓ 15

Ime i prezime: JURE PORTADA **Broj indeksa:**

3) $f(x) = x^3 + 3x - 4 \qquad x_0 = 1$

$f(1) = 1 + 3 - 4 = 0$

$f(x_0) + f'(x_0)(x - x_0) + f''(x_0) \cdot \frac{(x-x_0)^2}{2!} + f'''(x_0) \frac{(x-x_0)^3}{3!}$

?
1.

$f'(x) = 3x^2 + 3$

$f'(1) = 3 + 3 = 6$

$Tay = 0 + \cancel{4}\, 6(x-1) + \cancel{6}^{3} \cdot \frac{(x-1)^2}{\cancel{2}_1} \rightarrow x^2 - 2x + 1 \qquad + 6 \frac{(x-1)^3}{6}$

$= 6x - 6 + 3x^2 - 6x + 3$

$f''(x) = \cancel{3}\, 6x$

$= 3x^2 - 3$

$\underline{10}$

$f''(1) = 6 \cdot 1 = 6$

$f'''(x) = 6$

4) Ispitati ex

$z = x^2 + y^2 - xy - 2y + 1 \qquad T\left(\frac{2}{3}; \frac{4}{3}\right)$

$z_x = 2x - y$

$z_y = 2y - x - 2$

$2x - y = 0 \quad /\cdot 1 \qquad 2 \cdot x - \frac{4}{3} = 0$

$2y - x - 2 = 0 \quad /\cdot 2 \qquad 2x = \frac{4}{3}$

$2x - y = 0 \qquad x = \frac{\cancel{4}^2 / 3}{\cancel{2}_1 / 1} = \frac{2}{3}$

$-2x + 4y - 4 = 0$

$3y - 4 = 0$

$3y = 4$

$y = \frac{4}{3}$

$D = 2 \cdot 2 - 1 = 3$ ima Ex $\qquad \underline{20}$

$z_{xx} = 2 \qquad r_0 = 2 > 0$ min

$z_{xy} = -1 \qquad t_0 = -1$

$z_{yy} = 2 \qquad s_0 = 2$

$z = \left(\frac{2}{3}\right)^2 + \left(\frac{4}{3}\right)^2 - \frac{2}{3} \cdot \frac{4}{3} - 2\left(\frac{4}{3}\right) + 1$

$z = \frac{4}{9} + \frac{16}{9} - \frac{8}{9} - \frac{8}{3} + 1 = \frac{4 + 16 - 8 - 24 + 9}{9}$

$z = -\frac{3}{9} = -\frac{1}{3}$

IME I PREZIME: Jure Portada BROJ INDEKSA:

5) $\int \frac{x^3+1}{x^3+x}\,dx = \int \frac{x^3+x-x+1}{x^3+x}\,dx = \underbrace{\int \frac{x^3+x}{x^3+x}\,dx}_{\mathrm{I}} + \underbrace{\int \frac{-x+1}{x^3+x}\,dx}_{\mathrm{II}}$

$(x^3+x) = x(x^2+1)$

$\mathrm{I} = \int dx = x + C$

$\mathrm{II} = -\int \frac{x-1}{x(x^2+1)} = \int \frac{A}{x} + \int \frac{Bx+C}{(x^2+1)}$

$x-1 = Ax^2 + A + Bx^2 + Cx$

$x-1 = (A+B)x^2 + Cx + A$

$A+B=0 \qquad B=1$

$C=1$

$A=-1$

$A=-1$

$-\int \frac{x-1}{x(x^2+1)} = -\int \frac{dx}{x} + \int \frac{x+1}{(x^2+1)}\,dx$

$-\; = \left(-\ln(x) + \int \frac{x}{x^2+1}dx + \int \frac{dx}{x^2+1}\right)$

$= -\ln(x) + \int \frac{x}{x^2+1}\,dx + \int \frac{dx}{x^2+1}$

$= \begin{cases} 2x = dt \\ x\,dx = \frac{dt}{2} \end{cases}$

$= -\ln(x) + \frac{1}{2}\ln(2x+1) + \frac{1}{1}\arctan x$

$= \ln(x) - \frac{1}{2}\ln(2x+1) - \arctan x$

18

$\int \frac{x^3+1}{x^3+x} = \mathrm{I} + \mathrm{II} = x - \ln(x) + \frac{1}{2}\ln(2x+1) + \arctan x$

$+ \qquad - \qquad -$

IME I PREZIME: JURE PORTADA BROJ INDEKSA:

6) $y'' + 2y' = 2x$

$r^2 + 2r = 0$

$r(r+2) = 0$

$r_1 = 0$

$r_2 = -2$

$y_0(x) = C_1 e^{-2x} + C_2 e^0$

10

$\eta = a_2 x^2 + a_1 x + a_0$

$\eta' = 2a_2 x + a_1$

$\eta'' = 2a_2$

$2a_2 + 2(2a_2 x + a_1) = 2x$

$2a_2 + 4a_2 x + 2a_1 = 2x$

$2a_2 = 0$ $a_2 =$

$4a_2 = 2$

$2a_1 = 0$

?

$2a_2 + 2a_1 = 0 \Rightarrow a_1 = -\frac{1}{2}$

$\Uparrow$

$4a_2 = 2 \Rightarrow a_2 = \frac{1}{2}$

$\eta = \frac{1}{2} x^2 - \frac{1}{2} x$

Popuniti odmah!

IME I PREZIME: Mateja Mitrović   BROJ INDEKSA: 0269037541

DATUM: 22.09.2011.   VRIJEME: OD   DO

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

52

Broj ↓ bodova

1. Riješiti: $\int_0^1 x^2 \sin(2x^3 - 3)\,dx$ ~~10~~ 8
2. Izračunati površinu lika između parabole $y = x^2 - 3$ i pravca $y = 2x$. ~~15~~ 12
3. Odrediti Taylorov razvoj funkcije $f(x) = x^3 + 3x - 4$ oko točke $x_0 = 1$. ~~15~~ ~~14~~ 12
4. Ispitati ekstreme funkcije $f(x, y) = x^2 + y^2 - xy - 2y + 1$. 20
5. Riješiti: $\int \frac{x^3 + 1}{x^3 + x}\,dx$. ~~20~~
6. Riješiti diferencijalnu jednadžbu: $y'' + 2y' = 2x$. 20

IME I PREZIME: Mateja Mitrović BROJ INDEKSA: 0269037541

$1. \int_0^1 x^2\sin(2x^3-3)\,dx = \left|\begin{array}{l} 2x^3-3=t \\ 6x^2dx=dt \quad |:6x^2 \\ dx=\frac{dt}{6x^2}\end{array}\right| = \int_0^1 \cancel{x^2}\sin t\cdot\frac{dt}{6\cancel{x^2}} = \int_0^1 \sin t\cdot\frac{dt}{6} = \frac{1}{6}\int_0^1 \sin t\,dt = -\frac{1}{6}\cdot(\cos t)+C$

$= -\frac{1}{6}\cdot\cos(2x^3-3)\Big|_0^1 = -\frac{1}{6}\cdot\cos(2\cdot1^3-3) - \left(-\frac{1}{6}\cdot\cos(2\cdot0^3-3)\right) = -0.166+0.166=0$

8

X

VIDI NEKIĆ

2. $y=x^2-3 \quad y=2x$

$y=x^2-3$

$x^2-3=0$

$x^2=3 \,/\sqrt{\ }$

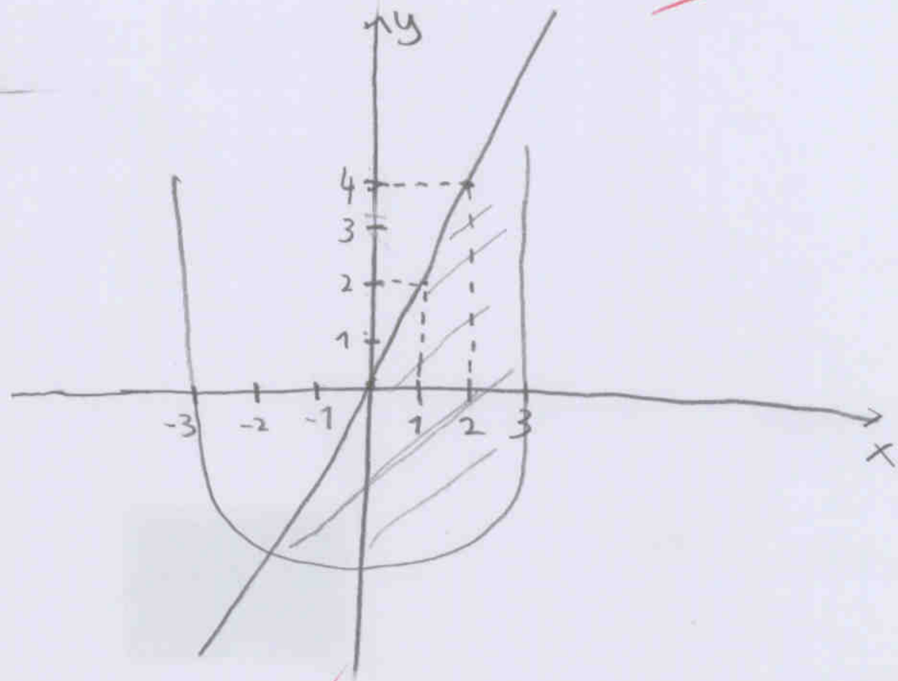
$x=\pm\sqrt{3}$

$x_1=-3$

$x_2=+3$

x	0	1	2
$y=2x$	0	2	4

$a>0 \ \cup$



$x^2-3=0$

$\underline{2x=0}$

$x^2-3=2x$

$x^2-3-2x=0$

$x^2-2x-3=0$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_{1,2}=\frac{2\pm\sqrt{(-2)^2-4\cdot1\cdot(-3)}}{2\cdot1}=$

$x_{1,2}=\frac{2\pm\sqrt{16}}{2}=\frac{2\pm4}{2}$

$x_1=\frac{2-4}{2}=\frac{-2}{2}=-1 \quad x_2=\frac{2+4}{2}=3$

$P=\int_{-1}^{3}(2x)-(x^2-3)\,dx$

$P=\int_{-1}^{3}2x-x^2+3\,dx=\int_{-1}^{3}-x^2+2x+3\,dx$

$P=\left(-\frac{x^3}{3}+\frac{2x^2}{2}+3x\right)\Big|_{-1}^{3}$

$P=\left(-\frac{3^3}{3}-\frac{2\cdot3^2}{2}+3\cdot3\right)-\left(-\frac{(-1)^3}{3}-\frac{2\cdot(-1)^2}{2}+2\cdot(-1)\right)$

$P=-9-\frac{8}{3}=\frac{-27-8}{3}=-\frac{35}{3}$

12

Ime i prezime: Mateja Mitković

Broj indeksa: 0269037541

3. $f(x) = x^3 + 3x - 4 \quad x_0 = 1$

$f'(x) = 3x^2 + 3$

$f'(x_0) = f'(1) = 3 \cdot 1^2 + 3 = 6$

$f''(x) = 6x$

$f''(x_0) = f''(1) = 6 \cdot 1 = 6$

$f'''(x) = 6$

$f^{IV}(x) = 0$

$f(1) = 1^3 + 3 \cdot 1 - 4 = 0$

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \frac{f^{IV}(x_0)}{4!}(x - x_0)^4$$

$$f(x) = 1 + \frac{6}{1!}(x-1) + \frac{\cancel{6}^{3}}{\underset{1 \cdot 2}{2!}}(x-1)^2 + \frac{\cancel{6}^{1}}{\underset{1 \cdot 2 \cdot 3}{3!}}(x-1)^3 + \frac{0}{4!}(x-1)^4$$

$$f(x) = 1 + 6(x-1) + 3(x-1)^2 + (x-1)^3 + (x-1)^4$$

$\frac{0}{4!} = 0$

~~14~~

12

4. $f(x,y) = x^2 + y^2 - xy - 2y + 1$

$f(x,y)_x = 2x - y$

$f(x,y)_y = 2y - x - 2$

$2x - y = 0$

$2y - x - 2 = 0$

$2x - y = 0$

$2x = y \mid :2$

$x = \frac{1}{2}y = \frac{1}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{3} = \frac{2}{3}$

$2y - \frac{1}{2}y - 2 = 0$

$\frac{3}{2}y = 2 \mid \cdot \frac{2}{3}$

$y = \frac{4}{3}$

$A\left(\frac{2}{3}, \frac{4}{3}\right)$

$f''(x,y)_{xx} = 2$

$f''(x,y)_{xy} = -1$

$f''(x,y)_{yx} = -1$

$f''(x,y)_{yy} = 2$

$$H_A = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 4 - 1 = 3 > 0$$

u točki $A\left(\frac{2}{3}, \frac{4}{3}\right)$ je lokalni minimum

$$\left(\frac{2}{3}\right)^2 + \left(\frac{4}{3}\right)^2 - \frac{2}{3} \cdot \frac{4}{3} - 2 \cdot \frac{4}{3} + 1 =$$

$$\frac{4}{9} + \frac{16}{9} - \frac{8}{9} - \frac{8}{3} + 1 = \frac{\cancel{12}^{4}}{\cancel{9}_{3}} - \frac{8}{3} + 1 = -\frac{4}{3} + 1 = \frac{-4 + 3}{3} = -\frac{1}{3}$$

20

IME I PREZIME: Mateja Mitrović

BROJ INDEKSA: 0269037541

$$\int \frac{x^3+1}{x^3+x}\,dx = \int dx - \int \frac{x^3+x}{-\frac{x}{x^3}}\,dx$$

$$(x^3+1):(x^3+x) = 1 - \frac{1}{x^3}$$

$$\underline{-x^3 \pm 1 \pm x}$$

$$-1 - x$$

$$\underline{\overset{+}{-1} - \frac{x}{x^3}}$$

$$-\frac{x}{x^3}$$

Popuniti odmah!

IME I PREZIME: BUTERIN ŠIME **BROJ INDEKSA:** 17-2-0099-2010

25

DATUM: VRIJEME: OD DO .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Riješiti: $\int_0^1 x^2 \sin(2x^3 - 3)\,\mathrm{d}x$	~~10~~ 5
2. Izračunati površinu lika između parabole $y = x^2 - 3$ i pravca $y = 2x$.	~~15~~
3. Odrediti Taylorov razvoj funkcije $f(x) = x^3 + 3x - 4$ oko točke $x_0 = 1$.	~~15~~
4. Ispitati ekstreme funkcije $f(x, y) = x^2 + y^2 - xy - 2y + 1$.	20
5. Riješiti: $\int \frac{x^3 + 1}{x^3 + x}\,\mathrm{d}x$.	20
6. Riješiti diferencijalnu jednadžbu: $y'' + 2y' = 2x$.	20

IME I PREZIME: BUTERIN ŠIME BROJ INDEKSA: 17-2-0049-2010

$$\int_0^1 x^2 \sin(2x^3-3)\,dx = \left\{\begin{array}{l} 2x^3-3=t \;/d \\ dx\, 6x^2 = dt \;/:6 \\ x^2 dx = \frac{dt}{6} \end{array}\right\}$$

$$\int_0^1 \sin t \cdot \frac{dt}{6}$$

$$\frac{1}{6}\int_0^1 \sin t \cdot dt$$

$$-\frac{1}{6}\cos(2x^3-3)+C$$

5

$$-\frac{1}{6}\cos(2\cdot 1^3-3)-(2\cdot 0^3-3)+C$$

$$-\frac{1}{6}\cos -1+3+C$$

$$-\frac{1}{6}\cos +2+C$$

IME I PREZIME: BUTERIN ŠIME BROJ INDEKSA:

(2) $f(x) = x^2 - 3$ – PARABOLA

$y = 2x$ – PRAVAC

$2x = 0$

$x = -2$

x	0	2
y	0	4

$x^2 - 3 = 0$

$x^2 = 3$

$x_0 = \frac{-b}{2a}$

$y_0 = \frac{4ac - b^2}{4a}$

$y = y$

$x^2 - 3 = 2x$

$x^2 - 2x - 3 = 0$

$\frac{-b \pm \sqrt{(b)^2 - 4 \cdot a \cdot c}}{2 \cdot a}$

$\frac{2 + \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2}$

$\frac{2 + \sqrt{16}}{2}$

$x_1 = 3$

$\frac{2 - \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2}$

$x_2 = \frac{2 - \sqrt{16}}{2}$

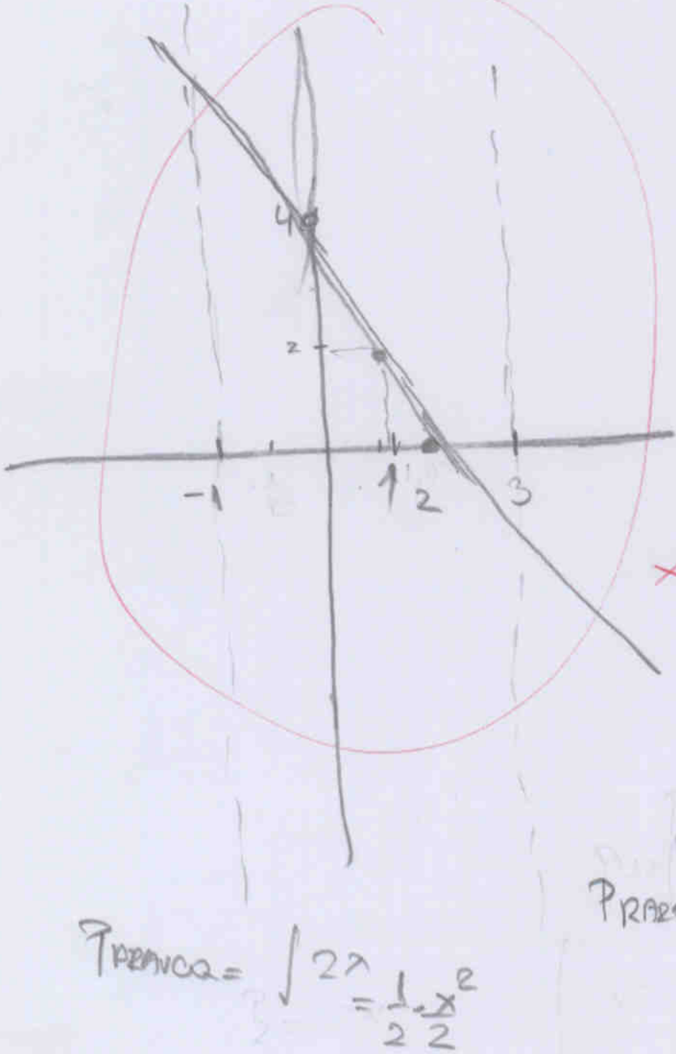
$x_2 = -1$

$\emptyset$

VIDI PORTADA

$\boxed{x_0 = \frac{2}{2 \cdot 1} \quad x_0 = 1 \quad y_0 = \frac{4 \cdot 1 \cdot (-3) - (2)^2}{4 \cdot 1} \quad y_0 = \frac{8}{4} \quad y_0 = 2}$

$T(1,2)$



$\times$

$P_{PRAVCA} = \int 2x = \frac{1}{2} \cdot \frac{x^2}{2}$

$P_{PARAB} = \int x^2 - 3 = -\frac{1}{3} \cdot \frac{x^3}{3}$

$P_{UK} = P_1 - P_2 \qquad -\frac{1}{3} \cdot \frac{x^3}{3} - \frac{1}{2} \cdot \frac{x^2}{2}$

$\frac{1}{3} \cdot \frac{(-2)^3}{3} - \frac{1}{2} \cdot \frac{(-2)^2}{2} = \frac{1}{3} \cdot \frac{(-8)}{3} - \frac{1}{2} \cdot \frac{4}{2} = \frac{-8}{9} - \frac{4}{2} = -2.88$

IME I PREZIME: BUTERIN SIME   BROJ INDEKSA: 17-2-0049-2010

③ $f(x,y) = x^3 + 3x - 4$

$f(x) = x^3 + 3x - 4$

$f'(x) = 3x^2 + 3$ ✓

$f''(x) = 6x$ ✓

$f'''(x) = 6$ ✓

$x_1 = 0$

$f(0) = 1^3 + 3 \cdot 1 - 4 = 0$ ✓

$f'(0) = 3 \cdot 1^2 + 3 = 6$ ✓

$f''(0) = 6 \cdot 1 = 6$ ✓

$f'''(0) = 6$ ✓

TAYLOROV RAZVOJ?

$\emptyset$

IME I PREZIME: BUTERIN SIME BROJ INDEKSA: 17-2-0049-2010

4) $f(x) = x^2 + y^2 - xy - 2y + 1$

$2\cancel{x} = 2x - y = 0$

$2y = 2y - x - 2 = 0$

$2x - y = 0$

$-y = -2x \quad /(-1)$

$y = 2x$

$2y - x - 2 = 0$

$2 \cdot 2x - x - 2 = 0$

$4x - x - 2 = 0$

$3x - 2 = 0$

$x = \frac{2}{3}$ //

$y = 2 \cdot \frac{2}{3}$

$y = \frac{4}{3}$ //

$T_2 = \left(\frac{2}{3}, \frac{4}{3}\right)$

$2x = 2$ - ro --- min

$2xy = -1$ - so

$2yy = 2$ - to

$D = ro \cdot to - (so)^2$

$D = 2 \cdot 2 - (-1)^2 \quad D = 4 - 1 = \underline{3}$

Ima ektrem

$Z_{min} = x^2 + y^2 - xy - 2y + 1$

$\left(\frac{2}{3}\right)^2 + \frac{4}{3}^2 - \frac{2}{3} \cdot \frac{4}{3} - 2 \cdot \frac{4}{3} + 1$

$\frac{4}{3} + \frac{8}{3} - \frac{8}{9} - \frac{8}{3} + 1 = \frac{12 + 16 - 8 - 16 + 9}{9} = 1.4$

20

Popuniti odmah!

Ime i prezime: MARKO VULELIJA **Broj indeksa:** 57660 (25)

Datum: Vrijeme: od do

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Riješiti: $\int\limits_0^1 x^2 \sin(2x^3-3)\,\mathrm{d}x$	~~10~~
2. Izračunati površinu lika između parabole $y=x^2-3$ i pravca $y=2x$.	~~15~~ 15
3. Odrediti Taylorov razvoj funkcije $f(x)=x^3+3x-4$ oko točke $x_0=1$.	~~15~~
4. Ispitati ekstreme funkcije $f(x,y)=x^2+y^2-xy-2y+1$.	~~20~~
5. Riješiti: $\int \frac{x^3+1}{x^3+x}\,\mathrm{d}x$.	~~20~~
6. Riješiti diferencijalnu jednadžbu: $y''+2y'=2x$.	~~20~~ 10

IME I PREZIME: MARKO VULELIJA BROJ INDEKSA: 57660

1.

$$\int_0^1 x^2 \sin(2x^3-3)\,dx$$

$u = x^2 \,/\, d$ $dv = \sin(2x^3-3) \,/ \int$ $\sin = u \,/\, d$

$du = 2x$ $v = \int \sin(2x^3-3)$ $-\cos = du$

$\frac{du}{2} = x$

$v \cdot$

$= u \cdot v - \int v \cdot du$

$= x^2 \cdot$

$\emptyset$

$= -0.043617$

IME I PREZIME: MARKO VULELIJA BROJ INDEKSA: 57660

2. $y = x^2 - 3$ parabola

$y = 2x$ pravac

$x^2 - 3 = 2x$

$x^2 - 3 - 2x = 0$

$x^2 - 2x - 3 = 0$

a b c

$x_{1/2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2} =$

$x_1 = \frac{2-4}{2} = -1$ $y_1 = 2 \cdot (-1) = -2$ $T_1(-1,-2)$

$x_2 = \frac{2+4}{2} = 3$ $y_2 = 2 \cdot 3 = 6$ $T_2(3,6)$

TJEME

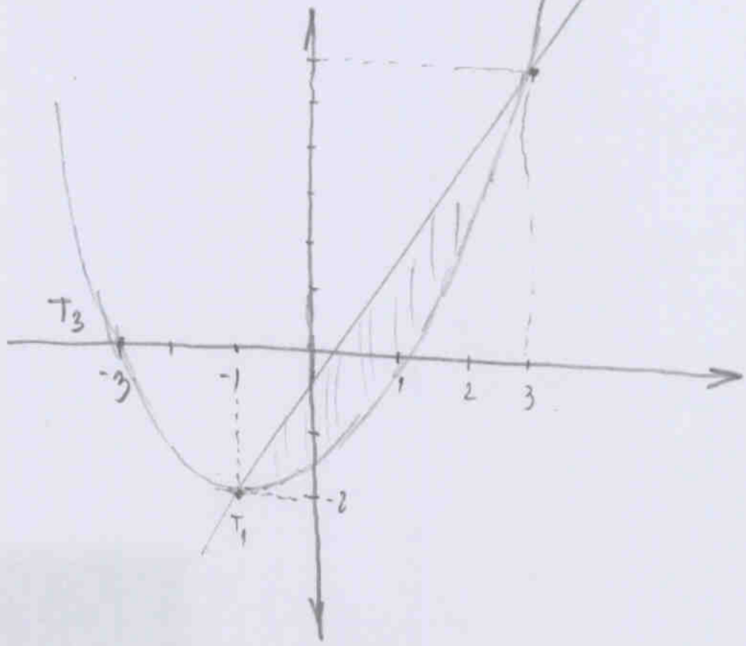
$y' = 0$

$y' = x^2 - 3$

$y' = 2x$ $y = 0^2 - 3 = -3$

$2x = 0$

$x = 0$



$P = \int_{-1}^{3} 2x - (x^2 - 3)dx = \int_{-1}^{3} (2x - x^2 + 3)dx = \int_{-1}^{3} 2x dx + \int_{-1}^{3} -x^2 dx + \int_{-1}^{3} 3 dx$

$= 2\int_{-1}^{3} x dx - \frac{x^3}{3}\Big|_{-1}^{3} + 3\int_{-1}^{3} dx = 2\frac{x^2}{2}\Big|_{-1}^{3} - \frac{x^3}{3}\Big|_{-1}^{3} + 3x\Big|_{-1}^{3} = \frac{32}{2} = \underline{10,666}$ ✓

3

15

IME I PREZIME: MARKO VULELIJA   BROJ INDEKSA: 57660

3. $f(x) = x^3 + 3x - 4$ oko točke $x_0 = 1$

$f(x_0) = 1^3 + 3 \cdot 1 - 4$
$= 1 + 3 - 4$
$= 0$

$f'(x) = 3x^2 + 3$
$f''(x) = 6x$

$\emptyset$

$f'(x) = (x^3 + 3x - 4)' = 3x + 3$ $\;\; 3x$ $f'(x_0) = 3 \cdot 1 + 3$
$= 3 + 3$
$= 6$

$\sum \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$

$= \frac{0 \cdot 1}{1!} \cdot (x-1)^0 + \frac{6 \cdot 1}{1} \cdot (x-1)' + \frac{6 \cdot 1}{2}(x-1)^2$
$+ \frac{6 \cdot 1}{2} \cdot (x-1)^3 + \frac{6 \cdot 1}{6} \cdot (x-1)^4$

$f''(x) = 3x^2 \cancel{x} = f''(x_0) = 3 \cdot 1 = 3$
$f'''(x) = 6$ $f'''(x_0) = 6$
$f^{IV}(x) = 0$ $f^{IV}(x_0) = 0$

$= 0 + 6x - 6 \frac{6(x^2+2x+1)}{2} + \frac{6(x^2 - x^2 + 3x + 1)}{x}$

$= 0 + 6x - 6 + 3(x^2 + 2x + 1) + x^2 - 3x^2 - 3 - 1$
$= 4 + x^3 + 3x$

4. $f(x,y) = x^2 + y^2 - xy - 2y + 1$

$\partial_x f = 2x - y$
$\partial_{xy} f = 0 - 1 = -1$
$\partial_{yx} f = -1$
$\partial_y f = 2y - x - 2$
$\partial_{yy} f = 2$

$\partial_x f = 0$
$\partial_x f = 0$

$2x - y = 0$
$2y - x - 2 = 0$

$-y + 2x = 0 \; / \cdot 2$
$2y - x - 2 = 0$

$-2y + 4x = 0$
$2y - x - 2 = 0$

$3x - 2 = 0$
$3x = 2 \; / : 3$
$x = \frac{2}{3}$

?

$\emptyset$

Ime i prezime: Broj indeksa:

5. $\int \frac{x^3+1}{x^3+x}\,dx = \int \frac{x^2+x+1}{x^3+x}\,dx$

IME I PREZIME: MARKO VULELIJA BROJ INDEKSA: 57660

6. $y'' + 2y' = 2x$

$r^2 + 2r = 2x$

$e^0 = 1$

$y_0 = C_1 + C_2 x e^2$

$n + r = 1 + 1 = 2$

$r_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 0}}{2} = \frac{2 \pm \sqrt{4}}{2} = \frac{2 \pm 2}{2}$

$r_1 = \frac{2-2}{2} = \frac{0}{2} = 0$

$r_2 = \frac{2+2}{2} = 2$

$r_1 = 0$

$r_2 = -2$

$n'' + 2n' = 2x$

$n = a_2 x^2 + a_1 x + a_0$

$n' = 2a_2 x + a_1$

$n'' = 2a_2$

$2a_2 + 2(2a_2 x + a_1) = 2x$

$2a_2 + 4a_2 x + 2a_1 = 2x$

$4a_2 x = 2x \quad /:2$

$a_2 = \frac{1}{2}$

$n = \frac{1}{2}x^2 - \frac{1}{2}x$

$2 \cdot \frac{1}{2} + 2a_1 = 0$

$1 + 2a_1 = 0$

$2a_1 = -1 /:2$

$a_1 = -\frac{1}{2}$

$y_0 = C_1 + C_2 x e^2 + \frac{1}{2}x^2 - \frac{1}{2}x$

10

0

Popuniti odmah!

Ime i prezime: Ivan Mandičić **Broj indeksa:** 56458

Datum: Vrijeme: od do .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

0

	Broj ↓ bodova
1. Riješiti: $\int\limits_0^1 x^2 \sin(2x^3 - 3)\,\mathrm{d}x$	~~10~~
2. Izračunati površinu lika između parabole $y = x^2 - 3$ i pravca $y = 2x$.	~~15~~
3. Odrediti Taylorov razvoj funkcije $f(x) = x^3 + 3x - 4$ oko točke $x_0 = 1$.	15
4. Ispitati ekstreme funkcije $f(x, y) = x^2 + y^2 - xy - 2y + 1$.	~~20~~
5. Riješiti: $\int \frac{x^3 + 1}{x^3 + x}\,\mathrm{d}x$.	~~20~~
6. Riješiti diferencijalnu jednadžbu: $y'' + 2y' = 2x$.	~~20~~

IME I PREZIME: Ivan Mandičić   BROJ INDEKSA: 56458

(6) $y'' + 2y' = 2x$

$y'' + 2\frac{dy}{dx} = 2x$

$2\left(\frac{dy}{dx}\right)'' + \frac{2dy}{dx} = 2x \; / dx$

$\cancel{\frac{dy}{dx} + 2dy = 2x\,dx} \; / \int$

$2dy + 2dy = 2x\,dx \; / \int$

$2\int dy + 2\int dy = 2\int x\,dx$

$2y + 2y = \cancel{2}\frac{x^2}{\cancel{2}}$

$4y = x^2$

$y = \frac{x^2}{4}$

$\emptyset$

(4) $f(x,y) = x^2 + y^2 - xy - 2y + 1$

$Df(x) = 2x - y$

$Df(y) = 2y - x - 2$

$2x - y = 0$

$2y - x - 2 = 0$

$2x = y$

$2y = x + 2$

$4x = x + 2$

$4x - x = 2$

$3x = 2$

$x = \frac{2}{3}$

$y = 2 \cdot \frac{2}{3} = \frac{4}{3}$

$\cancel{y = \frac{8}{6} = \frac{4}{3}}$

(1) $\int_0^1 x^2 \sin(2x^3 - 3)\,dx$ $\left\{\begin{matrix} t = 2x^3 - 3 & v = x^2 \\ dt = 6x^2 & dv = 2x \end{matrix}\right\}$ $\emptyset$

$t \cdot dv - \int v \cdot dt$

$(2x^3 - 3) \cdot (2x) - \int_0^1 x^2 \cdot 6x^2\,dx =$

$(4x^4 - 6x) - 6\int_0^1 x^4\,dx = (4x^4 - 6x) - 6\,\frac{x^5}{5}\Big|_0^1 = (4x^4 - 6x) - 6\left(\frac{1}{5} - \frac{0}{5}\right)$

$= (4x^4 - 6x) - \frac{6}{5} = 4x^4 - 6x - \frac{6}{5}$

IME I PREZIME: Ivan Mandičić

BROJ INDEKSA: 56458

(5) $\int \frac{x^3+1}{x^3+x}\,dx$

$(x^3+1) : (x^3+x) = 1$

$\cancel{x^3} \pm x$

$-x+1$

$\cancel{\int \frac{dx}{x^3+x}} + \int \frac{x^3+1}{-x+1}\,\cancel{dx}$

$\cancel{\left\{ \begin{array}{l} t = x^3+x \\ dt = 3x^2+1 \end{array} \right\} \int \frac{3x^2+1}{t}\,dt =}$

$x^3+1 : -x+1 = \cancel{-3}x^2 - x - 1$

$\cancel{-x^3} \mp x^2$

x^2+1

$\cancel{-x^2} \mp x$

$\cancel{x}+1$

$\cancel{-x} \mp 1$

2

$\cancel{\int \frac{dx}{-x+1}} + \cancel{\int}$

$\int \frac{dx}{x^3+x} + \int \frac{x^3+1}{-x+1}\,dx =$

(1) (2)

(1) $\int \frac{dx}{x^3+x} \left\{ \begin{array}{l} t = x^3+x \\ dt = 3x^2+1 \end{array} \right\} = \int \frac{3x^2+1}{t}\,dt = \cancel{\int}$

$= (3x^2+1) \int \frac{dt}{t} = (3x^2+1) \ln|t| = (3x^2+1) \ln|x^3+x|$

(2) $\int \frac{(-x^2-x-1)\,dx}{-x+1} + \int \frac{x^3+1}{2}\,dx = -\int \frac{(-x^2-x-1)}{t}\,dt + \frac{1}{2}\int x^3\,dx +$

$\left\{ \begin{array}{l} t = -x+1 \\ dt = -dx \end{array} \right\}$

$\frac{1}{2}\int dx = (x^2+x+1)\ln|-x+1| + \frac{1}{2}\frac{x^4}{4} + \frac{x}{2}$

$\underline{I} = (3x^2+1)\ln|x^3+x| + (x^2+x+1)\ln|-x+1| + \frac{x^4+4x}{8}$

Popuniti odmah!

IME I PREZIME: MIRO LUKIN BROJ INDEKSA: 54493-2007

DATUM: 22.09.2011 VRIJEME: OD DO .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Riješiti: $\int_0^1 x^2 \sin(2x^3 - 3)\, dx$ ~~10~~
2. Izračunati površinu lika između parabole $y = x^2 - 3$ i pravca $y = 2x$. ~~15~~
3. Odrediti Taylorov razvoj funkcije $f(x) = x^3 + 3x - 4$ oko točke $x_0 = 1$. 15
4. Ispitati ekstreme funkcije $f(x, y) = x^2 + y^2 - xy - 2y + 1$. 20
5. Riješiti: $\int \frac{x^3 + 1}{x^3 + x}\, dx$. 20
6. Riješiti diferencijalnu jednadžbu: $y'' + 2y' = 2x$. 20

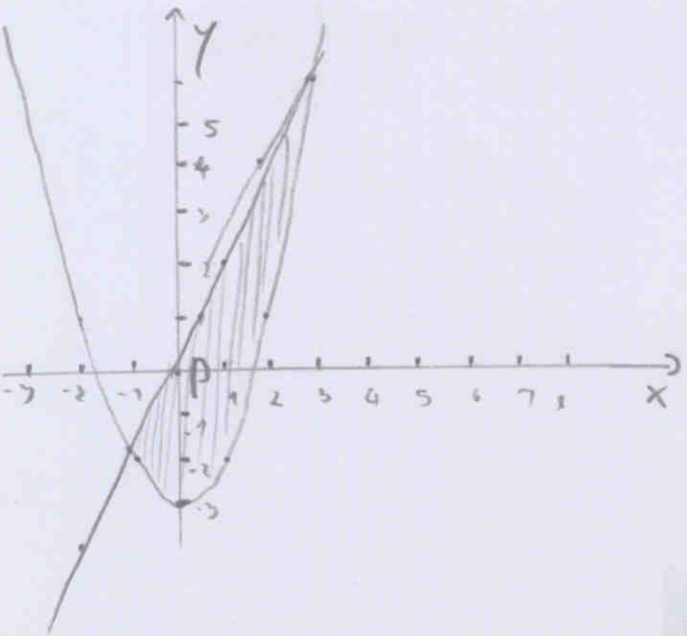
0

IME I PREZIME: MIRO LUKIN BROJ INDEKSA: 54493-2007

1) $\int_0^1 x^2 \sin(2x^3-3)\,dx$ $\emptyset$

2) $y = x^2 - 3$, $y = 2x$

x	-1	0	1	2	3	-2
y	-2	-3	-2	1	6	1



x	-2	-1	0	1	2	3
y	-4	-2	0	2	4	6

$x^2 - 3 = 2x$

$x^2 - 2x - 3 = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{2ac}}{2c}$$

$$= \frac{2 \pm \sqrt{2 \cdot 1 \cdot (-3)}}{2}$$

$$= \frac{2 \pm \sqrt{6}}{2}$$

$\emptyset$

P = ?