Popuniti odmah!

IME I PREZIME: SILVIJAN CVAR **BROJ INDEKSA:** ~~17-2-0066-2210~~ 0269052515

DATUM: 8.09.2011. **VRIJEME:** OD DO .

85

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Izračunati $\int_0^1 \sin^3 y \, \mathrm{d}y$. 15
2. Izračunati $\int e^{2x} x^2 \, \mathrm{d}x$. 15
3. Grafički prikazati funkciju $f(x,y) = \frac{x^2}{y}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim_{(x,y)\to(0,0)} f(x,y)$? 15
4. Istražiti domenu i ekstreme funkcije $f(x,y) = x^3 - 3xy + y^2$. 20
5. Pronaći opće rješenje problema: $y' + xy^2 + x = 0$. 20
6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = e^{x^2}$ oko točke $x_0 = 0$. ~~15~~ 0

① $\int_0^1 \sin^3 y\, dy = \int (\sin^2 y \cdot \sin y) dy = \int ((1-\cos^2 y)\cdot \sin y) dy$

$\int (\sin y - \sin y \cos^2 y)\, dy = \underbrace{\int \sin y\, dy}_{I_1} - \underbrace{\int \sin y \cos^2 y\, dy}_{I_2}$

$I_1 = \int \sin y\, dy = -\cos y$

$I_2 = \int \sin y \cos^2 y\, dy = \left\{ \begin{array}{l} \cos y = t \\ -\sin y\, dy = dt \\ \sin y\, dy = -dt \end{array} \right.$

$= \int t^2 \cdot (-dt) = -\int t^2\, dt = -\frac{t^3}{3} + C_{//} = -\frac{\cos^3 y}{3} + C = -\frac{1}{3}\cos^3 y_{//}$

$\boxed{I = -\cos y + \frac{1}{3}\cos^3 y}$ ✓

$\left(-\cos y + \frac{1}{3}\cos^3 y\right)\Big|_0^1 = \left(-\cos 1 + \frac{1}{3}\cos^3 1\right) - \left(-\cos 0 + \frac{1}{3}\cos^3 0\right)$

$= \left(-0{,}54 + \frac{1}{3}\cdot 0{,}15\right) - \left(-1 + \frac{1}{3}\cdot 1\right)$

$= (-0{,}54 + 0{,}05) - (-1 + 0{,}33)$

$= (-0{,}49) - (-0{,}67)$

$= -0{,}49 + 0{,}67 = 0{,}18_{//}$ ✓

15

IME I PREZIME: SILVIJAN CUAR BROJ INDEKSA: 17-2-0066-2010

(2) $\int e^{2x} x^2 dx = \begin{cases} u = x^2 & dv = e^{2x} dx \\ du = 2x\,dx & \int dv = \int e^{2x} dx \end{cases}$

$\int u\,dv = uv - \int v\,du$

$= x^2 \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} \cdot 2x\,dx$

$= x^2 \cdot \frac{1}{2} e^{2x} - \int e^{2x} \cdot x\,dx$ ✓

$I = x^2 \cdot \frac{1}{2} e^{2x} - I$

$v = \int e^{2x} dx = \begin{cases} 2x = t \\ 2dx = dt \,/\cdot \frac{1}{2} \\ d = \frac{1}{2} dt \end{cases}$

$v = \int e^t \cdot \frac{1}{2} dt$

$v = \frac{1}{2} \int e^t dt$

$v = \frac{1}{2} e^t = \frac{1}{2} e^{2x}$ ✓

PONOVO INTEGRACIJA

$u = x \qquad dv = e^{2x} dx$

$du = dx \qquad v = \int e^{2x} dx$

$v = \frac{1}{2} e^{2x}$

$= x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} dx$

$= x \cdot \frac{1}{2} e^{2x} - \frac{1}{2} \int e^{2x} dx$

$= x \cdot \frac{1}{2} e^{2x} - \frac{1}{2} \cdot \frac{1}{2} e^{2x}$ ✓

$= x \cdot \frac{1}{2} e^{2x} - \frac{1}{4} e^{2x}$ ✓

$I = x^2 \cdot \frac{1}{2} e^{2x} - \left(x \cdot \frac{1}{2} e^{2x} - \frac{1}{4} e^{2x}\right)$ ✓

15

NEDOSTAJU OVE ZAGRADE

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3.) $f(x,y) = \frac{x^2}{y}$

$\underline{D(f) = \{(x,y) : y \neq 0\}}$ ✓

$f(x,y) = c$

$\frac{x^2}{y} = c \,/\cdot y$

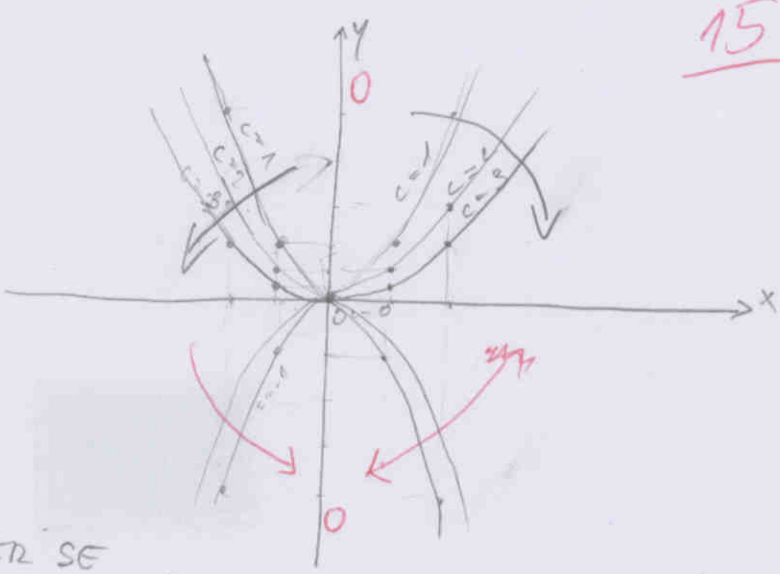
$x^2 = c \cdot y \Rightarrow \underline{\mathcal{J}m(f) = \mathbb{R}}$

RAZINSKE KRIVULJE.

$\frac{x^2}{y} = c$

$y = \frac{x^2}{c}$

$c = -1$	$y = -x^2$
$c = 0$	$y = \cancel{0}$
$c = 1$	$y = x^2$
$c = 2$	$y = \frac{x^2}{2} = \frac{1}{2}x^2$
$c = 4$	$y = \frac{x^2}{4} = \frac{1}{4}x^2$



15

LIMES $\underline{\text{NE POSTOJI}}$ JER SE U TOČKI $T(0,0)$ SIJEKU RAZLIČITE RAZINSKE KRIVULJE. ✓

x	-2	-1	0	1	2
y	4	1	0	1	4

$y = x^2$

x	-2	-1	0	1	2
y	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1

$\frac{1}{4}x^2$

$\frac{1}{4} \cdot (-2)^2$

$\frac{1}{4} \cdot 4$

x	-2	-1	0	1	2
y	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2

$\frac{1}{2}x^2$

$\frac{1}{2} \cdot 4 = 2$

x	-2	-1	0	1	2
y	-4	-1	0	-1	-4

$-x^2$

Ime i prezime: SILVIJAN CVAR **Broj indeksa:** 17-2-0066-2010

1. $f(x,y) = x^3 - 3xy + y^2$

$\underline{D(f) = \mathbb{R} \times \mathbb{R}}$

P.D.I.

$\partial_x f = 3x^2 - 3y$

$\partial_y f = -3x + 2y$

STACIONARNE TOČKE, TAMO GDJE JE:

$df(T) = 0$

$3x^2 - 3y = 0 \quad /:3$

$-3x + 2y = 0$

$x^2 - y = 0$

$-y = -x^2 \quad / \cdot (-1)$

$y = x^2$

$-3x + 2y = 0$

$-3x + 2x^2 = 0$

$2x^2 - 3x = 0$

$x(2x - 3) = 0$

$x_1 = 0$

$y_1 = 0$

$T_1(0,0)$

$2x - 3 = 0$

$2x = 3 \quad /:2$

$x = \frac{3}{2}$

$y = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$T_2\left(\frac{3}{2}, \frac{9}{4}\right)$

20

P.D.II

$\partial_{xx} f = 6x$

$\partial_{xy} f = \partial_{yx} f = -3$

$\partial_{yy} f = 2$

$T_1(0,0)$

$A = \partial_{xx} f(0,0) = 6 \cdot 0 = 0$

$B = \partial_{xy} f(0,0) = -3$

$C = \partial_{yy} f(0,0) = 2$

$\Delta = AC - B^2$

$= 0 \cdot 2 - (-3)^2$

$= 0 - (9)$

$\Delta = -9 < 0$

SEDLASTA TOČKA

$T_2\left(\frac{3}{2}, \frac{9}{4}\right)$

$A = \partial_{xx} f\left(\frac{3}{2}, \frac{9}{4}\right) = 6 \cdot \frac{3}{2} = 9 > 0$

$B = \partial_{xy} f\left(\frac{3}{2}, \frac{9}{4}\right) = -3$

$C = \partial_{yy} f\left(\frac{3}{2}, \frac{9}{4}\right) = 2$

$\Delta = AC - B^2$

$= 9 \cdot 2 - (-3)^2$

$= 18 - 9 = 9 > 0$

$\Delta > 0, A > 0$

⇒ U TOČKI $T_2\left(\frac{3}{2}, \frac{9}{4}\right)$ NALAZI SE LOKALNI MINIMUM.

$f\left(\frac{3}{2}, \frac{9}{4}\right) = \left(\frac{3}{2}\right)^3 - 3 \cdot \left(\frac{3}{2} \cdot \frac{9}{4}\right) + \left(\frac{9}{4}\right)^2$

$= \frac{27}{8} - 3 \cdot \left(\frac{27}{8}\right) + \frac{81}{16}$

$= \frac{27}{8} - \frac{81}{8} + \frac{81}{16} =$

$= \frac{-54}{8} + \frac{81}{16} = \frac{-108 + 81}{16}$

$= -\frac{27}{16}$

Ime i prezime: SILUDAN CUAR **Broj indeksa:** 17-2-0066-2010

(5) $y' + xy^2 + x = 0$

$$\frac{dy}{dx} + xy^2 + x = 0$$

$$\frac{dy}{dx} = -xy^2 - x$$

$$\frac{dy}{dx} = -x(y^2+1) \,/\cdot dx$$

$$\int \frac{dy}{y^2+1} = \int -x\,dx$$

$$\int \frac{dy}{y^2+1} = -\int x\,dx$$

$$\frac{1}{1}\arctan\frac{y}{1} = -\frac{x^2}{2} + c \quad ✓$$

$$\arctan y = -\frac{x^2}{2} + c \quad ✓$$

20

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(6) TAYLOROV RAZVOJ.

$f(x) = e^{x^2}$ $\quad f(0) = e^0 = 1 \quad x_0 = 0$

$f'(x) = e^{x^2} \cdot (x^2)'$ ✓

$= e^{x^2} \cdot 2$ ✗

$= 2 \cdot e^{x^2}$ $\quad f'(0) = 2 \cdot e^0 = 2$

$f'(x) = e^{x^2} \cdot 2x$

$f''(x) = \cancel{2' \cdot e^{x^2}} + 2 \cdot (e^{x^2})'$ $\quad f''(0) = 4 \cdot e^0 = 4$

$= 2 \cdot 2e^{x^2}$

$= 4 \cdot e^{x^2}$

0

$f'''(x) = \cancel{4' \cdot e^{x^2}} + 4 \cdot (e^{x^2})'$ $\quad f'''(0) = 8 \cdot e^0 = 8$

$= 4 \cdot 2e^{x^2}$

$= 8e^{x^2}$

VIDI PAŽLJIVO

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f^n(x_0)}{n!}(x-x_0)^n$$

$$e^{x^2} = 1 + \frac{2}{1}(x-0) + \frac{4}{2}(x-0)^2 + \frac{8}{6}(x-0)^3$$

$$e^{x^2} = 3(x-0) + 2(x-0)^2 + \frac{8}{6}(x-0)^3$$

Popuniti odmah!

Ime i prezime: Kristijan Paleka **Broj indeksa:** 57308 - 2009 22

Datum: 8.9.2011. **Vrijeme:** od do .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Izračunati $\int_0^1 \sin^3 y \, dy$.	~~15~~ 0
2. Izračunati $\int e^{2x} x^2 \, dx$.	~~15~~ 7
3. Grafički prikazati funkciju $f(x,y) = \frac{x^2}{y}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?	15
4. Istražiti domenu i ekstreme funkcije $f(x,y) = x^3 - 3xy + y^2$.	20
5. Pronaći opće rješenje problema: $y' + xy^2 + x = 0$.	20
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6. $f(x) = e^{x^2} \quad x_0 = 0$

$f(x) = e^{x^2} \Rightarrow f(0) = e^{0^2} = 1$ ✓

$f^{(1)}(x) = e^{x^2} \cdot 2x \Rightarrow f^{(1)}(0) = e^{0^2} \cdot 2 \cdot 0 = 0$ ✓

$f^{(2)}(x) = (e^{x^2} \cdot 2x) \cdot 2x + e^{x^2} \cdot 2 = 4x^2 e^{x^2} + 2e^{x^2} \Rightarrow f^{(2)}(x) = 2$ ✓

$f^{(3)}(x) = 8x \cdot e^{x^2} + 4x^2 \cdot e^{x^2} \cdot 2x + 2e^{x^2} \cdot 2x \Rightarrow f^{(3)}(0) = 0$ ✓

$= 8xe^{x^2} + 8x^3 e^{x^2} + 4xe^{x^2}$

$f^{(4)}(x) = 8 \cdot e^{x^2} + 8x \cdot e^{x^2} \cdot 2x + 24x^2 \cdot e^{x^2} + 8x^3 \cdot e^{x^2} \cdot 2x + 4 \cdot e^{x^2} + 4x \cdot e^{x^2} \cdot 2x$

$= \underbrace{8 \cdot e^{x^2}}_{8} + \underbrace{16x^2 e^{x^2}}_{0} + \underbrace{24x^2 e^{x^2}}_{0} + \underbrace{16x^4 e^{x^2}}_{0} + \underbrace{4e^{x^2}}_{4} + \underbrace{8x^2 e^{x^2}}_{0}$

$= 12$

$\Rightarrow f^{(4)}(0) = 12$

$e^{x^2} = 1 + \overbrace{\frac{0 \cdot (x-x_0)^1}{1!}}^{0} + \frac{2 \cdot (x-x_0)^2}{2!} + \overbrace{\frac{0 \cdot (x-x_0)^3}{3!}}^{0} + \frac{12 \cdot (x-x_0)^4}{4!} + \ldots$

$= 1 + \frac{2x^2}{2} + \frac{12x^4}{24}$

$= 1 + x^2 + \frac{x^4}{2}$ ✓

15

IME I PREZIME: KRISTIJAN PALEKA BROJ INDEKSA: 57308 - 2009

(1.) $\int_0^1 \underbrace{\sin^3 y}_{\sin^2 y \cdot \sin y} dy = \left| \begin{array}{ll} \sin y = u /' & \cos y\, dy = du \\ \sin^2 y\, dy = dv & v = \int \sin^2 y\, dy \\ & = -\frac{\cos y^3}{3} \end{array} \right|$ X X

VIDI CVAR

$= -\sin y \cdot \frac{\cos^3 y}{3} + \frac{1}{3} \int \cos^3 y \cos y\, dy$

$= -\sin y \cdot \frac{\cos^3 y}{3} + \frac{\sin^4 y}{12}$

$= -\sin(1-0) \cdot \frac{(\cos^3 1) - (\cos^3 0)}{3} + \frac{(\sin^4 1) - \sin^4 0}{12}$

$= -\frac{\pi}{2} + \frac{\pi}{6}$

$\cancel{0}$

(2.) $\int e^{2x} x^2 dx = \left| \begin{array}{ll} x^2 = u /' & 2x\, dx = du \\ e^{2x} dx = dv & v = \int e^{2x} dx \\ & = e^{2x} \end{array} \right|$ X

7

$= x^2 \cdot e^{2x} - 2 \int e^{2x} x\, dx = \left| \begin{array}{ll} x = u /' & dx = du \\ e^{2x} dx = dv & v = \int e^{2x} dx \\ & = e^{2x} \end{array} \right|$ X

$= x^2 \cdot e^{2x} - 2 \left(x \cdot e^{2x} - \int e^{2x} dx \right)$

$= x^2 e^{2x} - 2 \left(x e^{2x} - e^{2x} \right)$

$= x^2 e^{2x} - 2 x e^{2x} + 2 e^{2x}$

$= e^{2x} (x^2 - 2x + 2)$

Popuniti odmah!

IME I PREZIME: MARKO VULELIJA BROJ INDEKSA: 57660

15

DATUM: VRIJEME: OD DO

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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3.	Grafički prikazati funkciju $f(x,y) = \frac{x^2}{y}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim_{(x,y)\to(0,0)} f(x,y)$?	15
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1. $\int_0^1 \sin^3 y\,dy = \int_0^1 \sin^2 y \cdot \cos y\,dy = \left\{ \begin{matrix} \sin y = t \,/^d \\ \cos y\,dy = dt \end{matrix} \right\} = \int_0^1 t^2\,dt = \frac{t^3}{3} + C \Big|_0^1$

dt

$= \frac{\sin y^3}{3} + C \Big|_0^1$

×

VIDI GVAR

$= \frac{\sin 1^3}{3} - \frac{\sin 0^3}{3} = 0{,}2804 + C$

0

2. $\int e^{2x} x^2\,dx = x^2 \cdot \frac{1}{2} e^{2x} = \int \frac{1}{2} e^{2x} \cdot 2x\,dx$

$x^2 = u \,/^d$
$2x\,dx = du$

$v = \int e^{2x} dx$
$v = \frac{1}{2}\int e^t dt$
$= \frac{1}{2} e^t = \frac{1}{2} e^{2x}$

$2x = t \,/^d$
$2dx = dt \,/:2$
$dx = \frac{dt}{2}$

$x = u \,/^d$
$dx = du$

$v = \int e^{2x} dx = \frac{1}{2}$

$= \frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx$

$= \frac{1}{2} x^2 e^{2x} - \left[x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} dx \right]$

$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{2} \int e^{2x} dx$

$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{2} \cdot \frac{1}{2} e^{2x} + C$

✓ 15

Ime i prezime: MARKO VULEVIJA Broj indeksa: 57660

5.

$y'+xy^2+x=0$

$\frac{dy}{dx}+xy^2+x=0$

$\frac{dy}{dx}=-xy^2-x \;/dx\, y^2$

$dy\cdot y^2=-xdx-x$

$\varnothing$

4.

$f(x,y)=x^3-3xy+y^2$

$\varnothing$