Popuniti odmah!

Ime i prezime: IVAN VIDAKOVIĆ **Broj indeksa:** 57188

Datum: Vrijeme: od 9:56 do .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

20

~~36~~

Broj ↓ bodova

1. Izračunati $\int \frac{x^2+2x+2}{x^2+x-2}\,dx$. 15
2. Izračunati $\int x^2 \sin(x)\,dx$. 15
3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala:
$$\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,dx$$ 15
4. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$. 20
5. Pronaći opće rješenje problema: $y' + xy + x = 0$. 20
6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = 2x\cos x$ oko točke $x_0 = 0$. ~~15~~ 5

2) $\int x^2 \sin(x)\,dx$ $\quad u\cdot v - \int v\,du$

$u = x^2 \quad v = -\cos x$

$du = 2x\,dx \quad dv = \sin x$

$\left[\int x\cos x\,dx \quad u = x \quad du = 1\,dx \quad v = \sin x \quad dv = \cos x\,dx \quad x\cdot\sin x - \int \sin x\,dx \quad x\cdot\sin x + \cos x + C \right]$

$-x^2\cdot\cos x + \int \cos x \cdot 2x\,dx$ ✓

$-x^2\cdot\cos x + 2\int [x\cos x]\,dx$

$-x^2\cdot\cos x + 2(x\sin x + \cos x) + C$

$-x^2\cos x + 2x\sin x + 2\cos x + C$ ✓

15

6.) $f(x) = 2x\cos x \qquad x_0 = 0$

$f(0) = 2\cdot 0\cdot\cos x = 0$ ✓

$f'(x) = 2\cos x - 2x\sin x \Rightarrow f'(0) = 2\cdot\cos 0 - 2\cdot 0\sin 0 = \cancel{1}\ 2$

$f''(x) = 0\cdot\cos x - 2\sin x - (2\sin x + 2x\cos x) \Rightarrow f''(0) = -2\cdot\sin(0) - 2\sin 0 + 2\cdot 0\cdot\cos 0 = 0$ ✓

$\Downarrow\ 0 \quad - \quad \Downarrow\ 0 \quad \Downarrow\ 0$

$f'''(x) = \cancel{0\cdot\sin x} - 2\cos x - \cancel{0\cdot\sin x} - 2\cos x + 2\cos x - 2x\sin x$ ✓

PREDZNAK $\Rightarrow f'''(0) = -2\cos 0 - 2\cos 0 + 2\cos 0 - 2\cdot 0\sin 0$

$= -2 - 2 + 2 - 0 = -2$ ✗

$f''''(x) = \cancel{0\cdot\cos x} + 2\sin x \ \cancel{- 0\cdot\cos x} + 2\sin x + \cancel{0\cdot\cos x} - 2\sin x - 2\sin x - 2x\cos x$

$\Downarrow\ 0 \quad 0 \quad 0 \quad 0 \quad 0$

$f''''(0) = 0$ ✓

IME I PREZIME: IVAN VIDAKOVIĆ BROJ INDEKSA: 57188

6.) $f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots$

$f(x) = 0 + \frac{1}{1!}(x-0) + \frac{0}{2!}(x-0)^2 + \frac{-2}{3!}(x-0)^3 + \frac{0}{4!}(x-0)^4$

$= \frac{1(x-0)}{1!} + \cancel{\frac{0(x-0)^2}{2!}} + \frac{-2(x-0)^3}{3!} + \cancel{\frac{0(x-0)^4}{4!}}$

$= \frac{2x}{1!} + \frac{-2(x-0)^3}{3!} = \frac{2x}{1} - \frac{2x^3}{6}$ ✓

$\cancel{3}$ 5

VIDI PORTADA, KORAK, KRAJEV

Popuniti odmah!

IME I PREZIME: ANTE ŠUŠNJARA **BROJ INDEKSA:** 57679 25

DATUM: VRIJEME: OD 10.20 DO

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
1. Izračunati $\int \frac{x^2+2x+2}{x^2+x-2}\,\mathrm{d}x$.	~~15~~ 5
2. Izračunati $\int x^2 \sin(x)\,\mathrm{d}x$.	15
3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala: $\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$	15
4. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$.	~~20~~ 5
5. Pronaći opće rješenje problema: $y' + xy + x = 0$.	20
6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = 2x\cos x$ oko točke $x_0 = 0$.	15

1) $\int \frac{x^2+2x+2}{x^2+x-2}\,dx$

$\int \frac{x^2+x-2+x+4}{x^2+x-2}\,dx$

$\int \cancel{\frac{x^2+x-2}{x^2+x-2}}\,dx + \int \frac{x+4}{x^2+x-2}\,dx$ $\left[(x+\frac{1}{2})^2 - \frac{1}{4} - 2\right.$ $\left.(x+\frac{1}{2})^2 - \frac{9}{4}\right.$

$\frac{1}{2} - \frac{3}{2} = -\frac{2}{2} = -1$

$\frac{1}{2} + \frac{3}{2} = \frac{1+3}{2} = \frac{4}{2} = 2$

$-\frac{1}{4} - \frac{2}{1} = \frac{-1-8}{4} = -\frac{9}{4}$

$\frac{1}{2} + \frac{4}{1} = \frac{1+8}{2} = \frac{9}{2}$

$\int dx + \int \frac{x+4}{(x+\frac{1}{2})^2 - \frac{9}{4}}\,dx$

$x + \int \frac{x+\frac{1}{2}-\frac{1}{2}+4}{(x+\frac{1}{2})^2-\frac{9}{4}}\,dx$

$\int \frac{x^2+2x+2}{x+x-2}\,dx = x + \frac{1}{12}\ln\left|\frac{x-1}{x+2}\right| - \frac{9}{2}\left(\frac{1}{12}\ln\left|\frac{x-1}{x+2}\right|\right)$

$x + \int \frac{x+\frac{1}{2}}{(x+\frac{1}{2})^2-\frac{9}{4}}\,dx - \int \frac{\frac{9}{2}}{(x+\frac{1}{2})^2-\frac{9}{4}}\,dx$

$\left[\begin{array}{l} x+\frac{1}{2} = u \\ dx = du \\ a^2 = \frac{9}{4} / \sqrt{\ } \\ a = \frac{3}{2} \end{array}\right.$ $\left[\begin{array}{l} x+\frac{1}{2} = u \\ dx = du \end{array}\right.$

5

VIDI PORTADA

$\int \frac{u\,du}{u^2-\frac{9}{4}} = \left\{\begin{array}{l} t = u^2-\frac{9}{4} \\ dt = 2u\,du \end{array}\right\}$

$= \int \frac{\frac{dt}{2}}{t} = \frac{1}{2}\int \frac{dt}{t}$

$= \frac{1}{2}\ln t$

$= \frac{1}{2}\ln\left|u^2-\frac{9}{4}\right|$

$x + \int \frac{u}{u^2-\frac{9}{4}}\,du - \frac{9}{2}\int \frac{1}{u^2-\frac{9}{4}}\,du$

$x + \frac{1}{2\cdot\frac{3}{2}}\ln\left|\frac{u-a}{u+a}\right| - \frac{9}{2}\left(\frac{1}{2\cdot\frac{3}{2}}\ln\left|\frac{u-a}{u+a}\right|\right)$

$x + \frac{1}{\frac{6}{\frac{2}{1}}}\ln\left|\frac{x+\frac{1}{2}-\frac{3}{2}}{x+\frac{1}{2}+\frac{3}{2}}\right| - \frac{9}{2}\left(\frac{1}{\frac{6}{\frac{2}{1}}}\ln\left|\frac{x+\frac{1}{2}-\frac{3}{2}}{x+\frac{1}{2}+\frac{3}{1}}\right|\right)$

(4) IME I PREZIME: ANTE ŠUJNJARA BROJ INDEKSA:

$f(x,y) = y^3 - 3xy + x^2$

$\frac{f(x)}{dx} = -3y + 2x$

$\frac{f(x)}{dy} = 3y^2 - 3x$

$3y + 2x \,/\cdot 3$

$3y^2 - 3x \,/\cdot 2$

$9y + \cancel{6x}$

$6y^2 - \cancel{6x}$

$6y^2 + 9y + 0 = \ldots$ $a=6$, $b=9$, $c=0$

$y_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-9 \pm \sqrt{81 -}}{12}$

$y = \frac{-9 \pm 9}{12}$

$\boxed{y_1 = -\frac{16}{12} = -\frac{8}{6} = -\frac{4}{3}}$

$\boxed{y_2 = \frac{0}{12} = 0}$

$3y + 2x = 0$

$3 \cdot -\frac{8}{6} + 2x = 0$

$-\frac{24}{6} + 2x = 0$

$-4 + 2x = 0$

$2x = 4 \,/ : 2$

$x = \frac{4}{2}$

$x = 2$

$T_1 (2, -\frac{4}{3})$ ×

$0+2$

$\frac{d^2 f(x)}{d^2 x} = \frac{d}{dx}\left[\frac{f(x)}{dx}\right] = \frac{d}{dx}\left[3y + 2x\right]'$

$A = 2$

$\frac{d}{dy}\left[\frac{f(x)}{dy}\right] = \frac{d}{dy}\left[3y^2 - 3x\right] = 3 \cdot 2y$

$C = 6y$

$\frac{d}{dx}\left[\frac{f(x)}{dy}\right] = \frac{d}{dx}\left[3y^2 - 3x\right] = 3$

$B = 3$

$y = 0$

$3y^2 - 3x = 0$

$3 \cdot 0^2 - 3x = 0$

$-3x = 0 \,/ : -3$

$x = -\frac{0}{3}$

$x = 0$

$T_2 = (0,0)$ ✓

ZA $T_1 (2, -\frac{4}{3})$

$A = 2 > 0$ MINIMUM

$B = 3$

$C = 6 \cdot (-\frac{4}{3}) = -\frac{24}{3} = -8$

?

$D = A \cdot C - B^2$

$D = 2 \cdot -8 - 3^2$

$= -16 - 9$

$D = -25 < 0$ Nema Ekstrema

ZA $T_2 (0,0)$

$A = 2 >$ MINIMUM

$B = 3$

$C = 6 \cdot y = 6 \cdot 0$

$C = 0$

?

$D = A \cdot C - B^2$

$= 2 \cdot 0 - 3^2$

$= -9 < 0$ Nema ekstrema

BILA BI SEDLASTA TOČKA

5

IME I PREZIME: ANTE ŠUŠKJARA BROJ INDEKSA:

2) $\int x^2 \sin(x)\,dx$

$$\left[\begin{array}{l} x^2 = u \\ 2x\,dx = du \\ V = \int \sin(x)\,dx \\ V = -\cos x \end{array}\right.$$

$u\,dV = U \cdot V - \int V\,du$

$\int x^2 \sin(x)\,dx = x^2(-\cos x) - \int -\cos x \, 2x\,dx$

$= -x^2 \cos x + 2\int x \cos x \, dx$

$$\left[\begin{array}{l} x = u \\ dx = du \\ V = \int \cos x\,dx \\ V = \sin x \end{array}\right.$$

$= -x^2 \cos x + 2\left(x \sin x - \int \sin x \, dx\right)$

$= -x^2 \cos x + 2\left(x \sin x - (-\cos x)\right)$

$= -x^2 \cos x + 2\left(x \sin x + \cos x\right)$ ✓ 15

Popuniti odmah!

Ime i prezime: Mateja Pećarić **Broj indeksa:** 17-0032-2010 (10)

Datum: 08.09.2011 **Vrijeme:** od 10:00 do .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Izračunati $\int \frac{x^2+2x+2}{x^2+x-2}\,\mathrm{d}x$. 15
2. Izračunati $\int x^2 \sin(x)\,\mathrm{d}x$. 15
3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala:
$$\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$$ 15
4. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$. ~~20~~ 0
5. Pronaći opće rješenje problema: $y' + xy + x = 0$. ~~20~~ 0
6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = 2x\cos x$ oko točke $x_0 = 0$. ~~15~~ 10

2. $\int x^2 \sin(x)\,dx = \{$

0

VIDI PORTAPA
KOTZAR
KRAGJEV
SAVIĆ

Ime i prezime: Hateja Pećarić

Broj indeksa: 17-0032-2010

4. $f(x,y)=y^3-3xy+x^2$

$\frac{dy}{dx}=3y^2-3(x'\cdot y-x\cdot y')+0$

$=3y^2-3(0-x)+0$

$=3y^2+3x$ ×

$\frac{dy}{dx}=0-3(x'\cdot y-x\cdot y')+2x$

$=-3(y+0)+2x$

$=-3y+2x$

$3y^2+3x=0\ /:3$

$-3y+2x=0$

$y^2+x=0 \Rightarrow y^2=-x$

$-3y+2x=0 \qquad -x=y^2$

$x=\left(\frac{3}{2}\right)^2$

$-3y+2\cdot y^2=0 \qquad x=\frac{9}{4}$

$y(-3+2y)=0$

$y=0 \qquad -3=-2y$

$2y=3$

$y=\frac{3}{2}$

$\frac{df}{\partial x}=\underbrace{\frac{\partial}{\partial x}(y^2)}_{=0}-\underbrace{\frac{\partial}{\partial x}(3xy)}_{3y}+\underbrace{\frac{\partial}{x}(x^2)}_{2x}$

$\frac{df}{\partial y}=\ldots=3y^2-3x$

$S_1(0,0)$

$S_2\left(\frac{9}{4},\frac{3}{2}\right)$

∅

$S_1(0,0)$

$r=6y=0$

$s=3$

$-3y+2x$

$t=-3$

$r\cdot t-s^2=(-3)^2=9$ ×

$r>0$

sedlasta točka

$S_2\left(\frac{9}{4},\frac{3}{2}\right)$

$r=9$

$s=\frac{27}{4}$

$t=-\frac{9}{2}$

$r\cdot t-s^2=$

$9\cdot\left(-\frac{9}{2}\right)-\left(\frac{27}{4}\right)^2$

$-\frac{81}{2}-182.25$

$r\cdot t-s^2<0$

minimum ×

IME I PREZIME: Mateja Pećarić

BROJ INDEKSA: 17-0032-2010

5. $y' + xy + x = 0$

$$\frac{dy}{dx} = -x - xy \,/\cdot dx\,y \quad ?$$

$$y\,dy = -x - x\,dx \,/\int$$

$$\int y\,dy = \int -x - \int x\,dx$$

$$\ln|y| = -\frac{x^2}{2} - x$$

$$\ln|y| = \frac{-x^2 - 2x}{2}$$

$$\ln|y| = \frac{x(-x-2)}{2}$$

?

0

IME I PREZIME: Mateja Pećarić

BROJ INDEKSA: 17-0032-2010

$x_0=0$

6. $f(x)=2x\cdot\cos x \Rightarrow 0$

$f'(x)=(2x'\cdot\cos x-2x\cdot\cos x')$

$=2\cos x+2x\sin x \Rightarrow 2$ ✓

$f''(x)=-2\sin x+(2x'\cdot\sin x-2x\cdot\sin x')$

$=-2\sin x+2\sin x-2x\cos x$

$=-2x\cos x \Rightarrow 0$ ✓

$f'''(x)=(-2x'\cdot\cos x+2x\cdot\cos x')$

$=-2\cos x+2x\sin x) \Rightarrow -2$ ✗

$$0+\frac{2}{1!}(x-x_0)+\frac{0}{2!}(x-x_0)^2-\frac{2}{3!}(x-x_0)^3$$

$$2(x-0)-\frac{2}{6}(x-0)^3$$

10

$$2x-\frac{1}{3}(x-0)^3$$

Popuniti odmah!
Ime i prezime: ŠIME MATANOVIĆ **Broj indeksa:** 57655

Ø

Datum: **Vrijeme:** od 9:55 do .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj ↓ bodova
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3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala: $\int_{\pi}^{2\pi} \frac{\arctan x}{x}\,\mathrm{d}x$	15
4. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$.	20
5. Pronaći opće rješenje problema: $y' + xy + x = 0$.	20
6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = 2x\cos x$ oko točke $x_0 = 0$.	15

$$1.\ \int \frac{x^2+2x+2}{x^2+x-2} = \int \left(\frac{x^2}{x^2+x-2} + \frac{2x}{x^2+x-2} + \frac{2}{x^2+x-2} \right) dx =$$

$$= \int \left(\frac{1}{x-2} \quad \frac{2}{2x-2} \quad \frac{2}{x^2+x-2} \right) dx$$

VIDI PORTADA

$$= \ln|x-2| + 2\ln|\ldots$$

$$\int x^2 \sin(x)\,dx$$

$$3.\ \int_{\pi}^{2\pi} \frac{\arctan x}{x} dx = \left/ \begin{array}{l} t = \arctan x \\ dt = \frac{1}{1+x^2} dx \end{array} \right/ = \int_{\pi}^{2\pi} \frac{t}{x} dt$$

$$= \int_{\pi}^{2\pi} \frac{t}{x} \cdot \frac{1}{1+x^2} = \int_{\pi}^{2\pi} \frac{t}{x+x^3} = \int \left(\frac{t}{x} + \frac{t}{x^3} \right) dx$$

$$=$$

NUMERIČKOM INTEGRACIJOM!

IME I PREZIME: ŠIME MATANOVIĆ BROJ INDEKSA: 57655

$$2.\int x^2 \sin(x) = \left| \begin{matrix} u = x^2 & \int v = \sin x \\ du = 2x & \int v = \cos x \end{matrix} \right| = 2x \sin x - \int 2x \cdot \cos x$$

$$2x \sin x - 2 \int x \cos x = 2x \sin x - 2 \frac{x^2}{2} (-\sin x)$$

$$= 2x \sin x - x^2 \cdot (-\sin x) = -2x^3 (-\sin x^2)$$

IME I PREZIME: ŠIME MATANOVIĆ BROJ INDEKSA: 57655

4. $f(x,y) = y^3 - 3xy + x^2$

$T\left(\frac{1}{2}, \frac{1}{3}\right)$

$f(xy)_x = 2x - 3y$ ✓

$f(xy)_y = 3y^2 - 3x$ ✓

$2x - 3y = 0 \longrightarrow 2x = 3y \,/:2$

$x = \frac{3}{2}y \longrightarrow \frac{\cancel{3}}{2} \cdot \frac{1}{\cancel{3}} = \frac{1}{2}$

$3y^2 - 3x = 0$

$y(3y) = 0$

$3y = 0 /$

$y = \frac{1}{3}$

×

NIJE LONIĆ

$f''(xy)_{xx} = 2$

$f''(xy)_{yy} = 6y$