

Popuniti odmah!

JURE PORTADA

IME I PREZIME:

BROJ INDEKSA:

DATUM: 08.09.2011

VRIJEME: OD

DO

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓  
bodova

15

15

1. Izračunati  $\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx$ .

2. Izračunati  $\int x^2 \sin(x) dx$ .

3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala:

$$\int_{\pi}^{2\pi} \frac{\arctan x}{x} dx$$

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4. Istražiti ekstreme funkcije  $f(x, y) = y^3 - 3xy + x^2$ .

20 10

5. Pronaći opće rješenje problema:  $y' + xy + x = 0$ .

20

6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije  $f(x) = 2x \cos x$  oko točke  $x_0 = 0$ .

15

1)  $\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx = \int \frac{x^2 + x + x + 2 - 2 + 2}{x^2 + x - 2}$

$$\frac{(x^2 + 2x + 2) : (x^2 + x - 2) = 1}{x^2 + x - 2}$$

$\int \frac{x^2 + x - 2}{x^2 + x - 2} dx + \int \frac{x + 4}{x^2 + x - 2}$

$\int dx + \int \frac{x + 4}{x^2 + x - 2} =$

NA ZADNJOJ STRANICI!  
# 11

I =  $\int dx = x$

II =

2)  $\int x^2 \sin(x) dx = \begin{cases} u = x^2 & du = 2x dx \\ dv = \sin(x) dx & v = -\cos x \end{cases}$

$-x^2 \cos x + \int \cos x \cdot 2x dx = \begin{cases} u = 2x & du = 2 dx \\ dv = \cos x & v = \sin x \end{cases}$

$-x^2 \cos x + [2x \sin x - \int 2 \sin x dx] = -x^2 \cos x + [2x \sin x + 2 \cos x + C]$   
 $= -x^2 \cos x + 2x \sin x + 2 \cos x + C$  ✓

15

IME I PREZIME:

JURE PORTADA

BROJ INDEKSA:

4)

$$f(x,y) = y^3 - 3xy + x^2$$

$$z_x = -3y + 2x \quad \checkmark$$

$$z_y = -3x + 3y^2 \quad \checkmark$$

$$\begin{aligned} 2x - 3y &= 0 \rightarrow 2x = 3y \\ -3x + 3y^2 &= 0 \quad x = \frac{3}{2}y \end{aligned}$$

$$x_1 = \frac{3}{2} \cdot 0$$

$$x_1 = 0$$

$$x_2 = \frac{3}{2} \cdot \left(-\frac{3}{2}\right)$$

$$x_2 = -\frac{9}{4}$$

$$T(0,0)$$

$$z_{xx} = 2 = f_0 = 2 > 0 \text{ min}$$

$$z_{xy} = -3 = s_0 = -3$$

$$z_{yy} = 6y \quad t_0 = 0$$

$$D = -9 \rightarrow \text{nema Ex}$$

$$-3 \cdot \frac{3}{2}y + 3y^2 = 0$$

$$-\frac{9}{2}y + 3y^2 = 0 \quad | \cdot 2$$

$$-9y + 6y^2 = 0$$

$$3y(2y - 3) = 0$$

$$3y_1 = 0$$

$$y_1 = 0 \quad \checkmark$$

$$2y - 3 = 0$$

$$2y = 3 \quad \times$$

$$y = \frac{3}{2} \quad \times$$

$$T\left(-\frac{9}{4}, -\frac{3}{2}\right)$$

$$z_{xx} = 2 \quad f_0 = 2$$

$$z_{xy} = -3 \quad s_0 = -3$$

$$z_{yy} = 6y \quad t_0 = 6 \cdot \left(-\frac{3}{2}\right) = -9$$

$$D = -18 - 9 = -27$$

$$D = f_0 \cdot t_0 - (s_0)^2 = -18 - 9 = -27 \rightarrow \text{nema Ex}$$

10

SEDLASTA TOČKA  $\checkmark$

$$T_1(0,0) \quad \checkmark$$

$$T_2\left(-\frac{9}{4}, -\frac{3}{2}\right) \quad \times$$

$$5) y' + xy + x = 0$$

$$y' + xy = -x$$

$$e^{-\int f(x) dx} = e^{-\int x dx}$$

$$= e^{-\frac{x^2}{2}} \checkmark$$

$$e^{\int f(x) dx} = e^{\int x dx}$$

$$= e^{\frac{x^2}{2}} \checkmark$$

$$e^{-\frac{x^2}{2}} \cdot \left[ \int e^{\frac{x^2}{2}} \cdot (-x) dx \right]$$

$$e^{-\frac{x^2}{2}} \cdot \left[ -\int x e^{\frac{x^2}{2}} dx \right] \Rightarrow \begin{cases} \frac{x^2}{2} = t \\ \frac{1}{2} x^2 = t \quad | \cdot \\ \frac{1}{2} \cdot 2x dx = dt \\ x dx = dt \end{cases}$$

$$e^{-\frac{x^2}{2}} \left[ -\int e^t \cdot dt \right]$$

$$e^{-\frac{x^2}{2}} \left[ -e^{\frac{x^2}{2}} + c \right]$$

$$= -e^{-\frac{x^2}{2} + \frac{x^2}{2}} + ce^{-\frac{x^2}{2}}$$

$$= -e^0 + ce^{-\frac{x^2}{2}}$$

$$= -1 + ce^{-\frac{x^2}{2}} \checkmark$$

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$$f(x) = 2x \cdot \cos x$$

$$f(0) = 2 \cdot 0 \cdot \cos 0 = 0 \quad \checkmark$$

$$f'(x) = 2 \cos x + (-\sin x \cdot 2x) = 2 \cos x - 2x \sin x$$

$$f'(0) = 2, \quad \checkmark$$

$$f'' = \cancel{2 \cos x + (-\sin x) \cdot 2} - \cancel{(2 \sin x + \cos x \cdot 2x)}$$

$$f'' = -2 \sin x - (2 \sin x + 2x \cos x)$$

$$f'' = -2 \sin x - 2 \sin x - 2x \cos x = 0 \quad f''(0) = 0$$

$$f'' = -4 \sin x - 2x \cos x$$

$$f''' = -4 \cos x - (2 \cos x + (-\sin x) \cdot 2x)$$

$$f''' = -4 \cos x - 2 \cos x + \sin x \cdot 2x = -4 \cos x - 2 \cos x + 2x \sin x$$

$$f'''(0) = -6, \quad \checkmark \quad = -6 \cos x + 2x \sin x$$

$$f^{(4)} = 6 \sin x + 2 \sin x + 2x \cos x =$$

$$f^{(4)}(0) = 0 \quad \checkmark$$

$$f^{(4)} = 8 \sin x + 2x \cos x$$

$$f^{(4)} = +8 \cos x + 2 \cos x + 2x(-\sin x) = 8 \cos x + 2 \cos x - 2x \sin x$$

$$= 10 \cos x - 2x \sin x$$

$$f^{(4)}(0) = 10,$$

$$f^{(5)} = -10 \sin x - (2 \sin x + 2x \cos x) = -10 \sin x - 2 \sin x - 2x \cos x$$

$$= -12 \sin x - 2x \cos x$$

$$f^{(5)}(0) = 0$$

$$f^{(6)} = -12 \cos x - (2 \cos x + 2x(-\sin x)) = -12 \cos x - 2 \cos x + 2x \sin x$$

$$f^{(6)}(0) = -14 \quad \rightarrow$$

BRAVO ZA  
DERIVIRANJE,  
ALI GREŠKA  
NA KRAJU NE  
DOPUŠTA NIJEDAN  
BOD

IME I PREZIME:

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$$f'(2) \frac{(x-2)^0}{1} + f''(-6) \frac{(x-0)^2}{3!} + f'''(10) \frac{(x-0)^3}{5!} + f^{(4)}(-14) \frac{(x-0)^4}{7!} \quad \times$$

$$2(x-0)^1 - 6 \frac{(x-0)^3}{3!} + \dots = 2x - x^3$$

$$1) \int \frac{x^2+2x+2}{x^2+x-2} = \int \frac{x^2+x+x-2+4}{x^2+x-2} = \int \frac{x^2+x+2}{x^2+x-2} dx + \int \frac{x+4}{x^2+x-2} dx \quad \checkmark$$

I II

$$I = \int dx = x \quad \checkmark$$

$$II \int \frac{x+4}{x^2+x-2} dx = \int \frac{x+4}{(x-1)(x+2)} dx = \int \frac{A}{x-1} dx + \int \frac{B}{x+2} dx \quad \checkmark$$

$x^2+x-2 = \frac{-1 \pm \sqrt{1+8}}{2}$   
 ~~$(x-1)(x+2)$~~

$$x+4 = A(x+2) + B(x-1)$$

$$x+4 = Ax + 2A + Bx - B$$

$$x+4 = (A+B)x + (2A-B)$$

$$A+B=1 \quad | \cdot (-2)$$

$$CA-B=4$$

$$A - \frac{2}{3} = 1$$

$$-2A - 2B = -2$$

$$A = 1 + \frac{2}{3} \Rightarrow \frac{3+2}{3} = \frac{5}{3}$$

$$2A - B = 4$$

$$A = \frac{5}{3} \quad \checkmark$$

$$-3B = 2$$

$$B = -\frac{2}{3} \quad \checkmark$$

$$\int \frac{x+4}{x^2+x-2} = \int \frac{\frac{5}{3}}{x-1} dx + \int \frac{-\frac{2}{3}}{x+2} dx = \frac{5}{3} \ln(x-1) - \frac{2}{3} \ln(x+2) + C$$

$$\int \frac{x^2+2x+2}{x^2+x-2} = x + \frac{5}{3} \ln(x-1) - \frac{2}{3} \ln(x+2) + C \quad \checkmark \quad \underline{15}$$

Popuniti odmah!

IME I PREZIME:

Lovre Nikitović

BROJ INDEKSA:

17-2-0038-2010

55

DATUM:

VRIJEME: OD

DO

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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~~15~~

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$$2. \int x^2 \sin(x) dx = \left[ \begin{array}{l} u = x^2 \\ du = 2x dx \end{array} \quad \left. \begin{array}{l} dv = \sin(x) dx \\ v = \int \sin(x) dx \\ v = -\cos x \end{array} \right\} \right]$$

$$\int u dv = u \cdot v - \int v \cdot du$$

$$= x^2 \cdot (-\cos(x)) - \int (-\cos(x)) \cdot 2x dx$$

$$= -x^2 \cos(x) + 2 \int \cos(x) x dx = \left[ \begin{array}{l} u = x \\ du = dx \end{array} \quad \left. \begin{array}{l} dv = \cos(x) dx \\ v = \int \cos(x) dx \\ v = \sin x \end{array} \right\} \right]$$

$$= -x^2 \cos(x) + 2 \cdot (x \cdot \sin(x) - \int \sin(x) \cdot dx)$$

$$= -x^2 \cos(x) + 2(x \sin(x) - (-\cos(x))) + c$$

$$= -x^2 \cos(x) + 2(x \sin(x) + \cos(x)) + c$$

$$= (-x^2 \cos(x) + 2x \sin(x) + 2 \cos(x)) + c \quad \checkmark$$

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5.  $y' + xy + x = 0$

$$\frac{dy}{dx} = -xy - x$$

$$\frac{dy}{dx} = -x(y+1) / dx$$

$$dy = -x(y+1) dx / (y+1)$$

$$\frac{dy}{y+1} = -x dx / \int$$

$$\int \frac{dy}{y+1} = \int -x dx$$

$$\int \frac{dy}{y+1} = \left| \begin{array}{l} y+1 = t \\ dy = dt \end{array} \right|$$

$$\underline{\ln|y+1| = -\frac{x^2}{2} + c} \quad \checkmark$$

$$\int \frac{dt}{t} = \ln|t| = \ln|y+1|$$

20



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17-2-0035-2010

$$f(x) = 2x \cos x \quad x_0 = 0$$

$$\sin = \cos$$

$$\cos = -\sin$$

$$f(x_0) = f(0) = 2 \cdot 0 \cdot \cos 0 = 0 \cdot 1 = \boxed{0} \quad \checkmark$$

$$f'(x) = 2x' \cos x + 2x \cdot \cos(x)' = 2 \cos x + 2x(-\sin x)$$

$$= 2 \cos x - 2x \sin(x)$$

$$f'(0) = 2 \cos(0) - 2 \cdot 0 \cdot \sin(0) = 2 - 0 = \boxed{2} \quad \checkmark$$

$$f''(x) = 2' \cos(x) + 2 \cdot \cos(x)' - (2x' \sin(x) + 2x \cdot \sin(x)')$$

$$= 2 \cdot (-\sin(x)) - 2 \sin(x) + 2x \cdot \cos(x)$$

$$= -2 \sin(x) - 2 \sin(x) + 2x \cos(x)$$

$$f''(0) = -2 \sin(0) - 2 \sin(0) + 2 \cdot 0 \cdot \cos(0)$$

$$= 0 - 0 + 0 = \boxed{0} \quad \checkmark$$

$$f'''(x) = -2' \sin(x) + (-2) \cdot \sin(x)' - 2' \sin(x) + (-2) \cdot \sin(x)' + 2x' \cos(x) + 2x \cdot \cos(x)'$$

$$f'''(x) = -2 \cos(x) - 2 \cos(x) + 2 \cos(x) - 2x \sin(x)$$

$$f'''(x) = -2 \cos(x) - 2x \sin(x) \quad \times$$

$$f'(0) = -2 \cos(0) - 2 \cdot 0 \cdot \sin(0) = -2 - 0 = \boxed{-2}$$

$$f(x) = f(x_0) + (x-x_0) \cdot f'(x_0) + \frac{(x-x_0)^2}{2!} \cdot f''(x_0) + \frac{(x-x_0)^3}{3!} \cdot f'''(x_0) + \dots$$

$$f(x) = 2x \cos x = 0 + (x-0) \cdot 2 + \frac{(x-0)^2}{2!} \cdot 0 + \frac{(x-0)^3}{3!} \cdot (-2) \quad \times \quad \underline{10}$$

$$f(x) = 2x \cos x = 0 + 2(x-0) - 2 \cdot \frac{(x-0)^3}{3!}$$

$$f(x) = 2x \cos x = 2(x-0) - \frac{2(x-0)^3}{6} = 2(x-0) - \frac{(x-0)^3}{3} = 2x - \frac{(x-0)^3}{3} \quad \times$$



4.  $f(x,y) = y^3 - 3xy + x^2$

$\partial_x f = -3y + 2x$

$\partial_{xx} f = 2$

$\partial_{xy} f = -3$

$\partial_y f = 3y^2 - 3x$

$\partial_{yy} f = 6y$

$\partial_x f = 0$   
 $\partial_y f = 0$

$\partial_{xy} f = \partial_{yx} f$

$-3y + 2x = 0 \rightarrow 2x = 3y / (2)$

$3y^2 - 3x = 0$

$\frac{2}{3}x = y$

$y = \frac{2}{3}x$

$3 \cdot \left(\frac{2}{3}x\right)^2 - 3x = 0$

$3 \cdot \frac{4}{9}x^2 - 3x = 0$

$\frac{12}{9}x^2 - 3x = 0$

$\frac{4}{3}x^2 - 3x = 0$

$x \left(\frac{4}{3}x - 3\right) = 0$

$x_1 = 0$

$\frac{4}{3}x - 3 = 0$

$\frac{4}{3}x = 3 / (\frac{4}{3})$

$x = \frac{3 \cdot \frac{3}{4}}{3}$

$y_1 = \frac{2}{3} \cdot 0 = 0$

$y_2 = \frac{2}{3} \cdot \frac{9}{4} = \frac{18}{12} = \frac{3}{2}$

$x_2 = \frac{9}{4}$

Za tačku  $T_1(0,0)$

$A = \partial_{xx} f = 2$

$\Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{xy} f & \partial_{yy} f \end{vmatrix}$

$\Delta = \begin{vmatrix} 2 & -3 \\ -3 & 0 \end{vmatrix} = 0 - 9 = -9$

$A = 2 > 0$   
 $\Delta = -9 < 0$  } lokalan minimum ~~X~~

$T_1(0,0)$   
 $T_2(\frac{9}{4}, \frac{3}{2})$  ✓

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$f(x,y)_{min} = y^3 - 3xy + x^2 = 0^3 - 3 \cdot 0 \cdot 0 + 0^2 = 0$

Za tačku  $T_2(\frac{9}{4}, \frac{3}{2})$

$A = 2$

$\Delta = \begin{vmatrix} 2 & -3 \\ -3 & 9 \end{vmatrix} = 18 - (+9) = 18 - 9 = 9$

$A = 2 > 0$   
 $\Delta = 9 > 0$  } lokalan minimum ✓

$\frac{27}{8} - \frac{81}{8} + \frac{81}{16} = \frac{54 - 162 + 81}{16}$

$= \frac{-27}{16} = -1.6875$

$f(x,y)_{min} = y^3 - 3xy + x^2 = \left(\frac{3}{2}\right)^3 - 3 \cdot \left(\frac{9}{4}\right) \cdot \left(\frac{3}{2}\right) + \left(\frac{9}{4}\right)^2$

$f(x,y)_{min} = \frac{27}{8} - 3 \cdot \frac{27}{8} + \frac{81}{16} = \frac{27}{8} - \frac{81}{8} + \frac{81}{16}$

IME I PREZIME:

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17-2-0035-2010

3. 
$$\int_{\pi}^{2\pi} \frac{\arctan x}{x} dx$$

$$\Delta x = \frac{b-a}{n} = \frac{2\pi - \pi}{4} = \frac{3.141}{4} = 0.785$$

$a = \pi$   
 $b = 2\pi$   
 $n = 4$

k	0	1	2	3	4
$x_k$	0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
$\frac{\arctan x}{x}$	0	0.979	0.927	0.858	0.785

k	0	1	2	3	4
$x_k$	$\pi$	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	$2\pi$
$\frac{\arctan(x)}{x}$					

POGREŠNE TOČKE  $x_k$  !!!

$$y_0 = \frac{\arctan 0}{0} = 0$$

$$y_1 = \frac{\arctan(0.25)}{0.25} = 0.979$$

$$y_2 = \frac{\arctan(0.5)}{0.5} = 0.927$$

$$y_3 = \frac{\arctan(0.75)}{0.75} = 0.858$$

$$y_4 = \frac{\arctan(1)}{1} = 0.785$$

$$I = \frac{\Delta x}{2} [y_0 + y_n + 2 \cdot (y_1 + y_2 + y_3 + \dots)]$$

$$I = \frac{0.785}{2} [0 + 0.785 + 2 \cdot (0.979 + 0.927 + 0.858)]$$

$$I = 0.3925 [0.785 + 2 \cdot (2.764)]$$

$$I = 0.3925 [0.785 + 5.528]$$

$$I = 0.3925 \cdot 6.313 = \boxed{2.477}$$

Popunite odmah!

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BROJ INDEKSA: 57104

DATUM:

VRIJEME: OD

DO

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4.  $f(x, y) = y^3 - 3xy + x^2$

$\partial_x f = -3y + 2x$  ✓

$\partial_{xx} f = 2$  ✓

$\partial_{xy} f = -3$  ✓

$\partial_y f = 3y^2 - 3x$  ✓

$\partial_{yy} f = 6y$  ✓

$\partial_{yx} f = -3$

$\partial_x f = 0$

$\partial_y f = 0$

$-3y + 2x = 0 \quad | \cdot 3$

$3y^2 - 3x = 0 \quad | \cdot 2$

$-9y + 6x = 0$

$6y^2 - 6x = 0$

$-9y + 6y^2 = 0$

$y(-9 + 6y) = 0$

$y_1 = 0$

$-9 + 6y = 0$

$6y = 9$

$y = \frac{9}{6}$

$y_2 = \frac{3}{2}$

$T_1(0, 0)$  ✓

$T_2(\frac{9}{4}, \frac{3}{2})$  ✓

$-3 \cdot 0 + 2x = 0$

$2x = 0$

$x_1 = 0$

$-3 \cdot \frac{3}{2} + 2x = 0$

$-\frac{9}{2} + 2x = 0$

$2x = \frac{9}{2}$

$x = \frac{9}{4}$

$x = \frac{9}{4}$

$x = \frac{9}{4}$

$x = \frac{9}{4}$

$x_2 = \frac{9}{4}$

$x_2 = \frac{9}{4}$

$x_2 = \frac{9}{4}$

$A = |\partial_{xx}^2 \phi| = 2 > 0$  FUNKCIJA IMA MINIMUM

$\Delta = \begin{vmatrix} \partial_{xx}^2 \phi & \partial_{xy}^2 \phi \\ \partial_{yx}^2 \phi & \partial_{yy}^2 \phi \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ -3 & 6y \end{vmatrix} = 12y - 9$  ✓

$T_1 \left( \frac{3}{2}, 0 \right)$   
 $T_2 \left( \frac{9}{4}, \frac{3}{2} \right)$

$\phi(x,y) = y^3 - 3xy + x^2 = \left(\frac{3}{2}\right)^3 - 3 \cdot \frac{3}{2} \cdot \frac{3}{2} + \left(\frac{9}{4}\right)^2$   
 $= \frac{27}{8} - \frac{81}{8} + \frac{81}{16} = \frac{54 - 162 + 81}{16} = -\frac{168}{16} = -10.5$

$12 \cdot 0 - 9 = -9$   
 $12 \cdot \frac{3}{2} - 9 = 9$

U TOČKI  $T_1$  NEMA EKSTREMA  $\Delta < 0$  ✓ 20  
 U TOČKI  $T_2$  IMA MINIMUM  $\left(\frac{9}{4}, \frac{3}{2}, -10.5\right)$  ✓

6)  $f(x) = 2x \cos x$   $x_0 = 0$

$f(x_0) = 2 \cdot 0 \cdot \cos 0 = 0$

$f'(x) = 2 \cos x + (-\sin x \cdot 2x) = 2 \cos x - \sin x \cdot 2x$

$f'(x_0) = 2 \cos 0 - \sin 0 \cdot 2 \cdot 0 = 2$  ✓

$f''(x) = -2 \sin x - (\cos x \cdot 2x + 2 \sin x) = -2 \sin x - \cos x \cdot 2x - 2 \sin x$   
 $= -4 \sin x - \cos x \cdot 2x$

$f''(x_0) = -4 \sin 0 - \cos 0 \cdot 2 \cdot 0 = 0$

$f'''(x) = -4 \cos x - (-\sin x \cdot 2x + 2 \cos x) = -4 \cos x + \sin x \cdot 2x - 2 \cos x$   
 $= -6 \cos x + \sin x \cdot 2x = -6$

$f'''(x_0) = -6 \cos 0 + \sin 0 \cdot 2 \cdot 0 = -6$  ✓

$f^{(4)}(x) = 6 \sin x + (\cos x \cdot 2x + 2 \sin x) = 6 \sin x + \cos x \cdot 2x + 2 \sin x$   
 $= 8 \sin x + \cos x \cdot 2x$

$f^{(4)}(x_0) = 8 \sin 0 + \cos 0 \cdot 2 \cdot 0 = 0$

$$6) f(x) = 8 \cos x + (-\sin x 2x + 2 \cos x) = 8 \cos x - \sin x 2x + 2 \cos x$$

$$= 10 \cos x - \sin x 2x$$

$$f(x_0) = 10 \cos(0) - \sin(0) 2 \cdot 0 = 10$$

$$f'(x) = -10 \sin x - (\cos x 2x + 2 \sin x) = -10 \sin x - \cos x 2x - 2 \sin x$$

$$= -12 \sin x - \cos x 2x$$

$$f'(x_0) = -12 \sin(0) - \cos(0) 2 \cdot 0 = 0$$

$$f''(x) = -12 \cos x - (-\sin x 2x + 2 \cos x) = -12 \cos x + \sin x 2x - 2 \cos x$$

$$= -14 \cos x + \sin x 2x$$

$$f''(x_0) = -14 \cos(0) + \sin(0) 2 \cdot 0 = -14$$

$$f(x) = f(x_0) + \frac{(x-x_0)}{1!} \cdot f'(x_0) + \frac{(x-x_0)^2}{2!} \cdot f''(x_0) + \frac{(x-x_0)^3}{3!} \cdot f'''(x_0) +$$

$$\frac{(x-x_0)^4}{4!} \cdot f^{(4)}(x_0) + \frac{(x-x_0)^5}{5!} \cdot f^{(5)}(x_0) + \frac{(x-x_0)^6}{6!} \cdot f^{(6)}(x_0) + \frac{(x-x_0)^7}{7!} \cdot f^{(7)}(x_0)$$

$$f(x) = 0 + \frac{(x-0)}{1} \cdot 2 + \frac{(x-0)^2}{2} \cdot 0 + \frac{(x-0)^3}{6} \cdot (-14) + \frac{(x-0)^4}{24} \cdot 0 + \frac{(x-0)^5}{120} \cdot 10$$

$$+ \frac{(x-0)^6}{720} \cdot 0 + \frac{(x-0)^7}{5040} \cdot (-14)$$

$$= 2(x-0) - \frac{(x-0)^3}{6} + \frac{(x-0)^5}{12} + \frac{(x-0)^7}{60}$$

$$= 2x - x^3 + \dots$$

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$f(x) = 2x - x^3 + \dots$   
 $f(0) = 0$   
 $f'(0) = 2$   
 $f''(0) = 0$   
 $f'''(0) = -6$   
 $f^{(4)}(0) = 0$   
 $f^{(5)}(0) = 10$   
 $f^{(6)}(0) = 0$   
 $f^{(7)}(0) = -14$   
 $f^{(8)}(0) = 0$   
 $f^{(9)}(0) = 10$   
 $f^{(10)}(0) = 0$   
 $f^{(11)}(0) = -14$   
 $f^{(12)}(0) = 0$   
 $f^{(13)}(0) = 10$   
 $f^{(14)}(0) = 0$   
 $f^{(15)}(0) = -14$   
 $f^{(16)}(0) = 0$   
 $f^{(17)}(0) = 10$   
 $f^{(18)}(0) = 0$   
 $f^{(19)}(0) = -14$   
 $f^{(20)}(0) = 0$   
 $f^{(21)}(0) = 10$   
 $f^{(22)}(0) = 0$   
 $f^{(23)}(0) = -14$   
 $f^{(24)}(0) = 0$   
 $f^{(25)}(0) = 10$   
 $f^{(26)}(0) = 0$   
 $f^{(27)}(0) = -14$   
 $f^{(28)}(0) = 0$   
 $f^{(29)}(0) = 10$   
 $f^{(30)}(0) = 0$   
 $f^{(31)}(0) = -14$   
 $f^{(32)}(0) = 0$   
 $f^{(33)}(0) = 10$   
 $f^{(34)}(0) = 0$   
 $f^{(35)}(0) = -14$   
 $f^{(36)}(0) = 0$   
 $f^{(37)}(0) = 10$   
 $f^{(38)}(0) = 0$   
 $f^{(39)}(0) = -14$   
 $f^{(40)}(0) = 0$   
 $f^{(41)}(0) = 10$   
 $f^{(42)}(0) = 0$   
 $f^{(43)}(0) = -14$   
 $f^{(44)}(0) = 0$   
 $f^{(45)}(0) = 10$   
 $f^{(46)}(0) = 0$   
 $f^{(47)}(0) = -14$   
 $f^{(48)}(0) = 0$   
 $f^{(49)}(0) = 10$   
 $f^{(50)}(0) = 0$   
 $f^{(51)}(0) = -14$   
 $f^{(52)}(0) = 0$   
 $f^{(53)}(0) = 10$   
 $f^{(54)}(0) = 0$   
 $f^{(55)}(0) = -14$   
 $f^{(56)}(0) = 0$   
 $f^{(57)}(0) = 10$   
 $f^{(58)}(0) = 0$   
 $f^{(59)}(0) = -14$   
 $f^{(60)}(0) = 0$   
 $f^{(61)}(0) = 10$   
 $f^{(62)}(0) = 0$   
 $f^{(63)}(0) = -14$   
 $f^{(64)}(0) = 0$   
 $f^{(65)}(0) = 10$   
 $f^{(66)}(0) = 0$   
 $f^{(67)}(0) = -14$   
 $f^{(68)}(0) = 0$   
 $f^{(69)}(0) = 10$   
 $f^{(70)}(0) = 0$   
 $f^{(71)}(0) = -14$   
 $f^{(72)}(0) = 0$   
 $f^{(73)}(0) = 10$   
 $f^{(74)}(0) = 0$   
 $f^{(75)}(0) = -14$   
 $f^{(76)}(0) = 0$   
 $f^{(77)}(0) = 10$   
 $f^{(78)}(0) = 0$   
 $f^{(79)}(0) = -14$   
 $f^{(80)}(0) = 0$   
 $f^{(81)}(0) = 10$   
 $f^{(82)}(0) = 0$   
 $f^{(83)}(0) = -14$   
 $f^{(84)}(0) = 0$   
 $f^{(85)}(0) = 10$   
 $f^{(86)}(0) = 0$   
 $f^{(87)}(0) = -14$   
 $f^{(88)}(0) = 0$   
 $f^{(89)}(0) = 10$   
 $f^{(90)}(0) = 0$   
 $f^{(91)}(0) = -14$   
 $f^{(92)}(0) = 0$   
 $f^{(93)}(0) = 10$   
 $f^{(94)}(0) = 0$   
 $f^{(95)}(0) = -14$   
 $f^{(96)}(0) = 0$   
 $f^{(97)}(0) = 10$   
 $f^{(98)}(0) = 0$   
 $f^{(99)}(0) = -14$   
 $f^{(100)}(0) = 0$

$$2) \int x^2 \sin(x) dx = \left[ \begin{array}{l} u = x^2 \\ du = 2x dx \\ dv = \sin x dx \\ v = \int \sin x dx \\ v = -\cos x \end{array} \right]$$

$$= x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x dx = x^2(-\cos x) + \int \cos x \cdot 2x dx \left[ \begin{array}{l} 2x = u \\ 2 dx = du \\ dv = \cos x dx \\ v = \int \cos x \\ v = \sin x \end{array} \right]$$

$$= x^2(-\cos x) + (2x \cdot \sin x - \int \sin x \cdot 2 dx)$$

$$= x^2(-\cos x) + 2x \sin x - 2 \int \sin x$$

$$= x^2(-\cos x) + 2x \sin x + 2 \cos x + C$$



15

$$2 \cos x + (-\sin x \cdot 2x) = 2 \cos x$$

Popuniti odmah!

IME I PREZIME: ANĐEJA SAVIĆ

BROJ INDEKSA:

DATUM:

VRIJEME: OD

DO

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

45

Broj bodova

15

1. Izračunati  $\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx$ .

2. Izračunati  $\int x^2 \sin(x) dx$ .

15

3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala:

$$\int_{\pi}^{2\pi} \frac{\arctan x}{x} dx$$

15

4. Istražiti ekstreme funkcije  $f(x, y) = y^3 - 3xy + x^2$ .

20

5. Pronaći opće rješenje problema:  $y' + xy + x = 0$ .

20

6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije  $f(x) = 2x \cos x$  oko točke  $x_0 = 0$ .

15

$$2. \int x^2 \sin(x) dx = \left\{ \begin{array}{l} u = x^2 \quad du = 2dx \\ dv = \sin x \quad v = -\cos x \end{array} \right\} =$$

$$= x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x dx = -x^2 \cos x + 2 \int x \cos x dx =$$

$$= \left\{ \begin{array}{l} u = x \quad du = dx \\ dv = \cos x \quad v = \sin x \end{array} \right\} = -x^2 \cos x + 2 \left( x \sin x - \int \sin x dx \right)$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C \quad \checkmark$$

15



4.  $f(x,y) = y^3 - 3xy + x^2$

$dx f = -3y + 2x$

$dy f = 3y^2 - 3x$

$dx f = 0$

$dy f = 0$

$-3y + 2x = 0 \quad | \cdot 3$

$3y^2 - 3x = 0 \quad | \cdot 2$

$-9y + 6x = 0$

$6y^2 - 6x = 0$

$6xy - 9y = 0 \quad | \cdot 3$

$2y^2 - 3y = 0$

$y(2y - 3) = 0$

$y = 0 \quad 2y - 3 = 0$

$\Downarrow$   
 $x = 0 \quad \begin{matrix} 2y = 3 \\ y = \frac{3}{2} \end{matrix}$

$-3 \cdot \frac{3}{2} + 2x = 0$

$-\frac{9}{2} + 2x = 0$

$2x = \frac{9}{2}$

$x = \frac{9}{4}$

$T_1(0,0)$

$T_2\left(\frac{9}{4}, \frac{3}{2}\right)$

A  $dx^2 f = 2$

B  $dx y f = -3 + 2 = -1$

C  $dy^2 f = 6y$

$T(0,0) \Delta = AC - B^2 = 2 \cdot 0 - 1 = -1 < 0$

Lokali ekstrem

$A > 0 \Rightarrow$  lok. MINIMUM

$T\left(\frac{9}{4}, \frac{3}{2}\right) \Delta = AC - B^2 = 2 \cdot 6 \cdot \frac{3}{2} - 1$

$\Delta = 18 - 1 = 17 > 0 \Rightarrow$  SVAKAKOJ TOČKI

PREVIŠE

POGRESAKA



5.  $y' + xy + x = 0$

$y' = -xy - x$

$y' = -x(y+1)$

$\frac{dy}{y+1} = -x dx \int$

$\int \frac{dy}{y+1} = \int (-x) dx$

$y+1 = t$   
 $dy = dt$

20

$\int \frac{dt}{t} = -\int x dx$

$\ln|t| = -\frac{x^2}{2}$

$\ln|y+1| = -\frac{x^2}{2}$

$\ln|y+1| = -\frac{x^2}{2} + C$

6.  $f(x) = 2x \cos x$

$x_0 = 0$

$f(x) = 2x \cos x \quad f(0) = 0 \quad \checkmark$

$f'(x) = 2x \cdot (-\sin x) + 2 \cos x \quad f'(0) = 2 \quad \checkmark$

$f''(x) = -2x \cos x + 2(-\sin x) + 2 \cdot (-\sin x) \quad f''(0) = 0 \quad \checkmark$   
 $= -2x \cos x - 2 \sin x - 2 \sin x$   
 $= -2x \cos x - 4 \sin x$

$f'''(x) = -2 \cos x + 2x \sin x - 4 \cos x \quad f'''(0) = -2 - 4 = -6$

$2x \cos x \approx 0 + \frac{2}{1!}x + 0 + \frac{-6}{2!}x^3 = 2x - \frac{3x^3}{1}$

10

Popuniti odmah!

IME I PREZIME: DUJE KRALJICU

BROJ INDEKSA: 17-2-0015-2020

40

DATUM: VRIJEME: OD DO

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na suazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj bodova

15

15

1. Izračunati  $\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx$ .

2. Izračunati  $\int x^2 \sin(x) dx$ .

3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala:

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5. Pronaći opće rješenje problema:  $y' + xy + x = 0$ .

20

6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije  $f(x) = 2x \cos x$  oko točke  $x_0 = 0$ .

15

1)  $\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx = \int 1 + \frac{x+4}{x^2+x-2} dx = \int dx + \int \frac{x+4}{x^2+x-2} dx =$

$(x^2 + 2x + 2) : (x^2 + x - 2) = 1$

$x^2 + x - 2$

$\int \frac{x+4}{x^2+x-2} dx = \left[ \begin{aligned} t = x^2 + x - 2 \quad x = \frac{1}{2}(2x+1) - \frac{1}{2} \\ dt = (2x+1) dx \quad x+4 = \frac{1}{2}(2x+1) - \frac{1}{2} + 4 \end{aligned} \right]$

$\int \frac{\frac{1}{2}(2x+1) + \frac{7}{2}}{x^2+x-2} dx = \frac{1}{2} \int \frac{2x+1}{x^2+x-2} dx + \frac{7}{2} \int \frac{dx}{x^2+x-2} = \frac{1}{2} \int \frac{dt}{t} + \frac{7}{2} \int \frac{dx}{x^2+x-2}$

DAKJE...

$\int \frac{dx}{x^2+x-2} = \text{je TIP B} = \int \frac{dx}{(x+\frac{1}{2})^2 - \frac{1}{4} - 2} = \int \frac{dt}{t^2 - \frac{9}{4}} = \int \frac{dt}{(t+\frac{3}{2})(t-\frac{3}{2})}$

BOLJE ODMAH RASTAV NA PARC. RAZL.

$\frac{1}{x^2+x-2} = \frac{1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$  A=? B=?

RASTAV NA PARC. RAZLOMKE

3)

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{2\pi - \pi}{4}$$

$$\Delta x = \frac{2}{4} = \frac{1}{2}$$

k	0	1	2	3	4
x <sub>k</sub>	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
y <sub>k</sub>	0	13,75	11,70	8,37	7,58

$$\Delta x = \frac{2\pi - \pi}{4} = \frac{\pi}{4}$$

$$y = \frac{\arctan 0}{0} = 0 \quad ? \quad \text{DJELENJE ?}$$

S NULOM

5

$$y_1 = 13,75$$

$$y_2 = 11,70$$

$$y_3 = 8,37$$

$$y_4 = 7,85$$

$$\bar{I} \approx \frac{\Delta x}{2} [y_0 + y_4 + 2 \cdot (y_1 + y_2 + y_3)]$$

$$\bar{I} \approx \frac{1}{2} [0 + 7,85 + 2(13,75 + 11,70 + 8,37)]$$

$$\bar{I} \approx \frac{1}{4} [7,85 + 68,84]$$

$$\bar{I} \approx \frac{1}{4} \cdot 76,69$$

$$\bar{I} \approx 19,1725$$

$$2) \int x^2 \sin(x) dx = \left[ \begin{array}{l} u=x^2 \quad dv=\sin x dx \\ du=2x dx \quad v=-\cos x \end{array} \right] = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x dx$$

$$= -x^2 \cos x + 2 \int \cos x \cdot x dx = \left[ \begin{array}{l} u=x \quad dv=\cos x dx \\ du=dx \quad v=\sin x \end{array} \right]$$

$$= -x^2 \cos x + 2(x \cdot \sin x - \int \sin x dx) = -x^2 \cos x + 2(x \sin x + \cos x)$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C \quad \checkmark$$

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$$b) f(x) = 2x \cos x \quad x_0 = 0$$

$$f(x_0) = f(0) = 2 \cdot 0 \cdot \cos(0) = 0 \quad \checkmark$$

$$f'(x) = 2 \cdot \cos x + 2x(-\sin x) = 2 \cos x - 2x \sin x, \quad f'(x_0) = 2 \cdot 1 - 2 \cdot 0 \cdot 0 = 2 \quad \checkmark$$

$$f''(x) = -2 \sin x - (2 \sin x + 2x \cos x) \quad f''(x_0) = 0 - 0 + 0 = 0 \quad \checkmark$$

$$f'''(x) = -2 \cos x - 2 \cos x + 2 \cdot \cos x + 2x \sin x, \quad f'''(x_0) = -2 - 2 + 2 - 0 = -2 \quad \times$$

$$f(x) = f(x_0) + (x-x_0) \cdot f'(x_0) + \frac{(x-x_0)^2}{2!} \cdot f''(x_0) + \frac{(x-x_0)^3}{3!} \cdot f'''(x_0) + \dots$$

$$f(x) = 0 + (x-0) \cdot 2 + \frac{(x-0)^2}{2} \cdot 0 + \frac{(x-0)^3}{6} \cdot (-2) + \dots$$

$$f(x) = 0 + (x-0) \cdot 2 - \frac{(x-0)^3}{6} \cdot 2 \quad \times$$

$$= 2x - x^3$$

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$$4) f(x, y) = y^3 - 3xy + x^2$$

$$\partial_x f = -3y + 2x$$

$$\partial_x f = 0$$

$$\partial_{xx} f = 2$$

$$\partial_y f = 0$$

$$\partial_{xy} f = -3$$

$$-3y + 2x = 0 \Rightarrow -3y = -2x \quad | :(-3)$$

$$3y^2 - 3x = 0 \quad y = \frac{2}{3}x$$

$$\partial_y f = 3y^2 - 3x$$

$$3 \cdot \left(\frac{2}{3}x\right)^2 - 3x = 0$$

$$y_1 = \frac{2}{3} \cdot \frac{3}{2}$$

$$\partial_{yy} f = 6y$$

$$2x^2 - 3x = 0 \quad \times$$

$$y_1 = \frac{6}{6} = 1$$

$$\partial_{yx} f = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y_2 = \frac{2}{3} \cdot 0$$

$$x = \frac{3 \pm \sqrt{9-0}}{4}$$

$$y_2 = 0$$

$$x = \frac{3 \pm 3}{4}$$

$$T_1 \left( \frac{3}{2}, 1 \right) \quad \times$$

$$x_1 = \frac{6}{4} = \frac{3}{2}$$

$$T_2 (0, 0) \quad \checkmark$$

$$x_2 = \frac{0}{4} = 0$$

$$T_1 \left( \frac{3}{2}, 1 \right)$$

$$\partial_{yy} f = 6 \cdot y = 6 \cdot 1 = 6$$

$$A = \partial_{xx} f = 2$$

$$D = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ -3 & 6 \end{vmatrix} = 12 - 9 = 3$$

$A > 0$   
 $D > 0$  funkcija ima minimum u tački  $T \left( \frac{3}{2}, 1 \right)$

$$f(\text{min}) = y^3 - 3xy + x^2 = 1^3 - 3 \cdot \frac{3}{2} \cdot 1 + \left(\frac{3}{2}\right)^2 = 1 - \frac{9}{2} + \frac{9}{4} = -\frac{5}{4}$$

IME I PREZIME: DUJO KRALJIC

BROJ INDEKSA: 17-2-0015-2010

$$T_2(0,0)$$

$$\partial_{yy}f = 6y = 6 \cdot 0 = 0$$

$$A = \partial_{xx}f = 2$$

$$\Delta = \begin{vmatrix} \partial_{xx}f & \partial_{xy}f \\ \partial_{yx}f & \partial_{yy}f \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ -3 & 0 \end{vmatrix} = 0 - 9 = -9$$

$$A > 0$$

$\Delta < 0$  sedlasta točka ✓

10



Popuniti odmah!

IME I PREZIME: BERNARDO KOTLAR

BROJ INDEKSA:

DATUM: VRIJEME: OD 9:30

DO 11:30

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓  
bodova

~~15~~

15

~~15~~

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15

1. Izračunati  $\int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx$ .

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3. Nekom od metoda numeričke integracije (Simpsonova ili trapezna formula) približno odrediti vrijednost integrala:

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5. Pronaći opće rješenje problema:  $y' + xy + x = 0$ .

6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije  $f(x) = 2x \cos x$  oko točke  $x_0 = 0$ .

$$2. \int x^2 \sin x dx \quad / \quad \begin{array}{l} u = x^2 \\ du = 2x dx \end{array} \quad \begin{array}{l} v' = \sin x \\ v = -\cos x \end{array}$$

$$u \cdot v - \int v \cdot du = -x^2 \cos x - \int -\cos x \cdot 2x dx$$

$$-x^2 \cos x + 2 \int \cos x \cdot x dx \quad / \quad \begin{array}{l} u = x \\ du = dx \end{array} \quad \begin{array}{l} v' = \cos x \\ v = \sin x \end{array}$$

$$-x^2 \cos x + 2 \cdot (x \cdot \sin x - \int \sin x dx)$$

$$-x^2 \cos x + 2x \sin x + 2 \cos x + C \quad \checkmark \quad \underline{15}$$

$$4. f(x, y) = y^3 - 3xy + x^2$$

$$f'(x, y)_x = -3y + 2x$$

$$f'(x, y)_y = 3y^2 - 3x$$

$$-3y + 2x = 0 \quad | \cdot 3$$

$$3y^2 - 3x = 0 \quad | \cdot 2$$

$$-9y + 6x = 0$$

$$6y^2 - 6x = 0$$

$$6y^2 - 9y = 0$$

$$y - (6y - 9) = 0 \Rightarrow \text{DRUGO RJEŠENJE} \Rightarrow y = 0 \Rightarrow x = 0$$

$$6y = 9$$

$$y = \frac{9}{6} = \frac{3}{2} \checkmark$$

$$H = \begin{vmatrix} 2 & -3 \\ -3 & 6 \end{vmatrix} = 12 - 9 = 3 > 0$$

minimum je

$$\Delta = \begin{vmatrix} 2 & -3 \\ -3 & 6 \cdot \frac{3}{2} \end{vmatrix} = 2 \cdot 9 - (-3)^2 = 9$$

$$f''(x, y)_{xx} = 2 \checkmark$$

$$f''(x, y)_{xy} = -3 \checkmark$$

$$f''(x, y)_{yx} = -3 \checkmark$$

$$f''(x, y)_{yy} = 6y \checkmark$$

$$T_2(0, 0) ?$$

$$-3y + 2x = 0$$

2 25

$$2x = +3y$$

$$2x = +3 \cdot \frac{9}{6}$$

$$2x = +\frac{27}{6} \quad | : 2$$

$$x = +\frac{27}{12} = \frac{9}{4} \checkmark$$

$$A \left( \frac{27}{12}, \frac{9}{6} \right) \checkmark$$

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ima ekstrem

6.  $f(x) = 2x \cos x \quad x_0$

$$f(x_0) = 2x \cos x = 2 \cdot 0 \cdot \cos 0 = 0$$

$$\begin{aligned} f'(x) &= 2x \cos x = 2x' \cdot \cos x + 2x \cdot \cos x' \\ &= 2 \cos x + 2x \cdot (-\sin x) \\ &= 2 \cos x - 2x \sin x \end{aligned}$$

$$f'(0) = 2 \cdot 1 - 0 = 2$$

$$\begin{aligned} f''(x) &= (2' \cdot \cos x + 2 \cdot \cos x') - (2x' \cdot \sin x + 2x \cdot \sin x') \\ &= -2 \sin x - 2 \sin x + 2x \cos x \end{aligned}$$

$$f''(0) = 0 - 0 + 0 = 0$$

$$\begin{aligned} f'''(x) &= (-2' \cdot \sin x + (-2) \cdot \sin x') - (2x' \cdot \cos x + 2x \cdot \cos x') + (2x' \cdot \cos x + 2x \cdot \cos x') \\ &= -2 \cos x - 2 \cos x + (2 \cos x - 2x \sin x) \end{aligned}$$

$$= -2 \cos x - 2 \cos x + 2 \cos x - 2x \sin x$$

$$f'''(0) = -2 - 2 + 2 - 0 = -2x - 6$$

$$\begin{aligned} f(x) &= 0 + \frac{2}{1!} (x-0) + \frac{0}{2!} (x-0)^2 + \frac{-2}{3!} \cdot (x-0)^3 \\ &= 0 + 2x + 0 - \frac{2}{6} x^3 = 2x - \frac{1}{3} x^3 \quad \checkmark \quad 15 \end{aligned}$$

$$7. \int \frac{x^2 + 2x + 2}{x^2 + x - 2} dx$$

$$\begin{aligned} x^2 + 2x + 2 &= (x^2 + x - 2) + 7 + x \\ \frac{-x^2 + x}{x + 2} & \end{aligned}$$

$$\begin{aligned} x^2 + 2x + 2 &= x^2 + x - 2 + 7 + x \\ \frac{x^2 + 2x + 2 - (x^2 + x - 2)}{x + 2} &= \frac{7 + x}{x + 2} \end{aligned}$$

$$\int \frac{A}{x+2} - \frac{B}{x+1} dx$$

~~Ø~~

$$\Rightarrow I = \int \left( 1 + \frac{x+4}{x^2+x-2} \right) dx = \dots$$

$$3. \int_{\pi}^{2\pi} \frac{\arctan x}{x} dx$$

$$= \int_{\pi}^{2\pi} \arctan x \cdot x^{-1} dx$$

$\downarrow$   $\downarrow$   
 $u$   $v$

$$\begin{aligned} du &= \arctan x \\ dv &= \frac{1}{x^2} dx \\ v &= x^{-1} \end{aligned}$$

$$\arctan x = \int_{\pi}^{2\pi}$$

$$v = \int x^{-1} = v = x^0$$

NUMERIČKA INTEGRACIJA.