

Odmah popuniti ↓

IME I PREZIME: **IVES TOMIĆ**

BROJ INDEKSA:

50

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: DATUM

OD

DO

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

ooox

1. Primjenom Greenove formule izračunati integral

$$\oint_C y^2 dx + (x+y)^2 dy,$$

10

gdje je  $C$  kontura trokuta  $A(0,0)$ ,  $B(2,2)$  i  $C(1,3)$  prijedena u pozitivnom smislu (suprotno od kazaljke na satu).

2. Izračunati  $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$  gdje je  $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$  i  $\partial K$  rub kugle  $K$  radijusa 1 s centrom u točki  $T(2,1,0)$ , a koji je orijentiran vanjskom normalom.

3. Izračunati volumen tijela omeđenog plohama:  $x^2 + y^2 + z^2 = 25$ ,  $z = 4$ .

4. Izračunati

$$\int_{(1,\pi)}^{(2,3\pi)} 2x \sin y dx + (x^2 + 1) \cos y dy$$

20

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$f'''(t) - 4f'(t) = \cos(2t), \quad f(0) = f'(0) = f''(0) = 0.$$

20

5.  $f'''(t) - 4f'(t) = \cos(2t)$        $f(0) = f'(0) = f''(0) = 0$

$f''' \Rightarrow s^3 F(s) - \cancel{s^2 \cdot 0} - \cancel{s \cdot 0} - 0$

$f' \Rightarrow sF(s) - 0$

$s^3 F(s) - 4sF(s) = \frac{s}{s^2+4}$

$F(s)(s^3 - 4s) = \frac{s}{s^2+4}$

$F(s) = \frac{s}{(s^3 - 4s)(s^2+4)} = \frac{s}{s \underbrace{(s^2-4)}_{(s-2)(s+2)}(s^2+4)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} + \frac{D}{s^2+4}$

$s = A s^4 - 16A + B s(s+2)(s^2+4) + C s(s-2)(s^2+4) + D s^2(s^2+4)$

$s = A s^4 - 16A + B s^2 + 2B s^3 + 8B s + C s^2 - 2C s^3 - 8C s + D s^4 - 4D s^2 + 4D s$

$0 = A + D$

$0 = -16A$

$A = 0$

$0 = 0 + D$

$D = 0$

$0 = B + C - 4D$

$0 = 2B - 2C + E \quad / \cdot 4$

$1 = 8B - 8C - 4E$

$0 = 8B - 8C + 4E$

$1 = 8B - 8C - 4E$

$1 = 16B - 16C$

$0 = B + C \quad / \cdot 16$

$1 = 16B - 16C$

$0 = 16B + 16C$

$1 = 16B - 16C$

$1 = 16B$

$B = \frac{1}{16}$

$0 = B + C$

$0 = \frac{1}{16} + C$

$C = -\frac{1}{16}$

$0 = 2B - 2C + E$

$0 = \frac{2}{16} + \frac{2}{16} + E$

$0 = \frac{1}{8} + \frac{1}{8} + E$

$0 = \frac{2}{8} + E \quad / \cdot 4$

$D s^4 - 4D s^2 + 4D s$

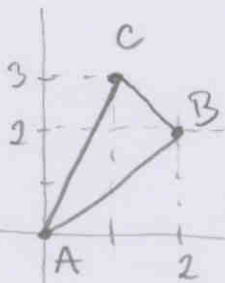
$$X(s) = \frac{1}{16} \frac{1}{s-2} - \frac{1}{16} \frac{1}{s+2} - \frac{1}{4} \frac{1}{s^2+4}$$

$$= \frac{1}{16} e^{2t} - \frac{1}{16} e^{-2t} - \frac{1}{4} \sin(2t) \quad \checkmark \underline{20}$$

①  $\oint_C \underbrace{y^2 dx}_P + \underbrace{(x+y)^2 dy}_Q$

$$\frac{dQ}{dx} = 2(x+y) \quad \checkmark$$

$$\frac{dP}{dy} = 2y \quad \checkmark$$



$+(\bar{x}, \bar{y})$   
 $B(2, 2)$   
 $C(1, 3)$

AB

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 0 = \frac{2 - 0}{2 - 0} (x - 0)$$

$$y - 0 = \frac{2}{2} (x - 0)$$

$$y - 0 = x - 0$$

$$\boxed{y = x} \quad x=y$$

AC

$$y - 0 = \frac{3 - 0}{1 - 0} (x - 0)$$

$$y - 0 = \frac{3}{1} (x - 0)$$

$$y - 0 = 3x$$

$$\boxed{y = 3x}$$

$$x = \frac{y}{3}$$

BC

$$y - 2 = \frac{3 - 2}{1 - 2} (x - 2)$$

$$y - 2 = -(x - 2)$$

$$y - 2 = -x + 2$$

$$y = -x + 2 + 2$$

$$\boxed{y = -x + 4}$$

$$x = -y + 4$$

IME I PREZIME:

INES TOMIĆ

BROJ INDEKSA:

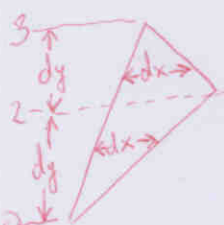
$$\iint_{D_0} 2(x+y) - 2y = \iint_{D_0} 2x + \cancel{2y} - \cancel{2y} = \iint_{D_0} 2x \, dx \, dy = \int_0^3 \int_0^{-x+4} 2x \, dx \, dy$$

$$\oint_C P dx + Q dy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx \, dy$$

TROKUT

$$= 2x$$

$$= \int_0^2 \int_0^{3-y} 2x \, dx \, dy + \int_2^3 \int_0^{y-2} 2x \, dx \, dy$$



= ...

$$= \int_0^3 (-x+4)^2 \, dx = \int_0^3 (-x^2 - 4x + 4^2) \, dx = \left[ -\frac{x^3}{3} - 2x^2 + 4x \right]_0^3 = 9 - 18 + 12 = 3$$

10

4.  $\left| \begin{matrix} 2x \sin y \\ (x^2+1) \cos y \end{matrix} \right|$

$$f(x,y) = -(x^2+1) \sin y$$

$$2 \sin y - (x^2+1) \sin y$$

$$\begin{aligned} & 2 \sin \pi - 2 \sin \pi \\ & 2 \sin 3\pi - 5 \sin 3\pi \end{aligned}$$

$$f(1, \pi) - f(2, 3\pi) = 0 - 0 = 0 \quad \checkmark$$

20

Odmah popuniti ↓

IME I PREZIME:

Jure Mazić

BROJ INDEKSA:

52915-2005

10

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: DATUM

OD

DO

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. Primjenom Greenove formule izračunati integral

$$\oint_C y^2 dx + (x+y)^2 dy,$$

gdje je C kontura trokuta  $A(0,0)$ ,  $B(2,2)$  i  $C(1,3)$  prijedena u pozitivnom smislu (suprotno od kazaljke na satu).

2. Izračunati  $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$  gdje je  $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$  i  $\partial K$  rub kugle  $K$  radijusa 1 s centrom u točki  $T(2,1,0)$ , a koji je orijentiran vanjskom normalom.

3. Izračunati volumen tijela omeđenog plohama:  $x^2 + y^2 + z^2 = 25$ ,  $z = 4$ .

4. Izračunati

$$\int_{(1,\pi)}^{(2,3\pi)} 2x \sin y dx + (x^2 + 1) \cos y dy$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu:

$$f'''(t) - 4f'(t) = \cos(2t), \quad f(0) = f'(0) = f''(0) = 0.$$

10

5)  $f'''(t) - 4f'(t) = \cos(2t)$

$f(0) = 0$   
 $f'(0) = 0$   
 $f''(0) = 0$

$$s^3 F(s) - s^2 f(0) - s f'(0) - f''(0) - 4[s F(s) - f(0)] = \frac{s}{s^2 + 4}$$

$$s^3 F(s) - 4F(s) = \frac{s}{s^2 + 4}$$

$$F(s)(s^3 - 4s) = \frac{s}{s^2 + 4}$$

$$F(s) = \frac{s}{s(s-2)(s+2)(s^2+4)}$$

$$\frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+4}$$

$$As^4 - 16A + Bs^4 + 4Bs^2 + 2Bs^3 + 8Bs + Cs^4 + 4Cs^2 - 2Cs^3 - 8Cs + Ds^4 - 4Ds^2 + Es^3 - 4Es$$

$$s^4 (A+B+C+D)$$

$$s^3 (2B-2C+E)$$

$$s^2 (4B+4C-4D)$$

$$s (8B-8C-4E)$$

$$A+B+C+D=0 \quad (1)$$

$$2B-2C+E=0 \quad (2)$$

$$4B+4C-4D=0 \quad (3)$$

$$8B-8C-4E=1 \quad (4)$$

$$A=0 \quad (5)$$

NE ZADONEJAVI  
JEDNAŽBU (2)

$$\Rightarrow F(s) = \frac{0}{s} + \frac{3}{s-2} + \frac{?}{s+2} + \frac{?s+?}{s^2+4}$$

$$f(t) = -\frac{1}{4} t e^{-2t} \quad \times$$

10

$A=0$

---

$E = -\frac{1}{4}$

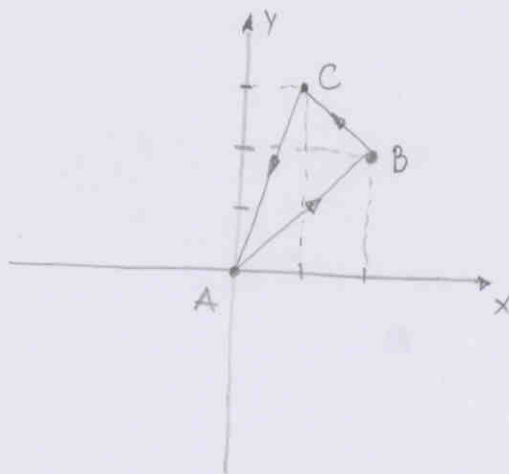
---

$B=C=0$

1)

$$\oint_C y^2 dx + (x+y)^2 dy$$

- A(0,0)
- B(2,2)
- C(1,3)



?