Odmah popuniti ↓

IME I PREZIME: Toni Prenda BROJ INDEKSA: 53465/2006

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: DATUM VRIJEME: OD DO

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

30

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

10

2. Neka je C cilindar zadan sa $C = \{(x, y, z) : (x+2)^2 + (y-3)^2 \leq 1,\ -1 \leq z \leq 1\}$. Izračunati plošni integral

$$\iint_{\partial C} 2x\,dydz$$

3. Zadana je krivulja s parametrizacijom $\mathbf{r}(t) = (\sin(2t), \cos(2t), t)$. Izračunati duljinu krivulje u dijelu koji odgovara parametaru $t \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

20

4. Zadan je dio stošca (oznaka Y) omeđen plohama $x^2 + y^2 = z^2$, $z = 1$ i $z = 4$. Izračunati $\int_Y xyz\,dxdydz$ prijelazom na cilindrične koordinate. (mala pomoć: $\sin x \cos x = \frac{\sin(2x)}{2}$)

0

5. Izračunati $\int_{\widehat{ABC}} ydx + dz$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(2,0,0)$, $B(0,1,0)$, $C(1,0,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu.

0

3. $r(t) = (\sin(2t), \cos(2t), t)$ $\quad t \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$\sin 2t$
$\cos 2t$
t

$r'(t) \begin{bmatrix} +\cos 2t \\ -\sin 2t \\ 1 \end{bmatrix}$ $\quad |r'(t)| = \sqrt{(-\cos 2t)^2 + \sin 2t^2 + 1^2}$ $\quad \|r'(t)\| = \sqrt{(\cos 2t)^2 + (-\sin 2t)^2 + 1^2}$

$r'(t) = \begin{bmatrix} +2\cos t \\ -2\sin t \\ 1 \end{bmatrix}$ $\quad |\|r'\|(t)| = \sqrt{(2\cos t)^2 + (2\sin t)^2 + 1}$ $\quad = \sqrt{\cos^2 2t + \sin^2 2t + 1}$

$= \sqrt{4\cos^2 t + 4\sin^2 t + 1} = \sqrt{4(\cos^2 t + \sin^2 t) + 1}$ $\quad = \sqrt{4(\cos^2 t + \sin^2 t) + 1}$

$= \sqrt{4(\cos^2 t + \sin^2 t) + 1}$ $\quad = \sqrt{5}$ $\quad = \sqrt{5}$

$= \sqrt{5}$

20

$$\sqrt{5} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} dt = \sqrt{5} \cdot t \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \frac{3\pi}{2} \cdot \sqrt{5} - \frac{\pi}{2} \cdot \sqrt{5} = 4{,}71 \cdot 2{,}23 - 1{,}57 \cdot 2{,}23 = \approx 9{,}84$$

$10.50 \quad - 0{,}66$

$= \sqrt{4+1}$

$= \sqrt{5}$

$r'(t) \begin{bmatrix} 2\cos 2t \\ -2\sin 2t \\ 1 \end{bmatrix}$ $\quad |\|r'\|(t)| = \sqrt{(2\cos 2t)^2 + (-2\sin 2t)^2 + 1^2}$

$\sqrt{4\cos^2 2t + 4\sin^2 2t + 1}$

$\sqrt{4(\cos^2 2t + \sin^2 2t) + 1}$

IME I PREZIME: TONI PRENAJA BROJ INDEKSA: 53465/2006

1. $X'''(t)+X'(t)=0 \qquad X(0)=X''(0)=1 \quad X'(0)=0$

$s^3X(s)-s^2x(0)-sx'(0)-x''(0)+sX(s)-sx(0)=0$

$s^3X(s)-s^2-1+sX(s)-1=0$

$X(s)(s^3+s)=s^2+2 \;/:s^3+s$

$X(s)=\frac{s^2+2}{\underbrace{s^3+s}_{s(s^2+1)}}$

10

$\frac{s^2+2}{s(s^2+1)}=\frac{A}{s}+\frac{Bs+C}{s^2+1}$

$s^2+2=A(s^2+1)+Bs^2+Cs$

~~$s^2+2=As^2+A+Bs$~~

$Y=A \quad s^2+2=As^2+A+Bs^2+Cs$

$0=B$

$2=A$

$1=A+B \rightarrow 1=2+B \Rightarrow B=1-2=\boxed{-1}$ ✓

$\boxed{0=C}$ ✓

$\boxed{2=A}$ ✓

$X(s)=\left\{2\cdot\frac{1}{s}-\frac{1}{s^2+1}\right\}$ ✓ ✗

~~$x(t)=\frac{2\cdot 1}{s}=\frac{1}{s^2+1}$~~ ✗ ✗ ✗

$x(t)=2\cdot t-t\cdot e^t$ ✗

$X(s)=\frac{2}{s}-\frac{s}{s^2+1}$

$\mathcal{L}^{-1}\left(\frac{2}{s}\right)=2$

$\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right)=\cos t$

$x(t)=2-\cos t$

IME I PREZIME: Toni Pleuda BROJ INDEKSA:

4. $x^2+y^2=z^2$ $z=1$ i $z=4$ $\int_Y xy\,z\,dx\,dy\,dz$

$\int \underbrace{x\,dx}_{P} + \int \underbrace{y\,dy}_{Q} + \int \underbrace{z\,dx}_{R}$

$\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$ $\frac{\partial Q}{\partial y}$ $\frac{\partial R}{\partial z}$

$\int$

IME I PREZIME: PRENDA TONI BROJ INDEKSA:

$n^5 + 2n^3 + n^3 + 2n$

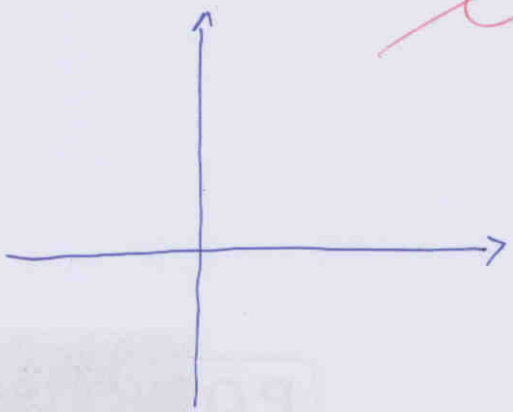
$n^5 + 3n^3 + 2n$

$(n+1)^2 = n^2 + 2n + 1$

5.

$\int_{\overrightarrow{ABC}} y\,dx + dz \qquad A(2,0,0)\ B(0,1,0)\ C(1,0,0)$

$\int_{2,0,0}^{0,1,0} y\,dx + dz$



$\sin 2t$

$\sqrt{(\sin 2t)^2 + (\quad)}$

$\sqrt{(2\sin t)^2 + (2\cos t)^2 + 1^2}$

$\sqrt{4\cos^2 t + 4\sin^2 t + 1}$

$\sqrt{4+4+1}$

$\sqrt{9} = 3$

$\cos 2t \cdot 2$

$-\sin 2t \cdot 2$

$\sqrt{(2\cos 2t)^2 + (-2\sin 2t)^2 + 1^2}$

$\sqrt{(\cos 2t)^1 \cdot 2 + (-\sin 2t)^2 \cdot 2 - 1}$

$\sqrt{4\cos^2 2t + 4\sin^3 2t + 1}$

$\sqrt{\cos^2 2t}$

$\sqrt{4(\cos^2 t + \sin 2t) + 1}$

$2\cos 2t$

$-2\sin 2t$

$\sqrt{4+1}$

$\sqrt{5}$

$\uparrow' \quad \cos t = -\sin t \qquad \underbrace{\sin^2 t + \cos^2 t}_{1}$

$\sin t = \cos t$

$\sin 2t' \Rightarrow \cos 2t \cdot 2$

$\cos 2t = -\sin 2t \cdot 2$