Odmah popuniti ↓

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OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: DATUM OD 8:00 DO .

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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1. Izračunati dvostruki integral:

$$\iint_S (x+y)\,\mathrm{d}x\mathrm{d}y\,,$$

gdje je S područje gornje poluravnine ($y \geq 0$) omeđeno kružnicom $(x-1)^2 + y^2 = 1$.

2. Izračunati $\int_{\widehat{ABC}} x\mathrm{d}x + y\mathrm{d}y + z\mathrm{d}z$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(0,0,1)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu.

3. Izračunati volumen tijela omeđenog valjkom $x^2 + y^2 = 4$ i ravninama $y + 1 = z$ i $y + 2 = z$.

0

4. Izračunati

$$\int_{(1,2\pi,0)}^{(1,\pi,\pi)} x\,\mathrm{d}x + z^2\cos y\,\mathrm{d}y + 2z\sin y\,\mathrm{d}z$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$x'''(t) - x''(t) = \mathrm{e}^t, \quad x'(0) = x''(0) = -1,\ x(0) = 0.$$

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(5) $x'''(t) - x''(t) = e^t \qquad x'(0) = x''(0) = -1, \quad x(0) = 0$

$x''(t) \circ\!\!-\!\!\bullet\; s^3X(s) - s^2\underbrace{x(0)}_{0} - s\underbrace{x'(0)}_{-1} - \underbrace{x''(0)}_{-1}$

$s^3X(s) + s + 1$

$x'(t) \circ\!\!-\!\!\bullet\; s^2X(s) - s\underbrace{x(0)}_{0} - \underbrace{x'(0)}_{-1}$

$s^2X(s) + 1$

$e^t \circ\!\!-\!\!\bullet \frac{1}{s-1}$

$s^3X(s) + s + 1 - (s^2X(s) + 1) = \frac{1}{s-1}$

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$X(s)(s^3 - s^2) = \frac{1}{s-1} - s - 1 \;/\cdot (s^3 - s^2) =$

$X(s) = \frac{1}{(s-1)(s^3-s^2)} - \frac{s-1}{s^3-s^2} = \frac{1}{s^2(s-1)(s-1)} - \frac{s-1}{s^2(s-1)} = \frac{1-(s-1)(s-1)}{s^2(s-1)(s-1)}$

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$X(s) = \frac{s^2-2s}{s^2(s-1)(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{D}{(s-1)^2} \;/\cdot s^2(s-1)(s-1)$

$s^2 - 2s = As(s-1)(s-1) + B(s-1)(s-1) + C(s^2(s-1)) + Ds^2$

$= As^3 - 2As^2 + As + Bs^2 - 2Bs + B + Cs^3 - Cs^2 + Ds^2$

$= s^3(A+C) + s^2(-2A+B-C+D) + s(A-2B) + B$

$A + C = 0 \Rightarrow -2 = C, \; \boxed{C = 2}$

$-2A + B - C + D = 1 \Rightarrow -4 + 0 - 2 = -D \;/\cdot -$

$A + 2B = -2 \qquad D = 4 + 2$

$D = 6$

$\boxed{B = 0}$

$\boxed{A = -2}$

$\boxed{C = 2}$

$\boxed{D = 6}$

$X(s) = -2\cdot\frac{1}{s} + \left(0\cdot\frac{1}{s^2}\right) + \left(2\cdot\frac{1}{s-1}\right) + \left(6\cdot\frac{1}{(s-1)^2}\right)$

$\circ\!\!-\!\!\bullet\; x(t) = -2 + 2e^t + 6te^t = x(t)$

$x(t) = -2 + 2e^t + 6te^t$

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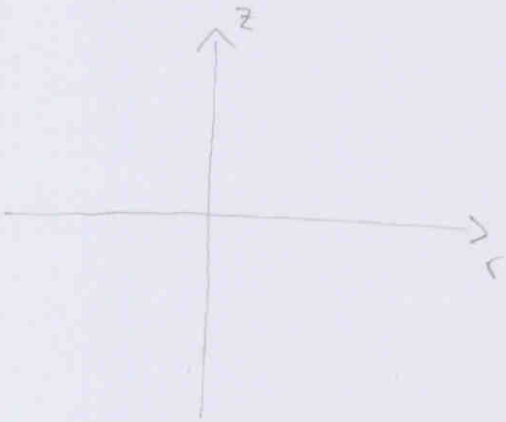
(3) $x^2+y^2=4$ $y+1=z$ $y+2=z$



$r=R$

$-x^2+y^2=R^2$

$z=z$

$y+1=y-1$

0