Odmah popuniti ↓

Ime i prezime: Marin Vulić **Broj indeksa:** 0035159546

(40)

Obavezno popuniti vrijeme rješavanja ispita: datum OD DO .

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. ooxx

1. X je zadan kao trokut s vrhovima $O(0,0)$, $A(-1,2)$ i $C(2,-1)$. Skicirati taj trokut i izračunati dvostruki integral

$$\iint\limits_X y\,\mathrm{d}x\mathrm{d}y$$

20

2. Neka je X dio kugle $x^2+y^2+z^2=16$ za koji vrijedi $z \le -2$. Označimo sa ∂X rub od X. Izračunati plošni integral

$$\iint\limits_{\partial \widehat{K}} x\,\mathrm{d}y\mathrm{d}z + z\,\mathrm{d}x\mathrm{d}z + y\,\mathrm{d}x\mathrm{d}y$$

3. Izračunati: $\int_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$, ako je $\mathbf{w}(x,y,z)=(y,z,x)$ i krivulja $\widehat{\Gamma}=\left\{(x,y,z)\,|\,x=\frac{1}{2}\cos t,\ y=\frac{1}{2}\sin t,\ z=\frac{\sqrt{3}}{2},\ t\in[0,\pi]\right\}$. ∅

4. Izračunati

$$\int\limits_{(2,2)}^{(1,1)} (y^2+2xy)\,\mathrm{d}x + (2xy+x^2)\,\mathrm{d}y$$

∅

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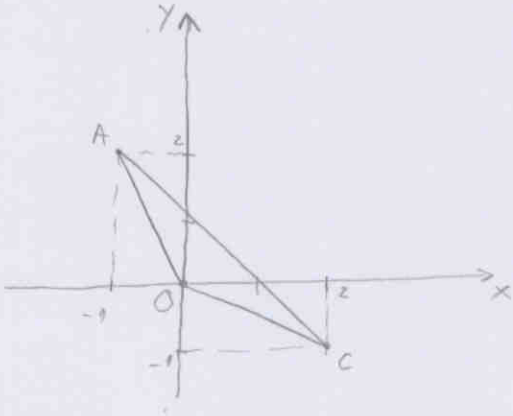
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t) - 2y''(t) = \mathrm{e}^t, \quad y(0) = y''(0) = 2,\ y'(0) = 2.$$

Ime i prezime: Marin Vulić **Broj indeksa:**

1. $O(0,0)$, $A(-1,2)$, $C(2,-1)$

$$\iint_X y\,dx\,dy$$



$\overline{OA} \ldots \quad y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-0=\frac{2-0}{-1-0}(x-0)$

$y=-2x \ldots \overline{OA} \Rightarrow$

$\overline{CO} \ldots \quad y-1=\frac{0+1}{0-2}(x-2)$

$y-1=-\frac{1}{2}x+1$

$y=-\frac{1}{2}x+2 \ldots \overline{CO}$

$\overline{AC} \ldots \quad y-2=\frac{-1-2}{2+1}(x+1)$

$y-2=-x-1$

$y=-x+1 \ldots \overline{AC} \Rightarrow x=1-y$

$$\int_{-1}^{2}\int_{-2x}^{-\frac{1}{2}x+2} y\,dx\,dy=\int_{-1}^{-x+1} dx\left(\frac{y^2}{2}\right)\Bigg|_{-2x}^{-\frac{1}{2}x+2}=\int_{-1}^{2} dx\left(\frac{-\frac{1}{2}x+2}{2}\right)^2-\left(\frac{-2x}{2}\right)^2=\int_{-1}^{2} dx\,(-x+1)^2+x^2=$$

$$=\int_{-1}^{2} dx\,(x^2-2x+1)+x^2=\int_{-1}^{2} 2x^2-2x+1\,dx=2\int_{-1}^{2} x^2dx-2\int_{-1}^{2} x\,dx+1\int_{-1}^{2} dx=$$

$$=2\frac{x^3}{3}\Bigg|_{-1}^{2}-2\frac{x^2}{2}\Bigg|_{-1}^{2}+1\,x\Bigg|_{-1}^{2}=2\left(\frac{8}{3}+\frac{1}{3}\right)-2\left(2+\frac{1}{2}\right)+1(2+1)=$$

$$=6-3+3=6$$ //

✓ 20

IME I PREZIME: Marin Vulić BROJ INDEKSA:

$$\int_{(2,2)}^{(1,1)} (y^2+2xy)dx + (2xy+x^2)dy$$

$$w = \begin{bmatrix} y^2+2xy \\ 2xy+x^2 \end{bmatrix} = -\operatorname{grad} f$$

$$\frac{df}{dx} = -y^2+2xy \int dx$$

$f =$?

$$\frac{df}{dy} = -2xy+x^2$$

$\varnothing$

$$\frac{d}{dy} = -2xy+x^2$$

3. $|r'| = \sqrt{\left(\frac{1}{2}\cos t\right)^2+\left(\frac{1}{2}\sin t\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}\cos^2 t+\frac{1}{4}\sin^2 t+\frac{3}{4}} = \sqrt{\frac{1}{4}(\cos^2 t+\sin^2 t)+\frac{3}{4}}$ ✓

$= \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$ ✓

$$\int_{\Gamma} (w \mid dr) = ?$$

$\varnothing$

$w = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

VIDI KRIVULJE INTEGRALE

IME I PREZIME: Marin Vulić   BROJ INDEKSA:

5. $y'''(t) - 2y''(t) = e^t$ $y(0) = y''(0) = 2$, $y'(0) = 2$

$$\frac{1}{s} + \frac{2}{3}\cdot\frac{1}{s^2} - \frac{11}{3}\frac{1}{s-2} + \frac{2}{3}\frac{1}{s-1}$$

$$1 + \frac{2}{3}t - \frac{11}{3}e^{2t} + \frac{2}{3}e^{t}$$

$y'''(t) = s^3F(s) - s^2 f(0) - s f'(0) - f''(0)$

$= s^3F(s) - 2s^2 - 2s - 2$ ✓

$y''(t) = s^2F(s) - sf(0) - f'(0)$

$= s^2F(s) - 2s - 2$ ✓

$1 + 2s^3 - 2s^2 - \cancel{2s} - 2s^2 + \cancel{2s} + 2$

$3 - 4s^2 + 2s^3$

JEDNA POGREŠKA U PREDZNAKU

DODIJELJEN PUNI BROJ BODOVA

20

$$s^3F(s) - 2s^2 - 2s - 2 - 2(s^2F(s) - 2s - 2) = \frac{1}{s-1}$$

$$s^3F(s) - 2s^2 - 2s - 2 - 2s^2F(s) + 4s + 4 = \frac{1}{s-1}$$ ✓

$$s^3F(s) - 2s^2F(s) = \frac{1}{s-1} + 2s^2 + 2s + 2 - 4s - 4$$ ✓

$$s^3F(s) - 2s^2F(s) = \frac{1}{s-1} + 2s^2 - 2s - 2$$ ✓

$$F(s)(s^3 - 2s^2) = \frac{1}{s-1} + 2s^2 - 2s - 2$$ ✓

$$F(s) = \frac{\frac{1}{s-1} + 2s^2 - 2s - 2}{s^3 - 2s^2} = \frac{1 \overset{+}{-} 2s^2(s-1) - 2s(s-1) - 2(s-1)}{(s^3 - 2s^2)(s-1)} \times = \frac{1 - 2s^3 + \cancel{2s^2} - 2s^2 + 2s - 2s + 2}{s^2(s-2)(s-1)} =$$

$3 - 4s^2 + 2s^3$

$$= \frac{-2s^3 + 3}{s^2(s-2)(s-1)} \times = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-2} + \frac{D}{s-1} ✓ \quad \cdot s^2(s-2)(s-1)$$ ✓

POSTUPAK U REDU.

$$A(s(s-2)(s-1)) + B((s-2)(s-1)) + C(s^2(s-1)) + D(s^2(s-2))$$

$$A(s^3 - 3s^2 + 2s) + B(s^2 - s - 2s + 2) + C(s^3 - s^2) + D(s^3 - 2s^2) = -2s^3 + 3$$

$$As^3 - 3As^2 + 2As + Bs^2 - 3Bs + 2B + Cs^3 - Cs^2 + Ds^3 - 2Ds^2 = -2s^3 + 3$$

$A + C + D = -2$ ✓ $-3A + B - C - 2D = 0$ ✓ $-3 + \frac{2}{3} - C - 2D = 0$

$2B = 3$ ✓ $2A - 3B = 0$ ✓ $-C - 2D = \frac{7}{3}$ ✓

$\boxed{B = \frac{2}{3}}$ $2A = 3B$ $C + D = -3$ ✓

$2A = 2$

$\boxed{A = 1}$ ✓ $-2D + D = \frac{7}{3} - 3$

$1 + C + \frac{2}{3} = -2$ $-D = \frac{7-9}{3}$

$C = -2 - 1 - \frac{2}{3}$

$C = -3 - \frac{2}{3} = \frac{-9-2}{3} = -\frac{11}{3}$ $\boxed{C = -\frac{11}{3}}$ ✓ $-D = \frac{-2}{3}$ $\boxed{D = \frac{2}{3}}$ ✓

Odmah popuniti ↓

IME I PREZIME: JAKŠA BLITVIĆ BROJ INDEKSA: 54805

35

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: DATUM OD DO .

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~~0~~

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15

5. $y'''(t)-2y''(t)=e^t \qquad y(0)=y''(0)=2 \qquad y'(0)=2$

$y'''(t) \circ\!\!-\!\!\bullet\ s^3Y(s)-s^2\overset{2}{y(0)}-s\overset{2}{y'(0)}-\overset{2}{y''(0)}$

$\boxed{s^3Y(s)-2s^2-2s-2}$

$2y''(t) \circ\!\!-\!\!\bullet\ s^2Y(s)-s\overset{2}{y(0)}-\overset{2}{y'(0)}$

$\boxed{s^2Y(s)-2s-2}$

$e^t \circ\!\!-\!\!\bullet\ \boxed{\frac{1}{s-1}}$

$s^3Y(s)-2s^2-2s-2-2(s^2Y(s)+2s+2)=\frac{1}{s-1}$ ×

$Y(s)(s^3-s^2)=\frac{1}{s-1}+2s^2+\cancel{2s}+\cancel{2}-\cancel{2s}-\cancel{2}$ ×

$Y(s)(s^3-s^2)=\frac{1}{s-1}+2s^2\ /:s^3-s^2$ →

(2)

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$$y(s) = \frac{1}{(s-1)(s^3-s^2)} + \frac{2s^2}{s^3-s^2} = \frac{1+2s^2(s-1)}{(s-1)(s^3-s^2)}$$

$$y(s) = \frac{1+2s^3-2s^2}{(s-1)s^2(s-1)} = \frac{1+2s^2(s-1)}{s^2(s^2-1)}$$

$$y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2-1} \quad / \cdot s^2(s^2-1)$$

$$y(s) = A(s(s^2-1)) + B(s^2-1) + (Cs+D)s^2$$

$$y(s) = As^3 - As + Bs^2 - B + Cs^3 + Ds^2$$

$$2s^3-2s^2+1 = As^3 - As + \underline{Bs^2} - B + Cs^3 + \underline{Ds^2}$$

$2 = A + C \rightarrow C = 2+0 \rightarrow \boxed{C=2}$

$-2 = B + D \rightarrow -2 = -1 + D \rightarrow D = -2+1$

$\boxed{0 = A}$ $\boxed{D=-1}$

$1 = -B$ $\boxed{B=-1}$

DOBAR POSTUPAK

15

$$y(s) = \frac{\cancel{0}}{\cancel{s}} + \frac{(-1)}{s^2} + \frac{2+(-1)}{s^2-1} = -\frac{1}{s^2} - \frac{2}{s^2-1}$$

$$= -\frac{1}{s^2} - 2\frac{1}{s^2-1^2}$$

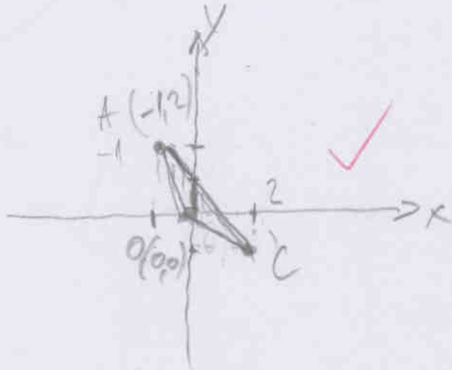
$$Y(s) \bullet\!\!-\!\!\circ\, y(t) = -t - 2\sin(t)$$

Ime i prezime: JAKŠA BLITVIĆ Broj indeksa: 54809

(3)

(1) $O(0,0)$ $A(-1,2)$ $C(2,-1)$

$$\iint_X y\,dx\,dy$$



OA)

$(0,0)$ $(-1,2)$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-0=\frac{2-0}{-1-0}(x-0)$$

$$y=-2x$$

OC

$(0,0)$ $(2,-1)$

$$y-0=\frac{-1-0}{2-0}(x-0)$$

$$y=-\frac{1}{2}x$$

AC

$(-1,2)$ $(2,-1)$

$$y-2=\frac{-1-2}{2-(-1)}(x-(-1))$$

$$y-2=\frac{-3}{3}x$$

$$y-2=-x$$

$$\boxed{y=-x+2}$$

→

IME I PREZIME: JAGŠA BLITVIĆ **BROJ INDEKSA:** 54805 5

nastavak 1

drugi dio $\int_{-1}^{0} dx \int_{-2x}^{-x+2} y\,dy$

$$\int_{-1}^{0} dx \; \frac{y^2}{2} \Big|_{-2x}^{-x+2} = \int_{-1}^{0} dx \; \frac{(-x+2)^2}{2} - \frac{(-2x)^2}{2}$$

$$\int_{-1}^{0} dx \; \frac{(-x+2)^2 + 4x^2}{2} = \frac{1}{2}\int_{-1}^{0} dx \; \left((-x+2)^2 + 4x^2\right)$$

$$\frac{1}{2}\int_{-1}^{0} -x^2 - 4x + 4 + 4x^2 = \frac{1}{2}\int_{-1}^{0} 3x^2 - 4x + 4\,dx$$

$$\frac{1}{2}\left(\cancel{3}\frac{x^3}{\cancel{3}} - \frac{4x^2}{\cancel{4}} + 4x\right)\Big|_{-1}^{0} = \frac{1}{2}\left(x^3 - 2x^2 + 4x\right)\Big|_{-1}^{0}$$

$$\frac{1}{2}\left(0 - \left((-1)^3 - 2\cdot(-1)^2 + 4\cdot(-1)\right)\right) = \frac{1}{2}\left(0 - (-1 - 2 - 4)\right)$$

$$= \frac{1}{2}\left(1+2+4\right) = \frac{1}{2}\cdot 7 = \underline{\underline{3\,\frac{1}{2}}} \text{ — drugi dio}$$

RJEŠENJE

$$-1 + 3\,\frac{1}{2} = \underline{\underline{2\,\frac{1}{2}}}$$

✓ 20

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6

2 $x^2+y^2+z^2=16$ $z \le -2$

$x=r\cos\varphi$ $\iint_{\partial K} x\,dy\,dz + z\,dx\,dz + y\,dx\,dy$

$y=r\sin\varphi$

$\boxed{R=4}$

$x^2+y^2+z^2=16$ $R^2=16$

$(r\cos\varphi)^2+(r\sin\varphi)^2+z^2=16$ $R=\sqrt{16}$

$r^2\cos^2\varphi+r^2\sin^2\varphi+z^2=16$

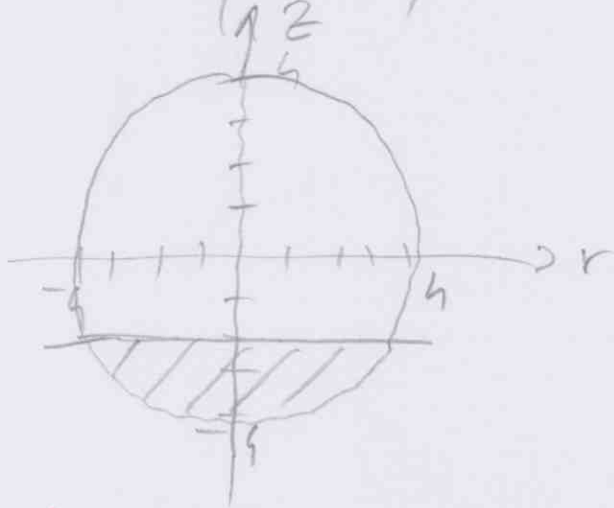
$r^2(\underbrace{\cancel{\cos^2\varphi+\sin^2\varphi}}_{1})+z^2=16$

$r^2+z^2=16 \rightarrow r=\sqrt{16-z^2}$

$r\in[0,\sqrt{16-z^2}]$

$\varphi\in[0,2\pi]$

$z\in(-4,-2]$



NIJE POKAZAN PRELAZAK S PLOŠNOG INTEGRALA NA VOLUMNI INTEGRAL

$\cancel{O}$

$$\int_{-4}^{-2}dz\int_0^{2\pi}d\varphi\int_0^{\sqrt{16-z^2}}r\,dr=\int_{-4}^{-2}dz\int_0^{2\pi}d\varphi\,\frac{r^2}{2}\Big|_0^{\sqrt{16-z^2}}$$

$$=\int_{-4}^{-2}dz\int_0^{2\pi}d\varphi\,\frac{(\cancel{\sqrt{16-z^2}})^{\cancel{2}}}{2}=\int_{-4}^{-2}dz\int_0^{2\pi}d\varphi\,\frac{16-z^2}{2}\longrightarrow$$

Ime i prezime: JAKŠA BLITVIĆ **Broj indeksa:** 54805

$$\frac{1}{2}\int_{-4}^{-2} dz \; 16-z^2 \, \varphi \Big|_0^{2\pi} = \frac{1}{2}\int_{-4}^{-2} dz \; 16-z^2 (2\pi)$$

$$2\pi \frac{1}{2}\int_{-4}^{-2} 16-z^2 \, dz = \pi \left(16z - \frac{z^3}{3}\right)\Big|_{-4}^{-2}$$

$$\pi \left(\left(16\cdot(-2) - \frac{(-2)^3}{3}\right) - \left(16\cdot(-4) - \frac{(-4)^3}{3}\right)\right)$$

$$\pi \left(-32 + \frac{8}{3}\right) - \left(-64 + \frac{16}{3}\right)$$

$$\pi \left(-32 + \frac{8}{3} + 64 - \frac{16}{3}\right) = \pi \left(32 + \frac{(8-16)}{3}\right)$$

$$\pi \left(32 - \frac{8}{3}\right) = \pi = \underline{\underline{29\frac{1}{3}}}$$