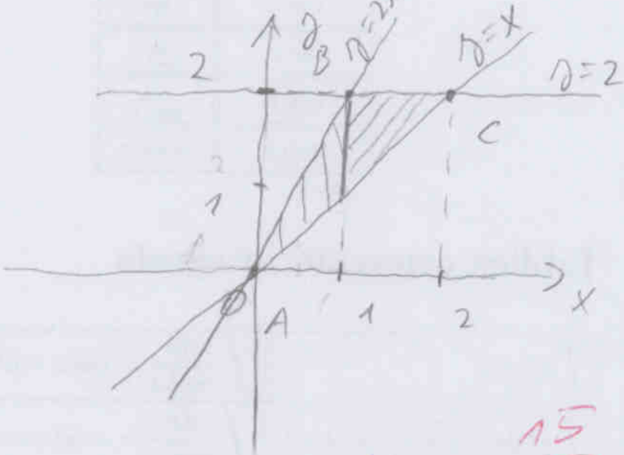
Popuniti odmah!
Ime i prezime: LOVRE LOVRIĆ **Broj indeksa:** 58080 – 2009 80
Datum: 30.06.2011. **Vrijeme:** od 08:00 do

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

	Broj bodova ↓	
1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,0)$, $B(1,2)$ i $C(2,2)$.	15	15
2. Zadano je $f(x) = \frac{1}{\sqrt{x+1}}$. Odrediti $\int_{-1}^{1} f(x)\,dx$. Skicirati graf funkcije f i površinu koja je određena integralom $\int_{-1}^{1} f(x)\,dx$.	15	15
3. Grafički prikazati funkciju $f(x,y) = \frac{x^3}{y}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?	15	10
4. Istražiti domenu i lokalne ekstreme funkcije $f(x,y) = x - y + \frac{1}{xy}$.	20	20
5. Riješiti diferencijalnu jednadžbu: $\sqrt[3]{x}\,y\,y' = 1 - x^2$	20	20
6. Pronaći partikularno rješenje koje zadovoljava sljedeće jednadžbe: $y'' + 4y = 0,\quad y(0) = 0,\ y'(0) = 2$	15	0

1.) $A(0,0)$
$B(1,2)$
$C(2,2)$



AB

$(y-y_1)\cdot(x_2-x_1)=(x-x_1)\cdot(y_2-y_1)$

$(y-0)\cdot(1-0)=(x-0)\cdot(2-0)$

$y=2x$

AC

$(y-0)\cdot(2-0)=(x-0)\cdot(2-0)$

$2y=2x$

$y=x$

BC

$(y-2)\cdot(2-1)=(x-1)\cdot(2-2)$

$y-2=0$

$y=2$

15

$P = P_1 + P_2 = \frac{1}{2} + \frac{1}{2} = \underline{\underline{1}}$

$P_1 = \int_0^1 (2x)-(x)\,dx =$

$P_1 = \int_0^1 x\,dx = \frac{1}{2}(x^2)\Big|_0^1 =$

$P_1 = \frac{1}{2}(1^2-0^2) = \frac{1}{2}$

$P_2 = \int_1^2 (2)-(x)\,dx = 2\int_1^2 dx - \int_1^2 x\,dx =$

$P_2 = 2\cdot(x)\Big|_1^2 - \frac{1}{2}(x^2)\Big|_1^2 =$

$P_2 = 2\cdot(2-1) - \frac{1}{2}(2^2-1^2) =$

$P_2 = 2 - \frac{3}{2} = \frac{1}{2}$

Ime i prezime: LOVRE LOVRIĆ **Broj indeksa:** 58080

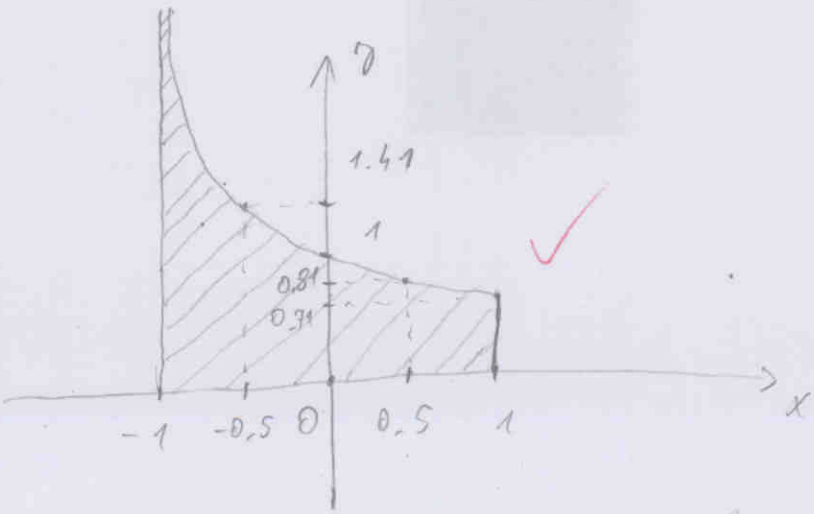
2.) $f(x) = \frac{1}{\sqrt{x+1}}$ $\int_{-1}^{1} f(x)\,dx$

$$P = \int_{-1}^{1} \frac{1}{\sqrt{x+1}}\,dx = \left\{ \begin{matrix} t = x+1 \\ dt = dx \end{matrix} \right\} = \int_{-1}^{1} \frac{1}{\sqrt{t}}\,dt = \int_{-1}^{1} t^{-\frac{1}{2}}\,dt =$$

$$P = 2\left(t^{\frac{1}{2}}\right)_{-1}^{1} = 2\cdot\left(\sqrt{x+1}\right)_{-1}^{1} = 2\cdot\left(\sqrt{1+1} - \sqrt{-1+1}\right) = \underline{\underline{2\cdot\sqrt{2}}}$$ ✓

x	-1	-0.5	0	0.5	1
$f(x) = \frac{1}{\sqrt{x+1}}$	$+\infty$	1.41	1	0.81	0.71

15



✓

IME I PREZIME: LOVRE LOVRIĆ BROJ INDEKSA: 58080

3.) $f(x,y) = \frac{x^3}{y}$

$D(f) = \{(x,y) : y \neq 0\}$

KODOMENA?

$D(f) = \{(\frac{x}{y}) : y \neq 0\}$ ✓

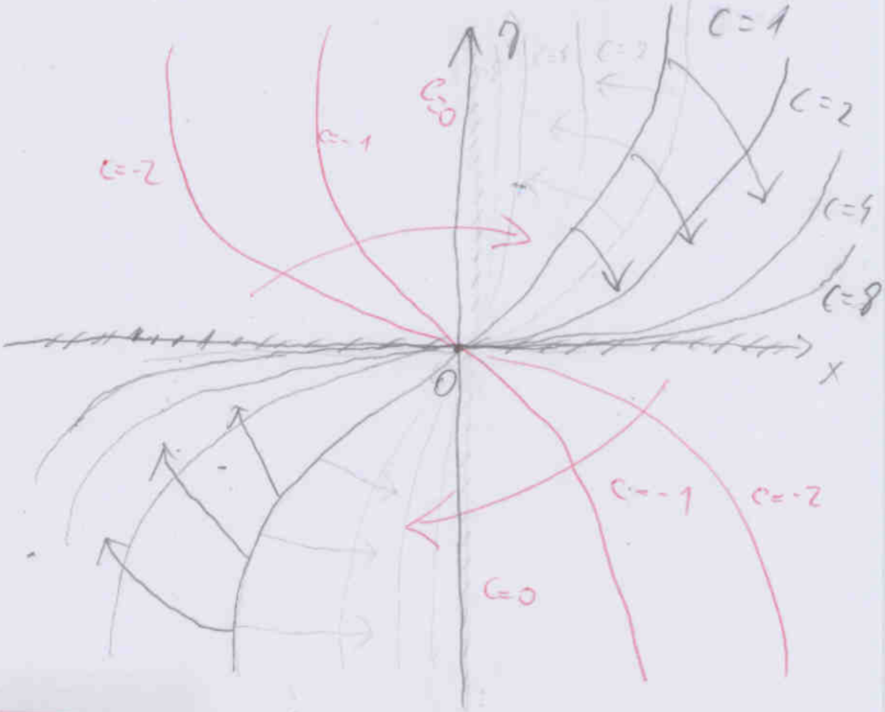
$\frac{x^3}{y} = C \Rightarrow y = \frac{x^3}{C}$

$\boxed{C=1}$ $y = x^3$

$\boxed{C=2}$ $y = \frac{x^3}{2}$

$\boxed{C=4}$ $y = \frac{x^3}{4}$

$\boxed{C=8}$ $y = \frac{x^3}{8}$



$\lim_{(x,y)\to(0,0)} \frac{x^3}{y} = 0$

LIMES $\lim_{(x,y)\to(0,0)} f(x,y)$ POSTOJI SAMO U NULTOČKI

ZATO JER SAMO KROZ NJU PROLAZI RAZINSKE KRIVULJE.

✗

LIMES NE POSTOJI JER SE U TOČKI (0,0) SIJEKU RAZLIČITE RAZINSKE KRIVULJE. S

10

IME I PREZIME: LOVRE LOVRIĆ BROJ INDEKSA: 58080

4.) $f(x,y) = x - y + \frac{1}{xy}$

$D(f) = \{(x,y): x \neq 0, y \neq 0\}$ ✓

$f_x(x,y) = 1 - \frac{1}{x^2 y} = 0 \Rightarrow -\frac{1}{x^2 y} = -1 \Rightarrow y = \frac{1}{x^2} \Rightarrow y = 1$

$f_y(x,y) = -1 - \frac{1}{xy^2} = 0 \Rightarrow -1 - \frac{1}{x \cdot \left(\frac{1}{x^2}\right)^2} = 0 \Rightarrow \frac{1}{x \cdot \frac{1}{x^4}} = -1 \Rightarrow x = -1$

$x^3 = -1$

$x = -1$

$f_{xx}(x,y) = \frac{2}{x^3 y}$

$f_{xy}(x,y) = \frac{1}{x^2 y^2}$

$f_{yy}(x,y) = \frac{2}{xy^3}$

$T_1(-1, 1)$

$\Delta = AC - B^2$

$\Delta = (-2) \cdot (-2) - (1)^2$

$\Delta = 4 - 1$

$\Delta = 3 > 0$

LOK. MAKSIMUM ✓

$A = \frac{2}{(-1)^3 \cdot 1} = -2 < 0$

$B = \frac{1}{(-1)^2 \cdot 1} = 1$

$C = \frac{2}{(-1) \cdot 1^3} = -2$

$f(-1, 1) = -1 - 1 + \frac{1}{(-1) \cdot 1} = -2 - 1 = \underline{\underline{-3}}$

20

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5.) $\sqrt[3]{x}\, y y' = 1 - x^2 \quad / : \sqrt[3]{x}\, y$

$$y' = \frac{1-x^2}{\sqrt[3]{x}\, y}$$

$$\frac{dy}{dx} = \frac{1-x^2}{\sqrt[3]{x}\, y}$$

$$y\,dy = \frac{1-x^2}{\sqrt[3]{x}}\,dx \quad / \int$$

$$\int y\,dy = \int \frac{1-x^2}{\sqrt[3]{x}}\,dx$$

$$\int y\,dy = \frac{y^2}{2}$$

$$\int \frac{1-x^2}{\sqrt[3]{x}}\,dx = \left\{ \begin{array}{l} x = t^3 \\ dx = 3t^2\,dt \end{array} \right\} =$$

$$3\int \frac{1-(t^3)^2}{t}\, t^2\,dt = 3\int (t - t^7)\,dt =$$

$$3\int t\,dt - 3\int t^7\,dt =$$

$$\frac{3}{2} t^2 - \frac{3}{8} t^8 = \frac{3}{2}\left(\sqrt[3]{x^2}\right) - \frac{3}{8}\left(\sqrt[3]{x^8}\right) + C$$

RIJEŠENJE DIF. JED.

$$\frac{y^2}{2} = \frac{3}{2}\left(\sqrt[3]{x^2}\right) - \frac{3}{8}\left(\sqrt[3]{x^8}\right) + C$$

$$y^2 = 3\left(\sqrt[3]{x^2}\right) - \frac{3}{4}\left(\sqrt[3]{x^8}\right) + C$$

20

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6.) $y'' + 4y = 0$ $\qquad y(0) = 0, \quad y'(0) = 2$

$\lambda^2 + 4 = 0$

$\lambda^2 = -4$

$\lambda_{1,2} = \pm 2i$ ✓

$y_H(x) = e^{ax}(C_1 \cos bx + C_2 \sin bx)$

$y_H(x) = e^{0 \cdot x}(C_1 \cos 2x + C_2 \sin 2x)$ ✓

$y_H(x) = C_1 \cos 2x + C_2 \sin 2x$ ✓

$y'(x) = -2C_1 \sin 2x + 2C_2 \cos 2x$

$y'(x) = C_1 \cdot \cos 2x - C_1 \sin 2x + C_2 \sin 2x + C_2 \cos 2x$ ✗

$2 = C_1 \cdot \cos 2 \cdot 0 - \cancel{C_1 \sin 2 \cdot 0} + \cancel{C_2 \sin 2 \cdot 0} + C_2 \cos 2 \cdot 0$

$2 = C_1 + C_2$

VIDI BATUR, MAGAŠ

$0 = C_1 \cdot \cos 2 \cdot 0 + \cancel{C_2 \sin 2 \cdot 0}$

$0 = C_1 \cdot \Rightarrow C_1 = 0$

$2 = 0 + C_2 \Rightarrow C_2 = 2$

0

PARTIKULARNO RJEŠENJE:

$y(x) = 0 \cdot \cos 2x + 2 \cdot \sin 2x$

$y(x) = 2 \sin 2x$

Popuniti odmah!

IME I PREZIME: HRVOJE BATUR **BROJ INDEKSA:** 17-2-0006-2010

DATUM: 30.06.2011 VRIJEME: OD 8:00 DO 9:03

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

~~80~~ 65

Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,0)$, $B(1,2)$ i $C(2,2)$. 15 15

2. Zadano je $f(x) = \frac{1}{\sqrt{x+1}}$. Odrediti $\int_{-1}^{1} f(x)\,dx$. Skicirati graf funkcije f i površinu koja je određena integralom $\int_{-1}^{1} f(x)\,dx$. 15 15

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4. Istražiti domenu i lokalne ekstreme funkcije $f(x,y) = x - y + \dfrac{1}{xy}$. 20 10

5. Riješiti diferencijalnu jednadžbu: $\sqrt[3]{x}\,y\,y' = 1 - x^2$ 20 20

6. Pronaći partikularno rješenje koje zadovoljava sljedeće jednadžbe: 15 ~~15~~ ~~0~~

$$y'' + 4y = 0, \quad y(0) = 0,\ y'(0) = 2$$

1. $A(0,0)$ $B(1,2)$ $C(2,2)$

$AB: (y-y_1)(x_2-x_1) = (x-x_1)(y_2-y_1)$

$(y-0)(1-0) = (x-0)(2-0)$

$\boxed{y=2x}$

$BC: (y-2)(2-1) = (x-1)(2-2)$

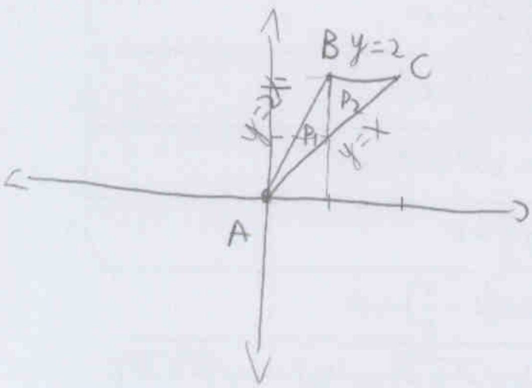
$(y-2) = 0$

$\boxed{y=2}$

$AC: (y-0)(2-0) = (x-0)(2-0)$

$2y = 2x$

$\boxed{y=x}$



$P_1 = \int_0^1 (2x - x)\,dx = \int_0^1 x\,dx$ ✓

$P_1 = \left[\frac{1}{2}x^2\right]_0^1 = \frac{1}{2}$

$P_2 = \int_1^2 (2-x)\,dx = \left[2x - \frac{x^2}{2}\right]_1^2$ ✓

$P_2 = (4-2) - \left(2 - \frac{1}{2}\right) = 2 - \frac{3}{2} = \frac{1}{2}$

$P_U = P_1 + P_2$

$P_U = \frac{1}{2} + \frac{1}{2}$

$P_U = 1$ ✓

15

Ime i prezime: HRVOJE BATUR **Broj indeksa:** 17-2-0006-2010

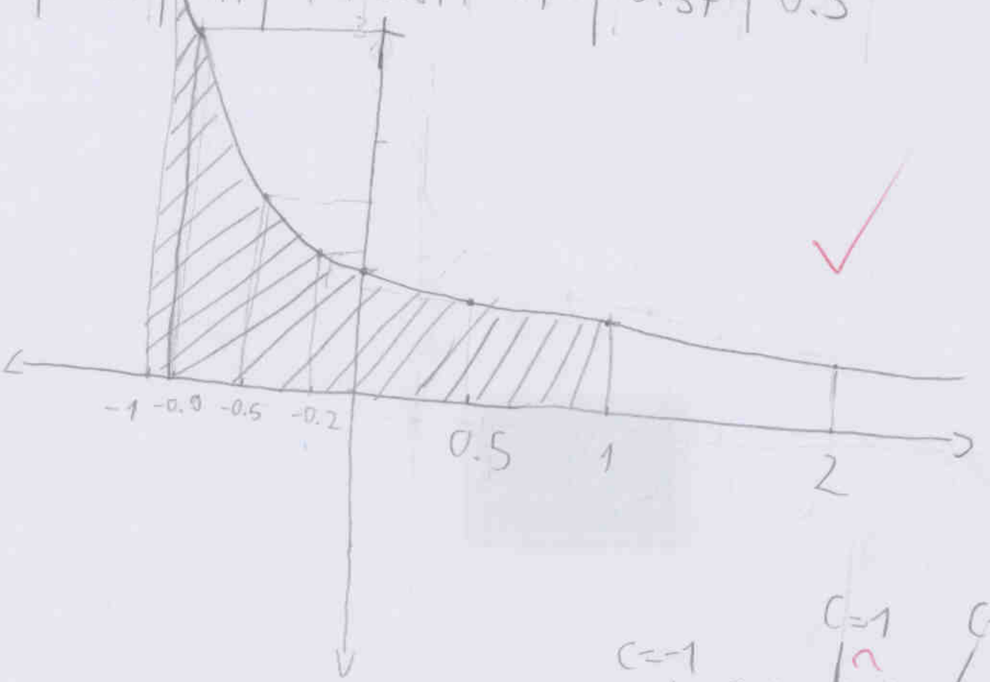
② $$\int_{-1}^{1} f(x) = \int \frac{1}{\sqrt{x+1}} = \int (x+1)^{-\frac{1}{2}}\, dx$$

$$= \left[\frac{(x+1)^{\frac{1}{2}}}{\frac{1}{2}}\right]_{-1}^{1} = \left[2\sqrt{x+1}\right]_{-1}^{1} = (2\sqrt{2}) - (2\sqrt{0})$$

$$= 2\sqrt{2}$$ ✓

$$= 2.828427$$

x	-0.9	-0.5	-0.2	0	0.5	1	2	3
$f(x)$	3.16	1.41	1.11	1	0.81	0.7	0.57	0.5



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✓

③ $$f(x,y) = \frac{x^3}{y}$$

$$c = \frac{x^3}{y}$$

$$x^3 = c \cdot y \qquad y = \frac{x^3}{c}$$

$$x = \sqrt[3]{c \cdot y}$$

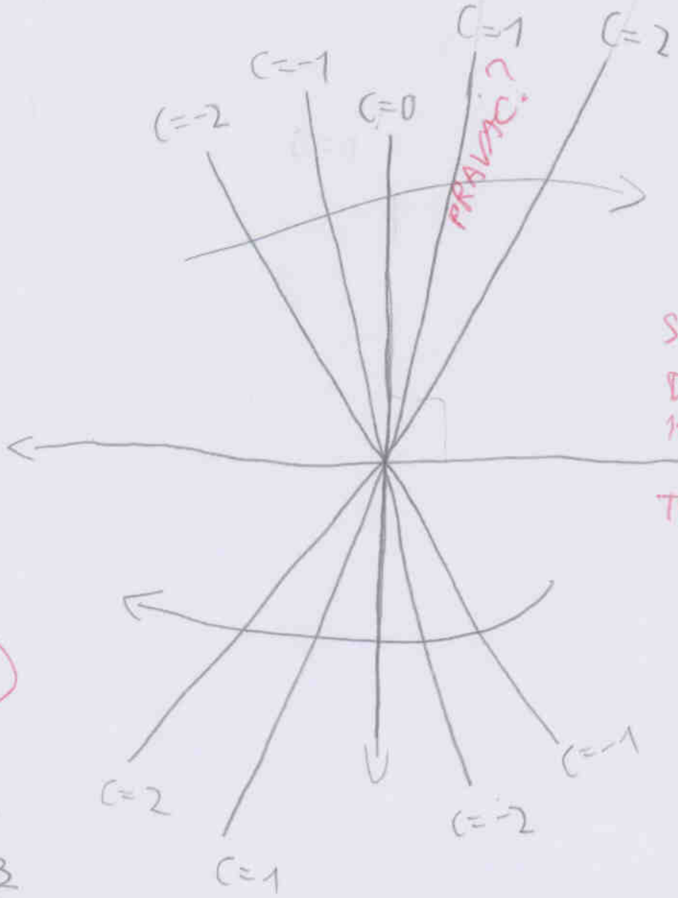
$c = 0$, $x = 0$

$c = 1$, $x = \sqrt[3]{y}$, $y = x^3$

$c = 2$, $x = \sqrt[3]{2y}$, $y = \frac{x^3}{2}$

$c = -1$, $x = -\sqrt[3]{y}$, $y = -x^3$

$c = -2$, $x = -\sqrt[3]{2y}$, $y = -\frac{x^3}{2}$



NACRTALI STE PRAVCE IAKO $y = x^3$ NIJE PRAVAC?

SMJER RASTA ✓

DOMENA?

KODOMENA?

TRAŽENI LIMES?

5

IME I PREZIME: HRVOJE BATUR **BROJ INDEKSA:** 17-2-0006-2010

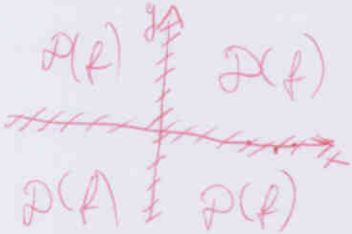
④ $f(x,y) = x - y + \frac{1}{xy}$

$D(f) = \mathbb{R} / \{0\}$

$y \neq 0$

$x \neq 0$

$\mathcal{D}(f) = \mathbb{R}\times\mathbb{R} \setminus \{x=0, y=0\}$



$\partial x = 1 + \frac{1}{y}\cdot\left(-\frac{1}{x^{-2}}\right) = 1 - \frac{1}{yx^2}$ ✓

$\partial y = -1 + \frac{1}{x}\cdot\left(-\frac{1}{y^2}\right) = -1 - \frac{1}{xy^2}$ ✓

$1 - \frac{1}{yx^2} = 0$

$\frac{1}{yx^2} = 1 \quad / y$

$\frac{1}{x^2} = y \quad / \sqrt{\ }$

$\frac{1}{x} = \sqrt{y}$

?

?

$\frac{1}{x} = \sqrt{1}$

$\boxed{x = +1}$ ×

$\boxed{x = -1}$

$T_1(-1,1)$

$T_2(1,1)$ ×

$-1 - \frac{1}{xy^2} = 0$

$\frac{1}{xy^2} = -1$

$\frac{1}{\sqrt{y}\cdot y^2} = -1$

$\frac{1}{\sqrt{y^5}} = -1$

$y^{-\frac{5}{2}} = -1 \quad /^2$

$y^{-5} = 1$

$\boxed{y = 1}$

$\partial xx f = -\frac{1}{y}\cdot(-2)\cdot x^{-3}$ (A)

$= \frac{2}{yx^3}$ ✓

$\partial xy f = -\frac{1}{x^2}\cdot(-1)\, y^{-2}$ (B)

$= \frac{1}{x^2y^2}$ ✓

$\partial yy f = -\frac{1}{x}\cdot(-2)\cdot y^{-3}$ (C)

$= \frac{2}{xy^3}$ ✓

	$T_1(-1,1)$	$T_2(1,1)$
A	-2	2
B	1	1
C	-2	2

$\Delta = 3$ ✓ | $\Delta = 3$

$T_1(-1,1)$ maksimum | $T_2(1,1)$ minimum

~~NIJE KRITIČNA TOČKA~~

~~$\partial_x f$~~

NIJE KRITIČNA TOČKA

$\partial_x f(1,1) = 0$

$\partial_y f(1,1) = -2 \neq 0$

10

IME I PREZIME: HRVOJE BATUR BROJ INDEKSA: 17-2-0006-2010

(5) $\sqrt[3]{x}\, y y' = 1-x^2 \quad / \cdot \frac{1}{\sqrt[3]{x}\, y}$

$y' = \frac{1-x^2}{\sqrt[3]{x}\, y}$

$\frac{dy}{dx} = \frac{1}{y} \cdot \frac{1-x^2}{\sqrt[3]{x}} \quad / \cdot y \cdot dx$

$y\, dy = \frac{1-x^2}{\sqrt[3]{x}}\, dx \quad / \int$

$\int y\, dy = \int \frac{1-x^2}{\sqrt[3]{x}}\, dx$

$\frac{y^2}{2} = \frac{3}{2}\sqrt[3]{x^2} - \frac{3}{8}\sqrt[3]{x^8} + C \quad / \cdot 2$ ✓

$y^2 = 3\sqrt[3]{x^2} - \frac{3}{4}\sqrt[3]{x^8} + C \quad / \sqrt{\ }$

$y = -\sqrt{3\sqrt[3]{x^2} - \frac{3}{4}\sqrt[3]{x^8} + C}$ ✓

$y = \sqrt{3\sqrt[3]{x^2} - \frac{3}{4}\sqrt[3]{x^8} + C}$ ✓

$\int \frac{1-x^2}{\sqrt[3]{x}}\, dx$

$= \int (1-x^2) \cdot x^{-\frac{1}{3}}\, dx$

$= \int (x^{-\frac{1}{3}} - x^{\frac{5}{3}})\, dx$

$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{x^{\frac{8}{3}}}{\frac{8}{3}} + C$

$= \frac{3}{2}\sqrt[3]{x^2} - \frac{3}{8}\sqrt[3]{x^8} + C$ ✓

20

Ime i prezime: HRVOJE BATUR **Broj indeksa:** 17-2-0006-2010

6. $y''+4y=0 \qquad y(0)=0,\ y'(0)=2$

$r^2+4=0$

$r^2=-4$

$r=\pm i$

$\pm 2i$

$y(x)=C_1\sin x + C_2\cos x$

$y(0)=C_1\sin(0)+C_2\cos(0)$

$0=C_1\cdot 0+C_2\cdot 1$

$0=C_2$

$\boxed{C_2=0}$

$y'(x)=C_1\cos(x)-C_2\sin(x)$

$2=C_1\cos(0)-C_2\sin(0)$

$2=C_1\cdot 1-C_2\cdot 0$

$2=C_1$

$\boxed{C_1=2}$

$y(x)=2\sin x$

~~10~~

15

~~ZADATAK VRIJEDI 15 BODOVA~~

~~⊖~~

Popuniti odmah!

IME I PREZIME: MARIN MAGAŠ **BROJ INDEKSA:** 17-2-0061-2010 58

DATUM: VRIJEME: OD 8:00 DO 9:20 .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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5. Riješiti diferencijalnu jednadžbu: $\sqrt[3]{x}\,y\,y' = 1 - x^2$ 20 10
6. Pronaći partikularno rješenje koje zadovoljava sljedeće jednadžbe: 15 0

$$y'' + 4y = 0, \quad y(0) = 0, \; y'(0) = 2$$

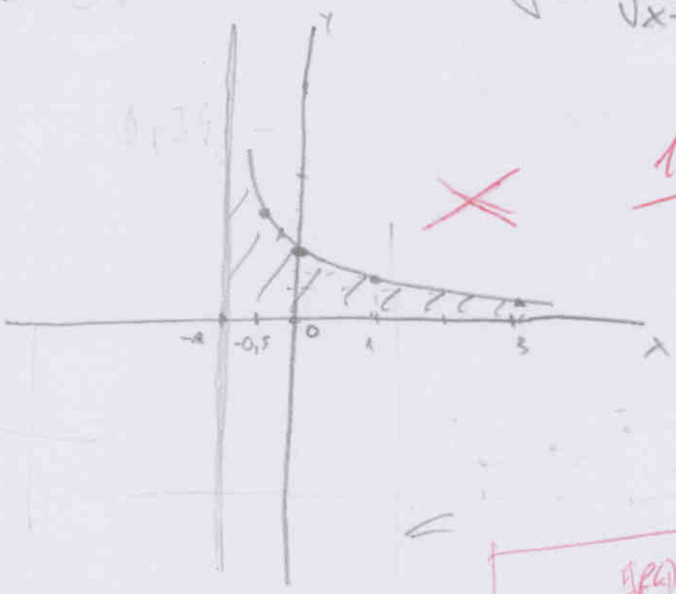
Ime i prezime: MARIN MAGAJ

Broj indeksa: 17-2-0061-2010

2. $\int \frac{dx}{\sqrt{x+1}} = \left(2\sqrt{x+1}\right)\Big|_{-1}^{1} + C = (2\sqrt{2}) - (0) = 2,82$ ✓

$f(x) = \frac{1}{\sqrt{x+1}}$

x	0	-1	-0,5	1	3
y	1	∞	1,41	0,7	0,5



10

5. $\sqrt[3]{x}\, y y' = 1 - x^2$

$y y' = \frac{1 - x^2}{\sqrt[3]{x}}$

$y \frac{dy}{dx} = \frac{1 - x^2}{\sqrt[3]{x}}$

$\int y\,dy = \int \frac{1 - x^2}{\sqrt[3]{x}}\,dx$ ✓

$\frac{y^2}{2} = \frac{x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C$ ✗

$\frac{y^2}{2} = \frac{3x^{\frac{2}{3}}}{2} - 3x^{\frac{1}{3}} + C \;/\cdot 2$ ✗

$y^2 = 3x^{\frac{2}{3}} - 6x^{\frac{1}{3}} + C$

$y = \pm\sqrt{3x^{\frac{2}{3}} - 6x^{\frac{1}{3}} + C_1}$

10

IME I PREZIME: MARIN MAGAŠ   BROJ INDEKSA: 17-2-0061-2010

6. $y'' + 4y = 0 \qquad y(0) = 0 \qquad y'(0) = 2$

$\lambda^2 + 4 = 0$

$\lambda^2 = -4$

$\lambda = \pm 2i$ ✓

$\alpha \pm \beta i$

$y = e^{0x}(C_1 \cos 2x + C_2 \sin 2x)$ ✓

$y = C_1 \cos 2x + C_2 \sin 2x)$ ✓

$y' = -2C_1 \sin 2x + 2C_2 \cos 2x$ ✓

$2 = -2C_1 + 2C_2$ ✗

$0 = C_1 + C_2 \Rightarrow C_1 = -C_2$

$2 = 4C_2 \qquad C_1 = -2$

$C_2 = 2$

$y = -2\cos 2x + 2\sin 2x$

$y = C_1 \cos 2x + C_2 \sin 2x$

$y(0) = C_1 \underbrace{\cos 0}_{=1} + C_2 \underbrace{\sin 0}_{=0}$

$y(0) = C_1 \Rightarrow C_1 = 0$

$y' = -2C_1 \sin 2x + 2C_2 \cos 2x$

$y'(0) = -2C_1 \underbrace{\sin 0}_{=0} + 2C_2 \underbrace{\cos 0}_{=1}$

$= +2C_2 \Rightarrow C_2 = 1$

$y(x) = \cancel{\cos 2x}$ $\sin(2x)$

---

3. $f(x,y) = \frac{x^3}{y}$

$\frac{x^3}{y} = C$

$x^3 = Cy$

$y = \frac{x^3}{C}$

$C = 1 \quad y = x^3$

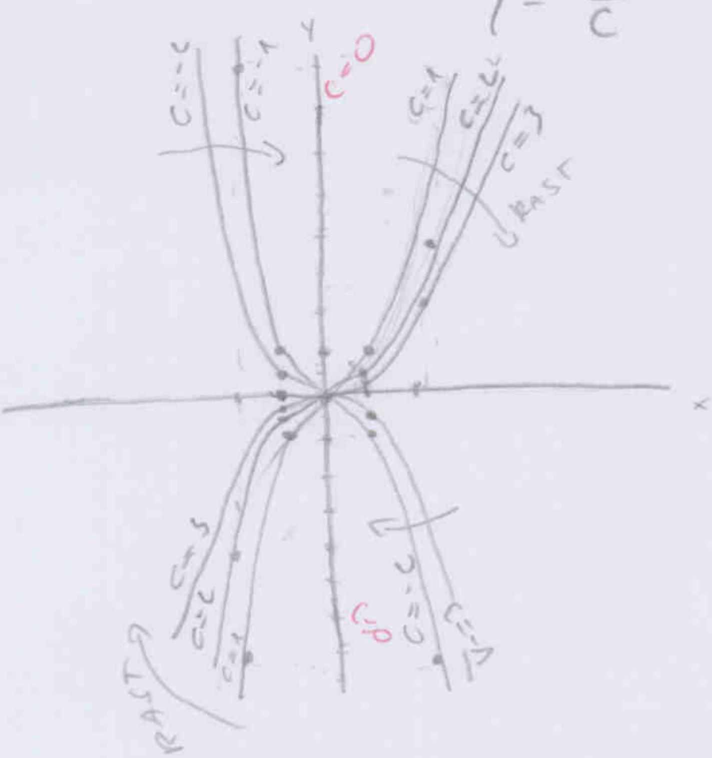
$C = 2 \quad y = \frac{x^3}{2}$

$C = 3 \quad y = \frac{x^3}{3}$

$C = -1 \quad y = -x^3$

$C = -2 \quad y = -\frac{x^3}{2}$

$C = -3 \quad y = -\frac{x^3}{3}$



DOMENA?

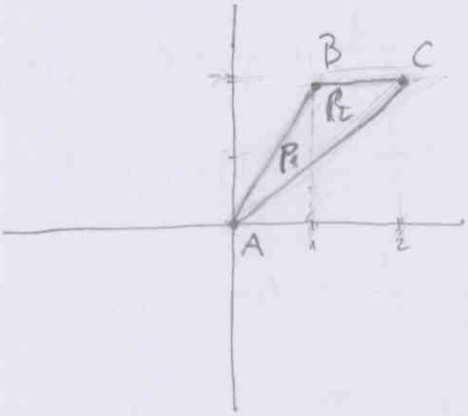
KODOMENA?

TRAŽEM LIMES?

8

IME I PREZIME: MARIN MAGAŠ   BROJ INDEKSA: 17-2-0061-2010

1. $A(0,0)$ $B(1,2)$ $C(2,2)$



AB: $(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$

$(y-0)(1-0)=(x-0)(2-0)$

$y=2x$

AC: $(y-0)(2-0)=(x-0)(2-0)$

$2y=2x$

$y=x$

BC: $(y-2)(2-1)=(x-1)(2-2)$

$(y-2)=0$

$y=2$

$P_2=\int_1^2(2-x)dx=$

$=\int_1^2 2\,dx-\int_1^2 x\,dx=\left(2x-\frac{x^2}{2}\right)_1^2$ $\left(4-\frac{4}{2}\right)-\left(2-\frac{1}{2}\right)$ $2-1{,}5=0{,}5$

$P_2=0{,}5$

$P_1=\int_0^1(2x-x)dx=\int_0^1 x\,dx=\left(\frac{x^2}{2}\right)_0^1=\frac{1}{2}$ $P_u=0{,}5+0{,}5=1$

15

Popuniti odmah!

Ime i prezime: Ante Btica **Broj indeksa:** 17-1-0019-2010 55

Datum: 30. 6. 2011. Vrijeme: od 8:00 do 8:45 .

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,0)$, $B(1,2)$ i $C(2,2)$. 15 15

2. Zadano je $f(x) = \frac{1}{\sqrt{x+1}}$. Odrediti $\int_{-1}^{1} f(x)\,\mathrm{d}x$. Skicirati graf funkcije f i površinu koja je određena integralom $\int_{-1}^{1} f(x)\,\mathrm{d}x$. 15 15

3. Grafički prikazati funkciju $f(x,y) = \frac{x^3}{y}$ pomoću razinskih krivulja. Koja je domena i kodomena ove funkcije? Strelicama označiti smjer rasta funkcije. Da li i zašto postoji limes $\lim\limits_{(x,y)\to(0,0)} f(x,y)$? 15

4. Istražiti domenu i lokalne ekstreme funkcije $f(x,y) = x - y + \frac{1}{xy}$. 20 5

5. Riješiti diferencijalnu jednadžbu: $\sqrt[3]{x}\,y\,y' = 1 - x^2$ 20 20

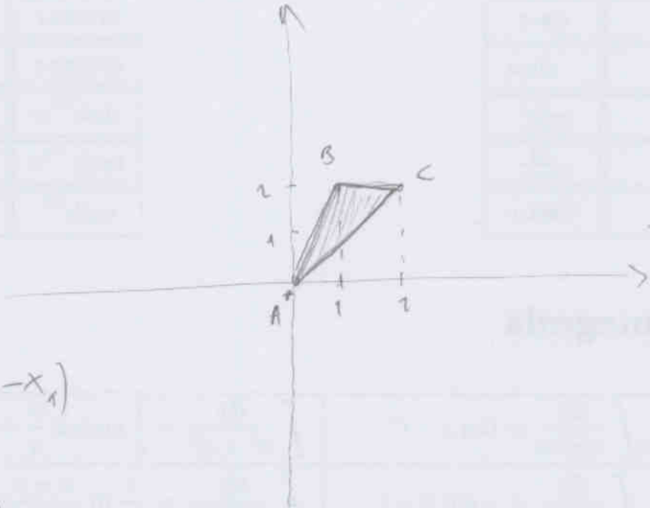
6. Pronaći partikularno rješenje koje zadovoljava sljedeće jednadžbe: 15 Ø

$$y'' + 4y = 0, \quad y(0) = 0,\ y'(0) = 2$$

(1.) $A(0,0)$

$B(1,2)$

$C(2,2)$



$AB: \; y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{2}{1}(x - 0)$

$p_1 \ldots\; y = 2x$

$AC: \; p_2 \ldots\; y = x$

$BC: \; p_3 \ldots\; y = 2$

$$P = \int_0^1 (2x - x)\,dx + \int_1^2 (2 - x)\,dx$$

$$P = \int_0^1 x\,dx + \int_1^2 2\,dx - \int_1^2 x\,dx$$

$$P = \frac{x^2}{2}\Big|_0^1 + 2x\Big|_1^2 - \frac{x^2}{2}\Big|_1^2$$

$4 - 2 = 2$ $\quad \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

$$P = \left(\frac{1^2}{2} - \frac{0^2}{2}\right) + (2\cdot 2 - 2\cdot 1) - \left(\frac{2^2}{2} - \frac{1^2}{2}\right)$$

$$P = \frac{1}{2} + 2 - \frac{3}{2} = \frac{1 + 4 - 3}{2} = \frac{2}{2} = 1$$

15

$P = 1$ površina trokuta omeđenog točkama A, B i C.

Ime i prezime: Ante Botić **Broj indeksa:** 17-1-0019-2010

(2) $f(x) = \frac{1}{\sqrt{x+1}}$

$$\int_{-1}^{1} \frac{1}{\sqrt{x+1}}\,dx = \int_0^2 \frac{dt}{\sqrt{t}} = \int_0^2 t^{-\frac{1}{2}}\,dt = \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\Big|_0^2 = 2\sqrt{t}\Big|_0^2 = 2\sqrt{2} - 2\sqrt{0} = 2\sqrt{2}$$ ✓

$t = x+1$

$dx = dt$

$x_1 = -1,\ t_1 = 0$

$x_2 = 1,\ t_2 = 2$

graf: $\mathcal{D}: x+1 > 0$

$x > -1 \quad x \in \langle -1, +\infty \rangle$

$x = 0 \quad y = 0$

$y = 1$

$x = -0{,}5$

15

(5.) $3\sqrt{x}\, y\, y' = 1 - x^2$

$x^{\frac{1}{3}} y \frac{dy}{dx} = 1 - x^2 \ / dx$

$x^{1/3} y\, dy = (1 - x^2)dx \ /: x^{1/3}$

$y\,dy = \frac{(1-x^2)dx}{x^{1/3}} \ / \int$

$\int y\,dy = \int \frac{(1-x^2)dx}{x^{1/3}}$

$(1-x^2) \cdot x^{-\frac{1}{3}}$

$x^{-\frac{1}{3}} - x^{+\frac{5}{3}}$

$\frac{y^2}{2} = \int x^{-\frac{1}{3}}dx - \int x^{\frac{5}{3}}dx$ ✓

$\frac{y^2}{2} = \frac{x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{x^{\frac{8}{3}}}{\frac{8}{3}} + C$ ✓

$\frac{y^2}{2} = \frac{3x^{\frac{2}{3}}}{2} - \frac{3x^{\frac{8}{3}}}{8} + C \ / 8$

$4y^2 = 12\sqrt[3]{x^2} - 3\sqrt[3]{x^8} + C_1$

$y = \sqrt{\frac{12\sqrt[3]{x^2} - 3\sqrt[3]{x^8}}{4} + C_1}$

20

Ime i prezime: Ante Batica **Broj indeksa:** 17-1-2019-2010

4.) $D_f(x,y) = x - y + \frac{1}{xy}$

5

$xy \neq 0$ $D: \mathbb{R} \setminus \{x=0, y=0\}$

$x \neq 0$

$y \neq 0$

Ekstremi:

$\partial_x = -1 + \frac{1}{x} \Rightarrow \frac{1}{x} = 1, x=1$

$\partial_y = 1 + \frac{1}{y} \Rightarrow \frac{1}{y} = -1, y=-1$

$T(-1,1)$

$A = \partial_{xx} =$

$C = \partial_{yy}$

$B = \partial_{xy}$

6.) $y'' + 4y = 0$ $y(0)=0$

$\lambda^2 + 4 = 0$ $y'(0)=2$?

$\lambda = \pm 2i$

$\cancel{0}$

8:05 – 9:50

Popuniti odmah!

Ime i prezime: GREGOR HAMARIĆ **Broj indeksa:** S7650

Datum: 30.06.2011. Vrijeme: od 8:05 do 9:50

30

MATEMATIKA 2: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,0)$, $B(1,2)$ i $C(2,2)$. 15 15
2. Zadano je $f(x) = \frac{1}{\sqrt{x+1}}$. Odrediti $\int_{-1}^{1} f(x)\,\mathrm{d}x$. Skicirati graf funkcije f i površinu koja je određena integralom $\int_{-1}^{1} f(x)\,\mathrm{d}x$. 15 15
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4. Istražiti domenu i lokalne ekstreme funkcije $f(x,y) = x - y + \dfrac{1}{xy}$. 20 0
5. Riješiti diferencijalnu jednadžbu: $\sqrt[3]{x}\,y\,y' = 1 - x^2$ 20 0
6. Pronaći partikularno rješenje koje zadovoljava sljedeće jednadžbe:

$$y'' + 4y = 0, \quad y(0) = 0,\ y'(0) = 2$$

15 0

IME I PREZIME: GREGOR HAMARIĆ BROJ INDEKSA: 57650

2. $f(x) = \frac{1}{\sqrt{x+1}}$

x	-1	-0,5	0	0,5	1
y	∅	1,41	1	0,82	0,71

$$\int_{-1}^{1} \frac{1}{\sqrt{x+1}}\,dx = \begin{bmatrix} x+1=t \\ dx=dt \end{bmatrix} = \int_{-1}^{1} \frac{1}{\sqrt{t}}\,dt =$$

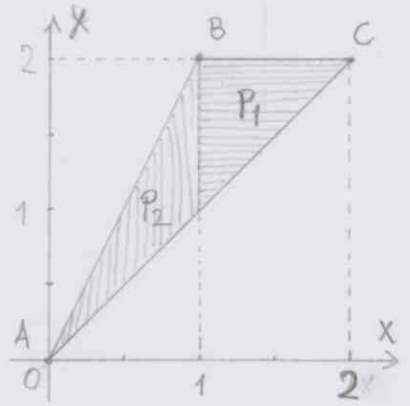
$$= \int_{-1}^{1} \frac{1}{t^{\frac{1}{2}}}\,dt = \int_{-1}^{1} t^{-\frac{1}{2}}\,dt = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C \Big|_{-1}^{1} =$$

$$= \frac{\sqrt{t}}{\frac{1}{2}} \Big|_{-1}^{1} = 2\sqrt{t} \Big|_{-1}^{1} = 2\sqrt{x+1} \Big|_{-1}^{1} =$$

$$= 2\sqrt{1+1} - 2\sqrt{-1+1} = \underline{\underline{2\sqrt{2}}}$$ ✓

15

1. $A=(0,0)$, $B=(1,2)$, $C=(2,2)$



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$AB \Rightarrow y - \cancel{0} = \frac{2-\cancel{0}}{\cancel{1}-\cancel{0}}(x - \cancel{0})$$

$$\underline{y = 2x}$$

$$BC \Rightarrow y - 2 = \frac{2-2}{2-1}(x-1)$$

$$y - 2 = 0$$

$$\underline{y = 2}$$

$$AC \Rightarrow y - \cancel{0} = \frac{2-\cancel{0}}{2-\cancel{0}}(x - \cancel{0})$$

$$\underline{y = x}$$

$$P_1 = \int_1^2 (2 - x)\,dx = \int_1^2 2\,dx - \int_1^2 x\,dx = 2x \Big|_1^2 - \frac{x^2}{2}\Big|_1^2 = 2x - \frac{x^2}{2}\Big|_1^2 = \left(4 - \frac{4}{2}\right) - \left(2 - \frac{1}{2}\right) =$$

$$= 2 - \frac{3}{2} = \underline{\underline{\frac{1}{2}}}$$

$$P_2 = \int_0^1 2x - x = \int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$

$$P = P_1 + P_2$$

$$\underline{\underline{P = 1}}$$ ✓ 15

IME I PREZIME: GREGOR HAMARIĆ BROJ INDEKSA: 57650

$\Delta = A \cdot C - B$

4. $f(x,y) = x - y + \frac{1}{xy}$

$\left(\frac{1}{x}\right)' = \frac{\cancel{1' \cdot x} - 1 \cdot x'}{x^2} = \frac{-1}{x^2}$

$f(x,y) = x - y + \frac{1}{x} \cdot \frac{1}{y}$

$\left(-\frac{1}{x^2}\right)' = \frac{0 \cdot x + 1 \cdot 2x}{x^4} = \frac{2x}{x^4}$

$f'_{(x)} = 1 - \frac{1}{x^2} \cdot \frac{1}{y} \rightarrow f'(xx) = \boxed{\frac{2x}{x^4} \cdot \frac{1}{y}}$ A $\quad f'(xy) = \boxed{\frac{1}{x^2} \cdot \frac{1}{y^2}}$ C

$f'(y) = -1 - \frac{1}{x} \cdot \frac{1}{y^2} \rightarrow f'(yy) = \boxed{\frac{1}{x} \cdot \frac{2y}{y^4}}$ B

VIDI BATUR, MAGAŠ
BOTICA

Ø

Ime i prezime: GREGOR HAMARIĆ **Broj indeksa:**

5. $\sqrt[3]{x} \cdot y \cdot y' = 1 - x^2$ $\emptyset$

6. $y'' + 4y = 0 \rightarrow y'' = -4y$

$y(0) = 0 \qquad y''(0) = -4 \cdot 0 = 0$ $\emptyset$

$y'(0) = 2$