Popuniti odmah!

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DATUM: 17.5. VRIJEME: OD DO

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

xoxo

Broj ↓ bodova

35

20

1. Odrediti determinantu matrice $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

2. Odrediti domenu i sve asimptote funkcije $f(x) = x + \sqrt{x^2 + x + 1}$

0

3. Ispitati konvergenciju reda $\sum \left(\frac{n^2 + 4n + 4}{2n + 2n^2} \right)^{2n}$

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4. Ispitati domenu, periodičnost, parnost i prvu derivaciju funkcije $g(x) = \ln(\sin(3x))$.

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2 + 2}{x^2 + 1}$.

$$\begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix} \quad 1\cdot(n)$$

$$\begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} -1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{vmatrix} \quad 1\cdot(n) =$$

$$1 \cdot \begin{bmatrix} -1 & 2 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} = -1 \cdot \begin{vmatrix} 3 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} =$$

$$= -1 \left[\cdot \begin{vmatrix} 3 & 2 & 0 & 3 & 2 \\ 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{vmatrix} \right]$$

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$\det(A) = 5$

$$= -1 \cdot \begin{bmatrix} 3 - 6 - 2 \end{bmatrix} = -1 \cdot \begin{bmatrix} -5 \end{bmatrix} = 5 \quad \checkmark$$

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5. $h(x)=\frac{x^2+2}{x^2+1}$

$x^2+1\geq 0$

$x=\pm\sqrt{-1}$

$x=\pm i\sqrt{1}$

1. Domena $D(f)=\mathbb{R}$
2. Periodičnost nije periodična
3. Parnost ; Parna je

$$h'(x)=\frac{2x\cdot(x^2+1)-(x^2+2)\cdot(2x)}{(x^2+1)^2}$$

$$h'(x)=\frac{2x^3+2x-(2x^3+4x)}{(x^2+1)^2}$$

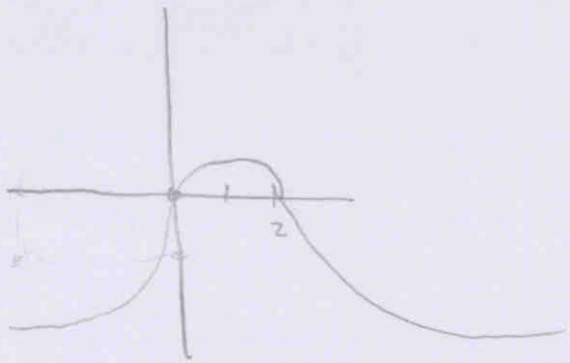
$$h'(x)=\frac{-2x}{(x^2+1)^2}$$

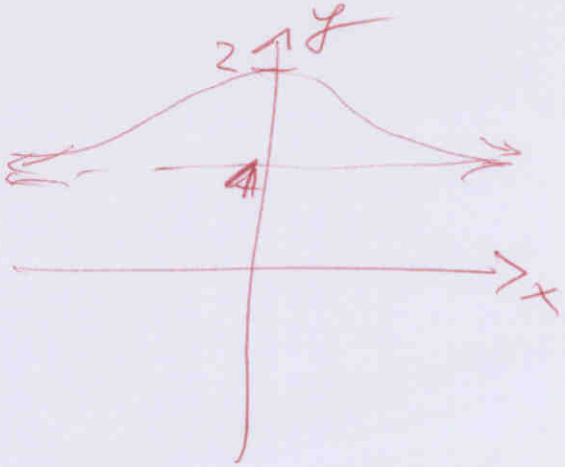
	$-\infty$		-2		0		1		$+\infty$
$h(x)$		+		+		−		−	

$-2x=0 \Rightarrow x=0$

$x=2$ Nultočka ✗

H. A. $\lim_{x\to\infty}\frac{x^2+2}{x^2+1}=1$





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2. $f(x) = x + \sqrt{x^2+x+1} \quad / \cdot \frac{-\sqrt{x^2+x+1}}{-\sqrt{x^2+x+1}}$

$$f(x) = -\frac{x\sqrt{x^2+x+1} - x^2+x+1}{-\sqrt{x^2+x+1}}$$

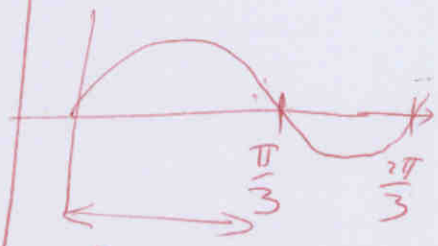
$$f(x) = \frac{-x\sqrt{x^2+x+1}}{-\sqrt{x^2+x+1}} - \frac{x^2+x+1}{-\sqrt{x^2+x+1}}$$

4. $g(x) = \ln(\sin(3x))$

1. $D(f) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$D(\ln) = \langle 0, +\infty \rangle$

$\sin(3x) > 0$



$D(f) = \langle 0, \frac{\pi}{3} \rangle$

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2. Parnost $g(x) = \ln(\sin(-3x))$

$g(x) = \ln(\sin(-3x))$

$g(-x) = \frac{1}{\sin(-3x)}$ ✗ Niti parna ni neparna ✓

$g(x) = \ln(\sin(3x))$

$g'(x) = \frac{1}{\sin(3x)} \cdot (\sin 3x)'$

$g'(x) = \frac{1}{\sin 3x} \cdot \cos 3x \cdot 3$ ✓

$g'(x) = \frac{3\cos 3x}{\sin 3x}$ ✓

$\sin x$ ima $P = \frac{2\pi}{3}$

$\sin x = \sin(x + 2\pi)$

$\sin 3x = \sin(3x + 2\pi)$

$f(x) = f(x+P)$

$\sin 3x = \sin(3(x+P))$

$= \sin(3x+3P)$

$3x + 2\pi = 3x + 3P$

$P = \frac{2\pi}{3}$

Periodičnost funkcija je periodična jer je trigonometrijska funkcija.

PERIOD ? $P =$

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$$2 \quad f(x) = x + \sqrt{x^2+x+1} \cdot \frac{-\sqrt{x^2+x+1}}{-\sqrt{x^2-x+1}}$$

$$f(x) = \frac{-x\sqrt{x^2+x+1} - (x^2+x+1)}{-\sqrt{x^2-x+1}}$$

$$f(x) = \frac{-x - x^2 - x - 1}{-1}$$

$$f(x) = \frac{-x^2-2x-1}{-1}$$

$$f(x) = x^2+2x+1$$

$$D(f) = \mathbb{R} \setminus \{-1\}$$

Vertikalna

a) $f(x) = \lim_{x \to -1} x^2+2x+1$

$$\lim_{x \to -1} 1-2+1 = \lim 0 = 0 \quad \text{V.A.}$$

Horizontalna

b) $f(x) = \lim_{x \to \infty} x^2+2x+1$

$$\lim_{x \to \infty} (x^2+2x+1)^2$$

$$\lim_{x \to \infty} 2x+2 = \infty \quad \text{D.H.A.}$$

$$f(x) = \lim_{x \to -\infty} x^2+2x+1 = [x \to -x] = \lim_{x \to +\infty} (-x^2-2x+1) = \lim_{x \to \infty} (-2x-2) = -\infty \quad \text{L.H.A.}$$

$$-2+3 = 4$$

$$\frac{2+3}{2} = \frac{-2}{2} + \frac{3}{2} = \frac{1}{2}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4-4}}{2}$$

$$x_{1,2} = \frac{-2 \pm \sqrt{0}}{2}$$

$$x_{1,2} = \frac{-2}{2}$$

$$x = -1$$

$x^2+x+1 \geq 0$

$x^2+x+1 = 0 \quad x_{1,2} = \dots$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$-2+4 = 2$$

$$= x(1-2)$$