17. 3. 2011.

Popuniti odmah!

IME I PREZIME: JURE SVILIČIĆ BROJ INDEKSA:

DATUM: VRIJEME: OD DO .

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

oxxx

Broj ↓ bodova

1. Ovisno od parametra λ odrediti rang matrice $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & \lambda \end{pmatrix}$ i riješiti matrični sustav 10

$$\mathbf{A}\,\mathbf{X} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

2. Odrediti modul (r) i argument (φ) kompleksnog broja $z = \dfrac{\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^4}{(-1+i)^6}$. ~~0~~

3. Istražiti konvergenciju reda: $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt{n^2-n}-n\right)$ 20

4. Odrediti drugu derivaciju funkcije: $f(x) = \mathrm{e}^{-x^2}$ 10

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $g(x) = \sqrt{x^2-x} - x$. ~~0~~

40

Ime i prezime: JURE SVILIČIĆ **Broj indeksa:**

1

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ -1 & 0 & \lambda & 1 \end{array}\right) \begin{array}{l} \\ R2-2R1 \\ R3+R1 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & \lambda+1 & 2 \end{array}\right] \cdot(-1) \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & \lambda+1 & 2 \end{array}\right] \begin{array}{l} R1-R2 \\ \\ R3-R2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & \lambda-1 & 1 \end{array}\right] \cdot \frac{1}{\lambda-1}$$

1^0 RANG MATRICE JE 2

$R(A) = 2$

SLUČAJEVI:

$1^0 \ \lambda - 1 = 0$

$2^0 \ \lambda - 1 \neq 0$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \frac{1}{\lambda-1} \end{array}\right] \begin{array}{l} R1+R3 \\ R2-2R3 \\ \\ \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{\lambda-1} \\ 0 & 1 & 0 & \frac{\lambda-3}{\lambda-1} \\ 0 & 0 & 1 & \frac{1}{\lambda-1} \end{array}\right]$$

2^0

PROVJERA:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & \lambda \end{pmatrix} \times \begin{pmatrix} \frac{1}{\lambda-1} \\ \frac{\lambda-3}{\lambda-1} \\ \frac{1}{\lambda-1} \end{pmatrix} = \begin{pmatrix} \frac{1}{\lambda-1} + \frac{\lambda-3}{\lambda-1} + \frac{1}{\lambda-1} \\ \frac{2}{\lambda-1} + \frac{\lambda-3}{\lambda-1} + 0 \\ -\frac{1}{\lambda-1} + 0 + \frac{\lambda}{\lambda-1} \end{pmatrix} = \begin{array}{l} \frac{\lambda-3+2}{\lambda-1} = \frac{\lambda-1}{\lambda-1} = \underline{1} \\ \frac{\lambda-1}{\lambda-1} = \underline{1} \\ \frac{-1+\lambda}{\lambda-1} = \underline{1} \end{array}$$

$$= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

10

Ime i prezime: JURE SULIĆIĆ **Broj indeksa:**

3. $$\sum_{n=1}^{\infty}\left(\sqrt{n^2-n}-n\right)=\infty-\infty$$

$$=\frac{\left(\sqrt{n^2-n}-n\right)\left(\sqrt{n^2-n}+n\right)}{\left(\sqrt{n^2-n}+n\right)}=\frac{\cancel{n^2}-n+\cancel{n\sqrt{n^2-n}}-\cancel{n\sqrt{n^2-n}}-\cancel{n^2}}{\sqrt{n^2-n}+n}=$$

$$=\frac{-n}{\sqrt{n^2-n}+n}=\lim_{x\to\infty}\frac{-\frac{n}{n}}{\sqrt{\frac{n^2}{n}-\frac{n}{n}}+\frac{n}{n}}=\frac{-1}{1+1}=-\frac{1}{2}\ ✓$$

$$-\frac{1}{2}\neq 0 \Rightarrow \text{DIVERGIRA}\ ✓$$

20

RED $\sum_{n=1}^{\infty}\left(\sqrt{n^2-n}-n\right)$ DIVERGIRA ✓

4. $$f(x)=e^{-x^2} \qquad (e^x)'=e^x$$

10

$$\left(e^{-x^2}\right)'=e^{-x^2}\cdot\left(-x^2\right)'=e^{-x^2}\cdot(-2x)\ ✓$$

$$\left[e^{-x^2}\cdot(-2x)\right]'=e^{-x^2}\cdot(-2x)\cdot(-2)$$
$$=e^{-x^2}\cdot 4x$$

✗

$$\left[e^{-x}\cdot(-2x)\right]'=\ldots$$

$$f'(x)=e^{-x^2}\cdot(-2x)$$

$$f''(x)=e^{-x^2}\cdot 4x$$

Ime i prezime: JURE SVILIČIĆ **Broj indeksa:**

5. $g(x) = \sqrt{x^2 - x} - x$

DOMENA

$\sqrt{x^2 - x} \geq 0$

$\underline{x^2 - x \geq 0 \text{ UVIJEK}}$ ✗

$\mathcal{D}(f): \langle -\infty, +\infty \rangle$

$x^2 - x = 0$?

$x(x-1) = 0$

$x_{1,2} = \dfrac{\quad}{\quad} =$

$x_1 = 0$

$x_2 = 1$

$x^2 - x \geq 0$ za

$x \in \langle -\infty, 0] \cup [1, +\infty\rangle$

$\mathcal{D}(f) =$

ASIMPTOTE

$$\lim_{x \to \infty} \sqrt{x^2 - x} - x = \frac{(\sqrt{x^2-x} - x)(\sqrt{x^2-x} + x)}{(\sqrt{x^2-x} + x)} = \frac{x^2 - x + \cancel{x\sqrt{x^2-x}} - \cancel{x\sqrt{x^2-x}} - \cancel{x^2}}{\sqrt{x^2-x} + x} =$$

$$= \frac{-x}{\sqrt{x^2-x} + x} = \frac{-\frac{x}{x}}{\sqrt{\frac{x^2}{x} - \frac{x}{x}} + x} = \frac{-1}{2} = -\frac{1}{2}$$

H.A	$x = -\frac{1}{2}$

D.H.A.

L.H.A

$$\lim_{x \to -\infty} \sqrt{x^2 - x} - x = \sqrt{} \quad \sqrt{x^2 + x} + x = +\infty$$

$= \lim\limits_{x \to +\infty}$

NEMA

$\underline{y = kx + l}$ KOSA ASIMPTOTA

$$k = \frac{\sqrt{x^2 - x} - x}{x} = \frac{\sqrt{\frac{x^2}{x} - \frac{x}{x}} - \frac{x}{x}}{\frac{x}{x}} = \frac{1-1}{1} = \frac{0}{1} = 0$$

NEMA KOSE ASIMPTOTE

$\emptyset$

L.K.A = ?

Ime i prezime: JURE SULIČIĆ **Broj indeksa:**

PARNOST / NEPARNOST

$f(x) = f(-x)$ $f(-x) = -f(x)$

$\sqrt{x^2-x} - x \neq \sqrt{-x^2+x} + x$ $\sqrt{x^2+x} + x = -\left(\sqrt{x^2+x} - x\right) = \sqrt{x^2+x} + x$ ✓

NIJE PARNA FUNKCIJA JE NEPARNA

DERIVACIJA

$$\boxed{\sqrt{x} = \frac{1}{2\sqrt{x}}}$$

$$\left(\sqrt{x^2-x} - x\right)' = \frac{1}{2\sqrt{x^2-x}} \cdot (x^2-x)' - 1$$

$$= \underline{\underline{\frac{2x-1}{2\sqrt{x^2-x}} - 1}}$$

$$\left(\frac{2x-1}{2\sqrt{x^2-x}} - 1\right)' = \frac{(2x-1)' \cdot 2\sqrt{x^2-x} - (2x-1)\cdot\left(2\sqrt{x^2-x}\right)'}{\left(2\sqrt{x^2-x}\right)^2} =$$

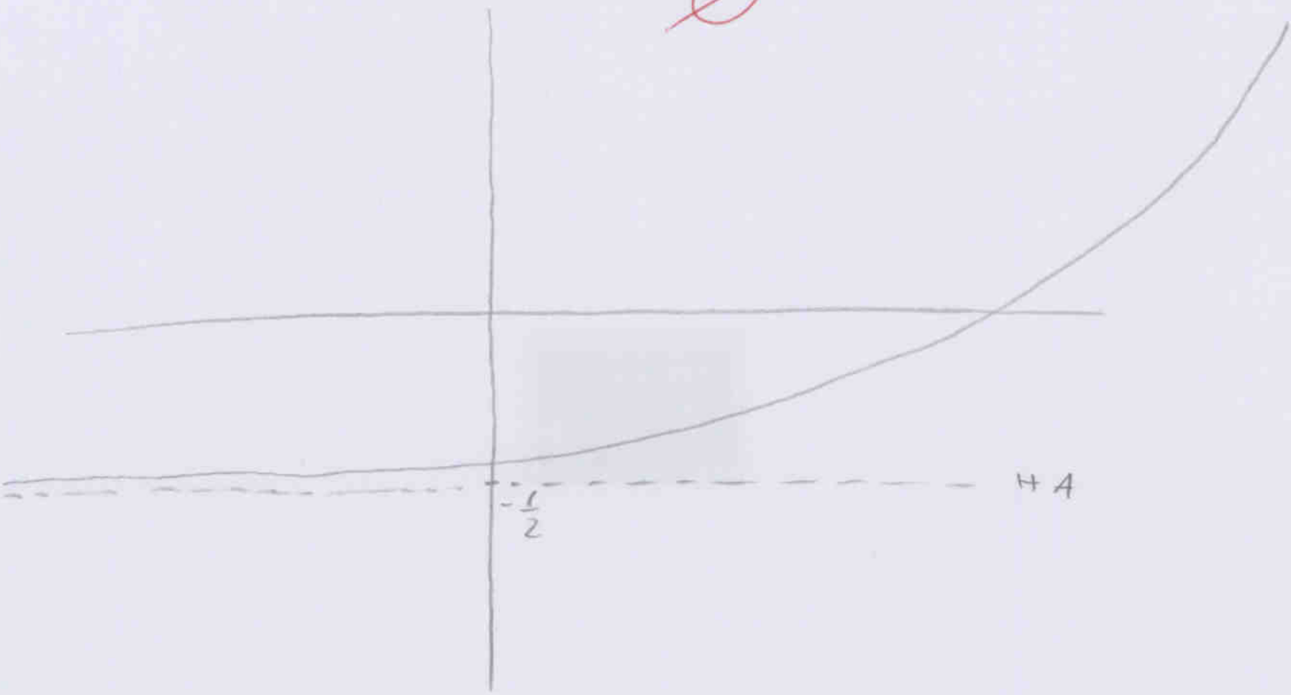
$$= \frac{4\sqrt{x^2-x} - (2x-1)\cdot 2 \cdot \frac{1}{2\sqrt{x^2-x}} \cdot (x^2-x)'}{\left(2\sqrt{x^2-x}\right)^2} =$$

$$= \frac{4\sqrt{x^2-x} - (2x-1) \cdot \frac{2}{2\sqrt{x^2-x}} \cdot 2x-1}{\left(2\sqrt{x^2-x}\right)^2} =$$

$$= \frac{4\sqrt{x^2-x} - \frac{2(2x-1)^2}{2\sqrt{x^2-x}}}{\left(2\sqrt{x^2-x}\right)^2}$$

Ime i prezime: JURE SULIĆ **Broj indeksa:**

	$-\infty$	$-\frac{1}{2}$	0	$+\infty$
f'	$-$	$-$	$-$	
f''	$+$	$+$	$+$	
f				



ČUVATI

Popuniti odmah!

IME I PREZIME: ŠIME MATANOVIĆ **BROJ INDEKSA:** 57655 30

DATUM: 17.03.2011 **VRIJEME:** OD DO

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. oxxx

Broj ↓ bodova

1. Ovisno od parametra λ odrediti rang matrice $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & \lambda \end{pmatrix}$ i riješiti matrični sustav 10

$$\mathbf{A}\,\mathbf{X} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

2. Odrediti modul (r) i argument (φ) kompleksnog broja $z = \dfrac{\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^4}{(-1+i)^6}$.

3. Istražiti konvergenciju reda: $\sum_{n=1}^{\infty} \left(\sqrt{n^2 - n} - n\right)$ $\cancel{0}$

4. Odrediti drugu derivaciju funkcije: $f(x) = e^{-x^2}$ 20

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $g(x) = \sqrt{x^2 - x} - x$.

1)
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ -1 & 0 & \lambda & 1 \end{array}\right] \sim \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & \lambda+1 & 2 \end{array}\right] /(-1) \sim \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & \lambda+1 & 2 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & \lambda-1 & 1 \end{array}\right] \frac{1}{\lambda-1} \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \frac{1}{\lambda-1} \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{\lambda-1} \\ 0 & 1 & 0 & \frac{\lambda-3}{\lambda-1} \\ 0 & 0 & 1 & \frac{1}{\lambda-1} \end{array}\right]$$ 10

2°

$\lambda - 1 = 0$

$\lambda = 1$

$r(A) = 3$

1° $\lambda - 1 = 0$

2° $\lambda - 1 \neq 0$ 10

4) $f(x) = e^{-x^2} = e^{-x^2} \cdot (-x^2)' = e^{-x^2} \cdot (-2x) = -2x e^{-x^2}$ ✓

$f''(x) = (-2x)'(e^{-x^2}) + (e^{-x^2})'(-x^2)'(-2x) = -2e^{-x^2} + -2x e^{-x^2}(-2x)$ ✓

$= -2e^{-x^2} + 4x^2 e^{-x^2}$ 20

3) $\lim_{n\to\infty} \sqrt{n^2 - n} - n = \lim_{n\to\infty} \sqrt{\frac{n^2}{n^2} - \frac{n}{n^2}} - \frac{n}{n} = 1 - 1 = 0$

$\lim_{n\to\infty} \sqrt{n^2 - n} - n = \left|\frac{\infty - \infty}{-\infty \to \infty}\right| = \lim_{n\to\infty} \sqrt{n^2 + n} + n = \lim_{n\to\infty} \sqrt{\frac{n^2}{n^2} + \frac{n}{n^2}} + \frac{n}{n} = 1 + 1 = 2$

$\lim_{n\to\infty} \frac{\sqrt{n^2 - n} - n}{\frac{n}{n}} = \lim_{n\to\infty} \frac{\sqrt{\frac{n^2}{n^2} \cdot \frac{n}{n}} - \frac{n}{n}}{\frac{n}{n}} = \frac{1-1}{1} = \frac{0}{1}$ $\cancel{0}$

IME I PREZIME: ŠIME MATANOVIĆ

BROJ INDEKSA: 57655

5)

$g(x)=\sqrt{x^2-x}-x$

$D(f)=R \setminus \langle 0,1\rangle$

$D(f)=\langle -\infty,0] \cup [1,+\infty\rangle$

1) DOMENA

$D_f\ x\in \mathbb{R}\setminus\{0,1\}$

$x^2-x=0$

$x(x-1)=0$

$x_1=0 \quad x-1=0$

$x_2=1$

2) PARNOST

$f(-x)=\sqrt{x^2+x}+x$, funkcija je niti parna niti neparna

3) PERIODIČNOST

Funkcija nije periodična jer ne sadrži trigonometrijska funkcije

4) ASIMPTOTE

H.A.

$\lim_{x\to+\infty}\sqrt{x^2-x}-x=\lim_{x\to+\infty}\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}-\frac{x}{x}=1-1=0$

$\lim_{x\to-\infty}\sqrt{x^2-x}-x=\left(\begin{matrix}x\to -x\\ -\infty\to+\infty\end{matrix}\right)=\lim_{x\to+\infty}\sqrt{x^2+x}+x=\lim_{x\to\infty}\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+\frac{x}{x}=1+1=2$

$x=2$

V.A.

$\lim_{x\to0}\sqrt{0-0}-0=0$

$\lim_{x\to1}\sqrt{(1)^2-1}-1=\sqrt{1-1}-1=\sqrt{0}-1=-1 \quad y=-1$

K.A.

$\lim_{x\to\pm\infty}\frac{\sqrt{x^2-x}-x}{x}=\lim_{x\to\infty}\frac{\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}-\frac{x}{x}}{\frac{x}{x}}=\frac{0}{1}=\pm\infty$

Ime i prezime: ŠIME MATANOVIĆ

Broj indeksa: 57655

$$\lim_{x\to-\infty}\frac{\sqrt{x^2-x}-x}{x} = \left|\begin{matrix} x\to -x \\ -\infty\to+\infty\end{matrix}\right| = \lim_{x\to+\infty}\frac{\sqrt{x^2+x}+x}{x} = \lim_{x\to\infty}\frac{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+\frac{x}{x}}{\frac{x}{x}} =$$

$$= \frac{2}{1} = \boxed{2}$$

$$\lim_{x\to+\infty}\sqrt{x^2-x}-x-2x = \lim_{x\to\infty}\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}-\frac{3x}{x} = -3+1 = \boxed{-2}$$

5.) KRITIČNE TOČKE

$$f(x)=\sqrt{x^2-x}-x =$$

$$f'(x)=\frac{1}{2\sqrt{x^2-x}}\cdot(x^2-x)'+(-x)' = \frac{2x-1}{2\sqrt{x^2-x}}-1 = \frac{x-1}{\sqrt{x^2-x}}-1$$

2) NULTOČKE

$$x^2-x-x=0$$
$$x^2-2x=0$$
$$x(x-2)=0$$
$$x=0 \qquad x-2=0$$
$$x=2$$

MONOTONOST

	$-\infty$	0	$+\infty$
$f'(x)$	$-$	$+$	
$f(x)$	$\searrow$	$\nearrow$	
	n.d.		

6.) TOČKE INFLEKSIJE

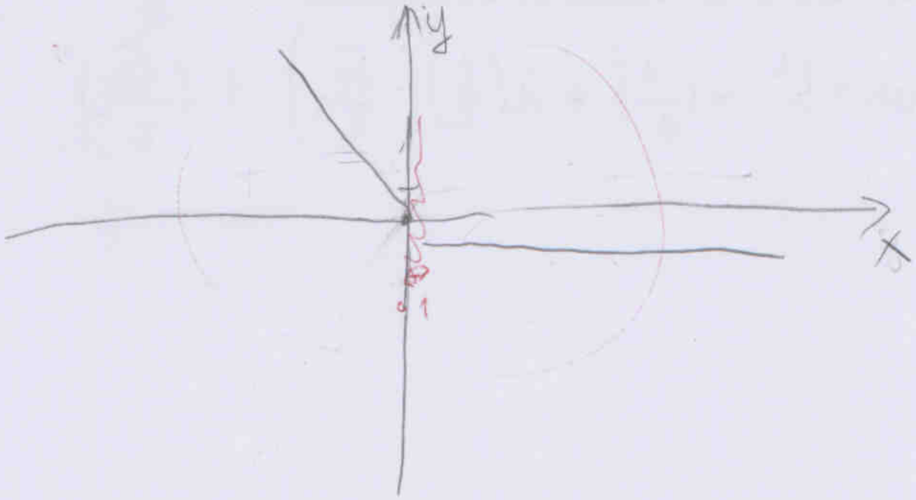
$$f''(x)=\left(\frac{x-1}{\sqrt{x^2-x}}-1\right)' = \frac{(x-1)'(\sqrt{x^2-x})-\left(\frac{1}{2\sqrt{x^2-x}}(x^2-x)'(x-1)\right)-(1)'}{\left(\sqrt{x^2-x}\right)^2} =$$

$$= \frac{\sqrt{x^2-x}-\frac{(2x-1)(x-1)}{2\sqrt{x^2-x}}}{x^2-x} = \frac{\sqrt{x^2-x}-\frac{2x^2-2x-x+1}{2\sqrt{x^2-x}}}{x^2-x} =$$

$$= \frac{\frac{\sqrt{x^2-x}}{1}-\frac{2x^2-3x+1}{2\sqrt{x^2-x}}}{\frac{x^2-x}{1}} \Rightarrow$$

nema točke infleksije

IME I PREZIME: ŠIME MATANOVIĆ BROJ INDEKSA: 57655



2) $$\frac{\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{4}}{(-1+i)^{6}} = \frac{1}{2}$$

$$(-1+i)^{6} = \binom{6}{0}(-1)^{6}(i)^{0} + \binom{6}{1}(-1)^{5}(i)^{1} + \binom{6}{2}(-1^{4})i^{2} + \binom{6}{3}(-1)^{3}i^{3} +$$

$$+\binom{6}{4}(-1)^{2}\cdot i^{4} + \binom{6}{5}(-1)^{1}\cdot i^{5} + \binom{6}{6}\cdot 1^{0}\cdot i^{6} = \binom{6}{6}$$

$= 1$

$$\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{2} = \frac{1}{4}-\frac{\sqrt{3}}{2}i+\left(\frac{\sqrt{3}}{2}i\right)^{2} =$$