

Popuniti odmah!

IME I PREZIME: *Albe Katan*

BROJ INDEKSA: *17-01-2010*

21

DATUM: *31.03.2010* VRIJEME: OD *08:15* DO *09:40*

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

XOX
Broj ↓
bodova

1. Koju relaciju zadovoljava inverz matrice? Proveriti tu relaciju za inverz matrice (ako postoji)

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

2. Pronaći sve kompleksne brojeve z takve da je $z^4 + |3i + 4| = \frac{5}{i^{31}}$.

3. Odrediti domenu i sve asimptote funkcije $f(x) = \ln\left(\frac{x}{1-x}\right)$.

4. Ispitati domenu, periodičnost, (ne)parnost i prvu derivaciju funkcije $g(x) = \sin(\cos(3x))$.

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5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije f iz zadatka 3.

5) $D_f = ?$

$$f(x) = \ln\left(\frac{x}{1-x}\right)$$

$$\begin{aligned} 1-x &\neq 0 \\ x &\neq 1 \\ x &\neq 0 \end{aligned}$$

$$D_f = \mathbb{R}^+ \setminus \{1\}$$

$\frac{x}{1-x} > 0$

	$-\infty$	0	1	$+\infty$
x	-	+	+	
$1-x$	+	+	-	
$\frac{x}{1-x}$	-	+	-	

$$\langle 0, 1 \rangle \cup \langle 1, +\infty \rangle$$

$$x \in \langle 0, 1 \rangle$$

ASIMPTOTE SE ISPITUJU NA RUBOVIMA DOMENE

horizontalne asimptote: nema horizontalnih asimptota!

vertikalne: ograničen je od 0 prema $+\infty$ i od 1

$$\text{baze: } k = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{\ln\left(\frac{x}{1-x}\right)}{x}$$



$$5) \text{ derivacija } f'(x) = \left[\ln\left(\frac{x}{1-x}\right) \right]' = \left(\frac{\frac{1}{x}}{\frac{1}{1-x}} \right) \cdot \frac{1}{(1-x)^2} = \frac{(1-x)}{x} \cdot \frac{1}{(1-x)^2} = \frac{1}{x(1-x)} = \frac{1}{x-x^2}$$

$$\left(\frac{x}{1-x}\right)' = \frac{(1-x) - x(-1)}{(1-x)^2} = \frac{1-x+x}{(1-x)^2}$$

$$f'(x) = \frac{1}{x-x^2} \quad \checkmark$$

$$(1-x)^2 = 0 \Rightarrow$$

$f'(x) = 0$ i $\frac{1}{x-x^2} \neq 0$ nikad nije jednak 0, prema derivacionem ulazu

$$\begin{aligned} x-x^2 &= 0 \\ x(1-x) &= 0 \Rightarrow \begin{cases} 1) \underline{x=0} \\ 2) \underline{1-x=0} \\ \quad \underline{x=1} \end{cases} \end{aligned}$$

$f(x)$	$-\infty$	0	1	$+\infty$
$f'(x)$	-	+	-	
		↗	↘	
	-1	90°	2	

$$6. f''(x) = \left(\frac{1}{x-x^2} \right)' = \frac{0 - (1-2x)}{(x-x^2)^2} = \frac{2x-1}{x^2-2x^3+x^4}$$

$$(x-x^2)^2 = 1-2x$$

$$f''(x) = \frac{2x-1}{x^2-2x^3+x^4}$$

$$f''(x) = 0$$

$$2x-1=0$$

$$2x=1$$

$$x = \frac{1}{2}$$

	0	$\frac{1}{2}$	1	$+\infty$
$f''(x)$	-	+	+	
$f(x)$	\cap	\cup	\cup	
	0,75	0,75	2	

$$x^2-2x^3+x^4 = 0$$

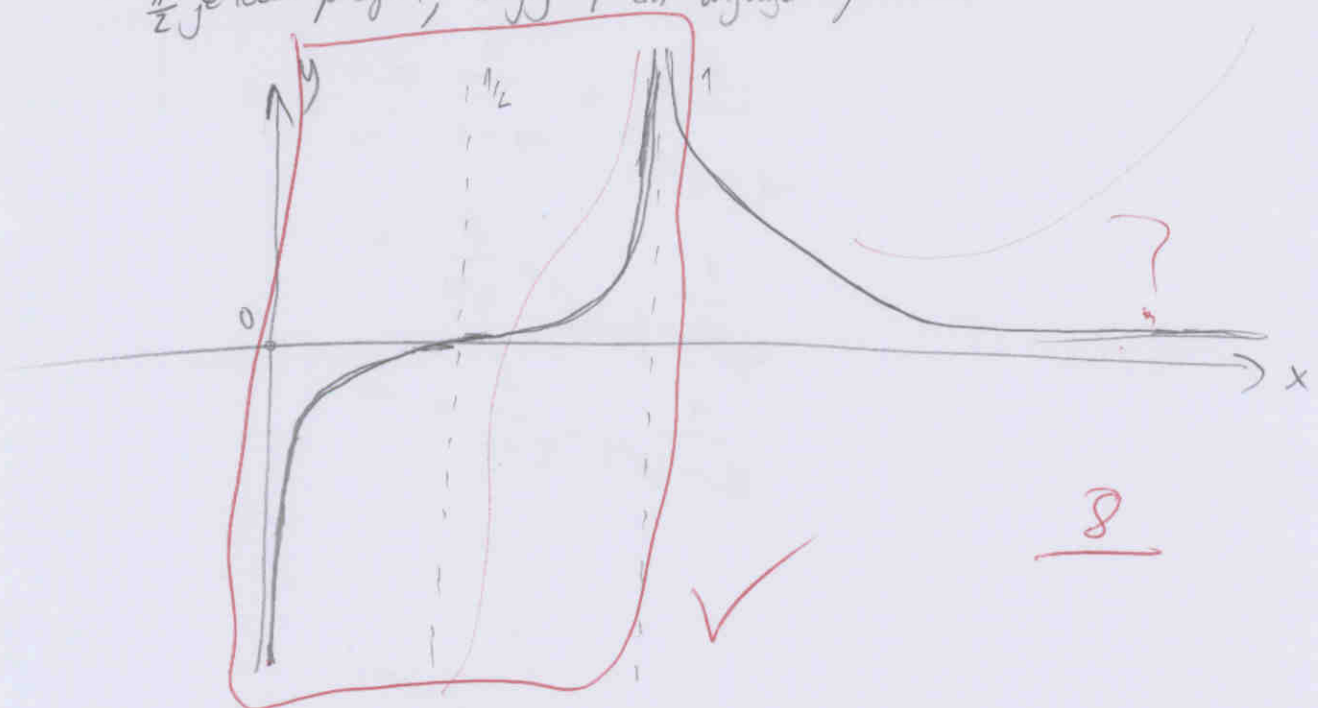
$$x^2(1-2x+x^2) = 0$$

$$1) x^2 = 0 \quad 2) x^2-2x+1 = 0$$

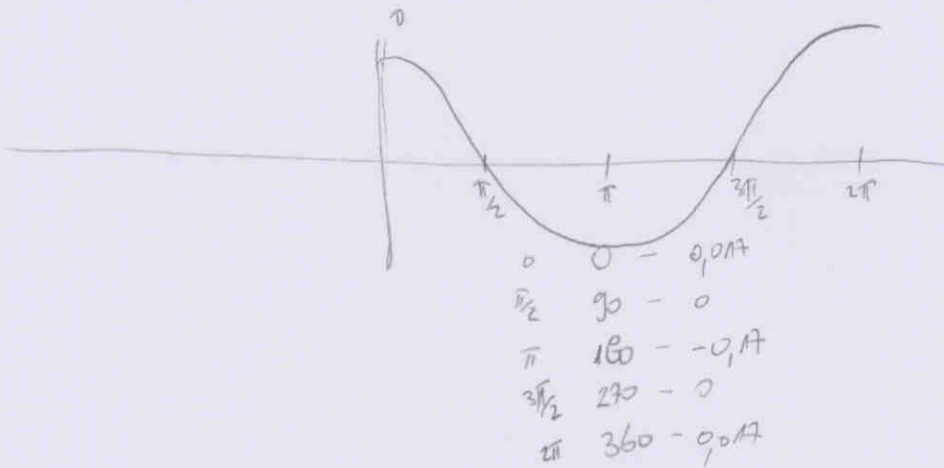
$$x = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4-4}}{2} = \frac{2 \pm 0}{2}, \quad x_{1,2} = 1$$

$\frac{1}{2}$ je tačka preгиба, u kojoj $f''(x)$ mijenja predznak



4



Alekcija je periodična
& temeljnim
periodom 2π X

$$f(x) = f(x+P)$$

$$\sin(\cos(3x)) = \sin(\cos(3(x+P)))$$

$$= \sin(\cos(3x+3P))$$

13

$$\cos(3x) = \cos(3x+3P)$$

ZNAMO

$$\cos x = \cos(x+2\pi)$$

UVRSTIMO 3x

$$\cos(3x) = \cos(3x+2\pi)$$

$$\cos(3x+3P) = \cos(3x+2\pi)$$

$$3x+3P = 3x+2\pi$$

$$3P = 2\pi$$

$$P = \frac{2\pi}{3}$$

IME I PREZIME:

BROJ INDEKSA:

$$\textcircled{1} \quad A \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array} \right] R_2: 2$$

$$\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \textcircled{0} & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}$$

IME I PREZIME:

BROJ INDEKSA:

① $z^4 + |3i+4| = \frac{5}{-i}$

$|3i+4| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$z^4 + 3i + 4 = \frac{5}{-i} \cdot \frac{1+i}{1+i}$

$z^4 + 3i + 4 = \frac{-5i}{1}$

X

~~⊙~~

$z^4 = -5i - 3i - 4$

$z^4 = -8i - 4$

$\varphi = 180 + \arctan \frac{y}{x} \Rightarrow \varphi = 180 + 63,435 = 243,43^\circ$

$y = -8$

$x = -4$

$r = \sqrt{(-8)^2 + (-4)^2} = \sqrt{64+16} = \sqrt{80} = 8,94$

$k=0,1,2,3$

↓
O.K.

$\omega_1 = \sqrt[4]{8,94} \left(\cos \frac{243,43^\circ}{4} + i \sin \frac{243,43^\circ}{4} \right) = 1,73 (0,487 + 0,87i) = 0,84 + 1,5i$

$\omega_2 = 1,73 \left(\cos \frac{243,43^\circ + 360^\circ}{4} + i \sin \frac{243,43^\circ + 360^\circ}{4} \right) = 1,73 (-0,487 + 0,87i) = -0,84 + 1,5i$

$\omega_3 = 1,73 \left(\cos \frac{243,43^\circ + 720^\circ}{4} + i \sin \frac{243,43^\circ + 720^\circ}{4} \right) = 1,73 (-0,487 - 0,87i) = -0,84 - 1,5i$

$\omega_4 = 1,73 \left(\cos \frac{1323,43^\circ}{4} + i \sin \frac{1323,43^\circ}{4} \right) = 1,73 (0,487 - 0,87i) = 0,84 - 1,5i$

② $y(x) = \sin(\cos(3x))$

① domain

$D_f = ? \quad \left. \begin{array}{l} D_{\cos} = \mathbb{R} \\ D_{\sin} = \mathbb{R} \end{array} \right\} D_f = \mathbb{R} \quad \checkmark$

② deriv.

$y'(x) = (\sin(\cos(3x)))'$

$y'(x) = \frac{1}{\cos(\cos(3x))} \cdot \left(-\frac{1}{\sin(3x)} \right) \cdot 3 = -\frac{3}{\cos^2(3x) \cdot \sin(3x)} \quad \checkmark$

③ p/n

$\sin(\cos(3x)) \neq \sin(\cos(-3x))$

$\sin(\cos(-3x)) \neq \sin(\cos(3x))$ NIJE PARNA X

$f(x) = -f(-x)$

$\sin(\cos(-3x)) \neq -\sin(\cos(3x))$

NIJE PARNA

funkcija je neparna

Popuniti odmah!

IME I PREZIME:

BUTERIN

BROJ INDEKSA:

DATUM: 31.3.

VRIJEME: OD

DO

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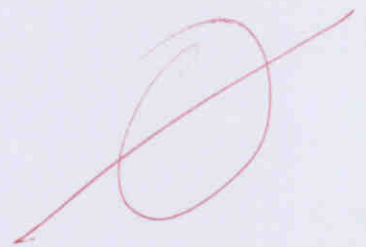
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①

$$A = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array} \right] R_4 - R_1$$



$$A = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array} \right] R_4 - R_1$$

$$A = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & -1 & -1 & -1 & 0 \end{array} \right] R_2 \cdot \frac{1}{3}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & -1 & -1 & -1 & 0 \end{array} \right] R_2 - 2R_3$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & -\frac{5}{3} & -2 & -2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & -1 & -1 & -1 & 0 \end{array} \right] R_1 \cdot R_3 \cdot \frac{1}{2}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array} \right] R_1 - R_4$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & -\frac{5}{3} & -2 & -2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & -1 & -1 & -1 & 0 \end{array} \right] R_4$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

IME I PREZIME:

BROJ INDEKSA:

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$$2^4 + |3i + 4| = \frac{5}{i^{31}}$$

$$-i^4$$

$$i^{31} = i^{-4} \cdot i^2 \cdot i^1$$

$$= 1 \cdot (-1) \cdot i$$

$$= -i$$

