Popuniti odmah! DANIJEL SORIĆ

IME I PREZIME: **BROJ INDEKSA:**

DATUM: VRIJEME: OD DO

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

35

xooo

Broj ↓ bodova

1. Ispitati domenu i odrediti sve asimptote funkcije $f(x) = \dfrac{\sqrt{x^2-3}-(3+x)}{x+2}$. — 15
2. Izračunati u kompleksnim brojevima: $\sqrt[3]{\dfrac{4+3i}{5i}} = ?$ — 20
3. Koliko iznosi $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{3^n}-\frac{1}{4^n}\right)$?
4. Ispitati tok i nacrtati skicu grafa funkcije: $g(x) = 2e^{-x^2}$.
5. Izračunati prvu derivaciju i pronaći sve ekstreme funkcije $h(x) = \sqrt[3]{x^2-x-1}$.

2. $\sqrt[3]{\dfrac{4+3i}{5i}} = \sqrt[3]{\dfrac{4+3i}{5i}\cdot\dfrac{-5i}{-5i}} = \sqrt[3]{\dfrac{-20i-15i^2}{-25i^2}} = \sqrt[3]{\dfrac{-20i+15}{25}} = \sqrt[3]{\dfrac{-20i}{25}+\dfrac{15}{25}} = \sqrt[3]{\dfrac{3}{5}-\dfrac{4}{5}i}$

$r = \sqrt{\left(\frac{3}{5}\right)^2+\left(-\frac{4}{5}\right)^2}$ $\varphi = \arctan\frac{y}{x}$

$r = \sqrt{\frac{9}{25}+\frac{16}{25}}$ $\varphi = \arctan\frac{-\frac{4}{5}}{\frac{3}{5}}$

$r = \sqrt{\frac{25}{25}}$ $\varphi = \arctan -\frac{20}{15}$

$r = \sqrt{1}$ $\varphi = \arctan -\frac{4}{3}$

$r = 1$ ✓ $\varphi = -0{,}92$ ✓

$z_1 = \sqrt[3]{r}\cdot\left(\cos\frac{\varphi+k\cdot 2\pi}{n} + i\sin\frac{\varphi+k\cdot 2\pi}{n}\right)$

$z_1 = \sqrt[3]{1}\cdot\left(\cos\frac{-0{,}92+0\cdot 2\pi}{3} + i\sin\frac{-0{,}92+0\cdot 2\pi}{3}\right)$

$z_1 = 1\cdot\left(\cos\frac{-0{,}92}{3} + i\sin\frac{-0{,}92}{3}\right)$

$z_1 = 1\cdot(\cos(-0{,}3066) + i\sin(-0{,}3066))$

$z_1 = 1\cdot(0{,}9533 + i\cdot(-0{,}3018))$

$z_1 = 0{,}9533 - 0{,}3018i$ ✓

DANIJEL SORIĆ

IME I PREZIME: BROJ INDEKSA:

$$z_2 = \sqrt[3]{r} \cdot \left(\cos\frac{\varphi + k\cdot 2\pi}{3} + i\sin\frac{\varphi + k\cdot 2\pi}{3}\right)$$

$$z_2 = \sqrt[3]{1} \cdot \left(\cos\frac{-0{,}92 + 1\cdot 2\pi}{3} + i\sin\frac{-0{,}92 + 1\cdot 2\pi}{3}\right)$$

$$z_2 = 1 \cdot \left(\cos\frac{5{,}363}{3} + i\sin\frac{5{,}363}{3}\right)$$

$$z_2 = 1 \cdot (\cos 1{,}7876 + i\sin 1{,}7876)$$

$$z_2 = 1 \cdot (-0{,}2151 + i \cdot 0{,}9765)$$

$$z_2 = -0{,}2151 + 0{,}9765\,i \quad \checkmark$$

20

$$z_3 = \sqrt[3]{r} \cdot \left(\cos\frac{\varphi + k\cdot 2\pi}{3} + i\sin\frac{\varphi + k\cdot 2\pi}{3}\right)$$

$$z_3 = \sqrt[3]{1} \cdot \left(\cos\frac{-0{,}92 + 2\cdot 2\pi}{3} + i\sin\frac{-0{,}92 + 2\cdot 2\pi}{3}\right)$$

$$z_3 = 1 \cdot \left(\cos\frac{11{,}646}{3} + i\sin\frac{11{,}646}{3}\right)$$

$$z_3 = 1 \cdot (\cos 3{,}882 + i\sin 3{,}882)$$

$$z_3 = 1 \cdot ((-0{,}738) + i \cdot (-0{,}6745))$$

$$z_3 = -0{,}738 - 0{,}6745\,i \quad \checkmark$$

IME I PREZIME: DANIJEL SORIĆ BROJ INDEKSA:

1. $f(x) = \frac{\sqrt{x^2-3} - (3+x)}{x+2}$

$Df(x) = \mathbb{R} \setminus \{-2\}$ ✓ 15

$x+2 = 0$

$x = -2$

H.A. $\lim_{x\to+\infty} \frac{\sqrt{x^2-3} - (3+x)}{x+2} \quad /:x$

$= \frac{\sqrt{\frac{x^2}{x^2} - \frac{3}{x^2}} - \left(\frac{3}{x} + \frac{x}{x}\right)}{\frac{x}{x} + \frac{2}{x}} = \frac{\sqrt{1} - 1}{1} = \frac{1-1}{1} = \frac{0}{1} = 0$ ✓

$\lim_{x\to-\infty} \frac{\sqrt{x^2-3} - (3+x)}{x+2} = \lim_{x\to+\infty} \frac{\sqrt{(-x)^2-3} - (3-x)}{-x+2} \quad /:x$

$= \frac{\sqrt{\frac{x^2}{x^2} - \frac{3}{x^2}} - \left(\frac{3}{x} - \frac{x}{x}\right)}{\frac{-x}{x} + \frac{2}{x}}$

NEMA V.A. ?

$= \frac{\sqrt{1}+1}{-1} = \frac{2}{-1} = -2$ ✓ $\quad y = -2$

NE TRAŽITI KOSE ASIMPTOTE KADA STE DOBILI HORIZONTALNE

K.A. $\frac{\sqrt{x^2-3} - (3+x)}{x+2}$

$k: \lim_{x\to+\infty} \frac{\sqrt{x^2-3} - (3+x)}{x+2} = \lim_{x\to+\infty} \frac{\sqrt{x^2-3} - (3+x)}{x} \quad /:x^2$

$= \frac{\sqrt{\frac{x^2}{x^4} - \frac{3}{x^4}} - \left(\frac{3}{x^2} + \frac{x}{x^2}\right)}{\frac{x^2}{x^2} + \frac{2x}{x^2}} = \frac{0}{1} = 0$

$l: \lim_{x\to+\infty} \frac{\sqrt{x^2-3} - (3+x)}{x+2} - 0 \cdot x = \lim_{x\to+\infty} \frac{\sqrt{x^2-3} - (3+x)}{x+2} \quad /:x$

$= \frac{\sqrt{\frac{x^2}{x^2} - \frac{3}{x^2}} - \left(\frac{3}{x} + \frac{x}{x}\right)}{\frac{x}{x} + \frac{2}{x}}$

$= \frac{1-1}{1} = \frac{0}{1} = 0$ ✓ ✓

V.A.

$k_2: \lim_{x\to-\infty} \frac{\sqrt{x^2-3} - (3+x)}{x+2} = \lim_{x\to+\infty} \frac{\sqrt{(-x)^2-3} - (3-x)}{-x+2} = \lim_{x\to+\infty} \frac{\sqrt{(-x)^2-3} - (3-x)}{-x} \quad /:x^2$

$= \frac{\sqrt{\frac{x^2}{x^2} - \frac{3}{x^2}} - \left(\frac{3}{x^2} - \frac{x}{x^2}\right)}{\frac{x^2}{x^2} - \frac{2x}{x^2}} = \frac{1}{1} = 1$ ✗

$l_2: \lim_{x\to-\infty} \frac{\sqrt{x^2-3} - (3+x)}{x+2} - x = \lim_{x\to+\infty} \frac{\sqrt{(-x)^2-3} - (3-x)}{-x+2} + x = \lim_{x\to+\infty} \frac{\sqrt{x^2-3} - (3x) - x^2 + 2x}{-x+2}$

$= \frac{1}{0} = +\infty$

IME I PREZIME: BROJ INDEKSA:

5. $h(x) = \sqrt[3]{x^2 - x - 1}$

$$y' = \frac{1}{3\sqrt[3]{(x^2-x-1)^2}} = \frac{1}{3\cdot(x^2-x-1)^{\frac{2}{3}}} \cdot (2x-1)$$

$$h'(x) = \left((x^2-x-1)^{\frac{1}{3}}\right)'$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$x^2 - x - 1 = 0$

$$x_{12} = \frac{1 \pm \sqrt{1^2 - 4\cdot 1\cdot(-1)}}{2\cdot 1}$$

$$x_{12} = \frac{1 \pm \sqrt{1+4}}{2}$$

$$x_{12} = \frac{1 \pm \sqrt{5}}{2}$$

$$x_{12} = \frac{1 \pm 2{,}23}{2}$$

$$x_1 = \frac{1 + 2{,}23}{2} = 1{,}615$$

$$x_2 = \frac{1 - 2{,}23}{2} = -0{,}615$$

$D(f) = ?$

Popuniti odmah! Franc Zenić

IME I PREZIME: **BROJ INDEKSA:**

DATUM: 31.3.2011. VRIJEME: OD DO .

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

xooo

Broj ↓ bodova

5

1. Ispitati domenu i odrediti sve asimptote funkcije $f(x) = \dfrac{\sqrt{x^2-3}-(3+x)}{x+2}$.

2. Izračunati u kompleksnim brojevima: $\sqrt[3]{\dfrac{4+3i}{5i}} = ?$

3. Koliko iznosi $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{3^n}-\frac{1}{4^n}\right)$?

4. Ispitati tok i nacrtati skicu grafa funkcije: $g(x) = 2e^{-x^2}$.

5. Izračunati prvu derivaciju i pronaći sve ekstreme funkcije $h(x) = \sqrt[3]{x^2-x-1}$.

X

3. $\sum_{n=0}^{\infty}\left(\frac{1}{3^n}-\frac{1}{4^n}\right) = \left(\frac{1}{1}-\frac{1}{1}+\frac{1}{3}-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}\dots\right) = \left(0+\frac{1}{12}+\frac{7}{144}\right| = \frac{19}{144}$

$= \left(\frac{1}{3}-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}\dots\right| = \frac{12+7}{144} = \frac{19}{144} \approx 0.13194$

$= \left(\frac{1}{3}+\frac{1}{9}\dots\right| - \left(\frac{1}{4}+\frac{1}{16}+\dots\right|$

$a_1 = \frac{1}{3}$ $a_2 = \frac{1}{9}$

$q = \frac{a_2}{a_1} = \frac{1}{3}$ Red je konvergentan

$S_n = \frac{a_1}{1-q}$ $S_n = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$

$\left(\frac{1}{3}\right)$ $\left(\frac{\frac{1}{4}}{\frac{3}{4}}\right) = \frac{1}{3}$ $\left(\frac{\frac{2}{9}}{\frac{1}{9}}\right) = 1$

$\frac{1}{3}-\frac{1}{4} = \frac{4-7}{12} = \frac{1}{12}$

$\frac{1}{4}-\frac{1}{16} = \frac{16-4}{144} = \frac{7}{144}$

$= \frac{1}{1}-\frac{1}{1} = 0$

$a_1 = \frac{1}{4}$ $a_2 = \frac{1}{16}$

$q = \frac{\frac{1}{16}}{\frac{1}{4}} = \frac{1}{4}$ Red je konvergentan

$S_n = \frac{a_1}{1-q} = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

$\frac{\frac{1}{2}}{3}$

$\sum_{n=0}^{\infty}\frac{1}{3^n} = \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

$\sum_{n=0}^{\infty}\frac{1}{4^n} = \dots = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$

$\frac{3}{2}-\frac{4}{3} = \frac{9-8}{6} = \frac{1}{6}$

IME I PREZIME: Frane Tenis

BROJ INDEKSA:

1 $f(x) = \frac{\sqrt{x^2-3} - (3+x)}{x+2}$ $\quad x+2=0$, $x=-2$

$f(-2) = \frac{\sqrt{4-3} - (3-2)}{-2+2} = \frac{1-1}{-2+2} = \frac{0}{0}$

$D_f = \mathbb{R} \setminus \{-2\}$ ✓

$f(x) = \lim_{x \to -2} \frac{\sqrt{x^2-3} - (3+x) \mid :x^2}{x+2 \mid :x^2} = \lim_{x \to -2} \frac{\sqrt{1} - 0}{0+0} = \frac{1}{0} = +\infty$ V.A. ✗

$\overset{(1-1)}{=} \overset{\frac{0}{0}}{\lim_{x \to -2}} \overset{L'H}{\frac{\frac{2x}{2\sqrt{x^2-3}} - 1}{1}} = \frac{-4}{2\sqrt{1}} - 1 = -3$

$= \frac{\sqrt{\frac{x^2}{x^2} - \frac{3}{x^2}} - \left(\frac{3}{x} + 1\right)}{1 + \frac{2}{x}} = \frac{1-1}{1} = \frac{0}{1} = 0$ D.H.A

D.H.A

$f(x) = \lim_{x \to +\infty} \frac{\sqrt{x^2-3} - (3+x) \mid :x^2}{x+2 \mid :x^2} \cdot x = \frac{\sqrt{1} - 0}{0} = \infty$ N.D.H.A ✓

L.H.A $\quad f(x) = \lim_{x \to -\infty} \frac{\sqrt{x^2-3} - (3+x)}{x+2} = |x \to (-x)| =$

$\lim \frac{\sqrt{x^2-3} - (3-x) \mid :x^2}{-x+2 \mid :x^2} \cdot x = \frac{1-0}{-0} = -\infty$ N.L.H.A

$\frac{\sqrt{\frac{x^2}{x^2} - \frac{3}{x^2}} - \left(\frac{3}{x} - \frac{x}{x}\right)}{-\frac{x}{x} + \frac{2}{x}} = \frac{1+1}{-1} = -2$ L.H.A.

-1

$4-3 \quad \frac{1}{-2+2} = \frac{0}{0} = \infty$

D.k.H.A.

$f(x) = \sqrt{x} \cdot \frac{x}{f(x)}$

IME I PREZIME: Franc Zenić

BROJ INDEKSA:

2 $\sqrt[3]{\frac{4+3i}{5i}} = ?$

$\frac{4+3i}{5i} \cdot \frac{-5i}{-5i} = \frac{-20i+15}{25} = \frac{15}{25} - \frac{20i}{25} = \frac{3}{5} - \frac{4}{5}i$ ✓

$z = \sqrt[3]{\frac{3}{5} - \frac{4}{5}i}$

$|r| = \sqrt{\frac{9}{25} + \frac{16}{25}}$

$r = \sqrt{\frac{25}{25}}$

$r = 1$ ✓

$\tan\varphi = \frac{-\frac{4}{5}}{\frac{3}{5}} = \frac{-4}{3}$

$\varphi = -53^\circ 7' 48''$

$\varphi = 306^\circ 52' 11''$

5

$|z| = \sqrt[3]{1} \cdot \left(\cos\frac{\varphi + 2k\pi}{3} + i\sin\frac{\varphi + 2k\pi}{3}\right)$

$k=0 \quad |z| = 1 \cdot \left(\cos\frac{-53^\circ 7' 48'' + 0}{3} + i\sin\frac{53^\circ 7' 48'' + 0}{3}\right)$

$= 0.952 + 0.304i$

$k=1 \quad |z| = 1 \cdot \left(\cos\frac{-53^\circ 7' 48'' + 2\pi}{3} + i\sin\frac{53^\circ 7' 48'' + 2\pi}{3}\right)$ ($2\pi \approx 360^\circ$)

$= 0.940 + 0.338i$

$k=2 \quad |z| = 1 \cdot \left(\cos\frac{-53^\circ 7' 48'' + 4\pi}{3} + i\sin\frac{53^\circ 7' 48'' + 4\pi}{3}\right)$

$= 1 \cdot (0.927 + 0.37i$

$= 0.927 + 0.372i$

IME I PREZIME: Frane Ženić

BROJ INDEKSA:

4. $g(x) = 2e^{-x^2}$.

1 Domena $D(f) = \mathbb{R}$ ✓

$D(f) = \mathbb{R}$ ✓

2 Periodičnost nije periodična

3 Parnost $g(-x) = 2e^{-(-x)^2}$

$g(-x) = 2e^{-x^2}$ ✓ Neparno je

$= g(x)$ PARNA

4. $g'(x) = -2x\,2e^{-x^2}$

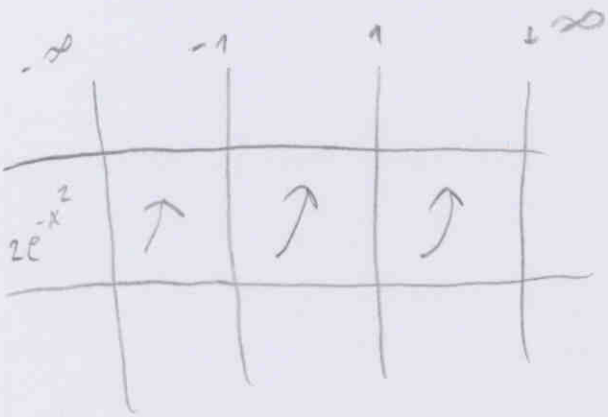
$= -4x e^{-x^2}$ ✓

≥ 0

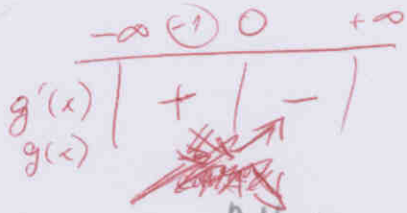
$g(x) = 2e^{-x^2}$

$g(0) = 2e^{-0^2}$

$g(0) = 2$ ~~Ek Nultočka~~



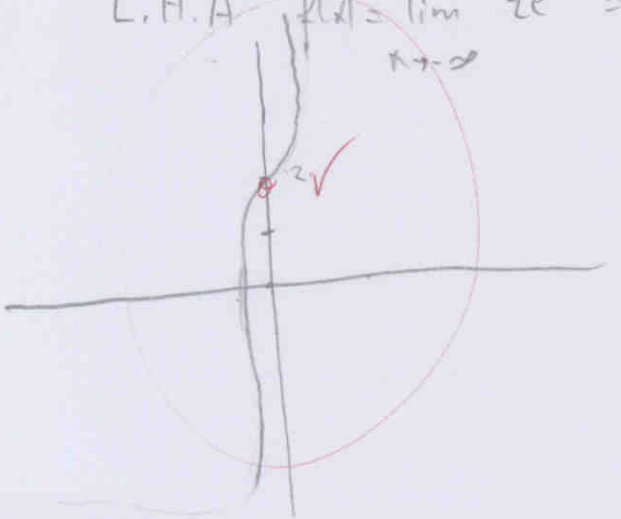
$g'(x) = 0$ za $x = 0$



5. Asimtote Nema V.A. jer je $D(f) = \mathbb{R}$

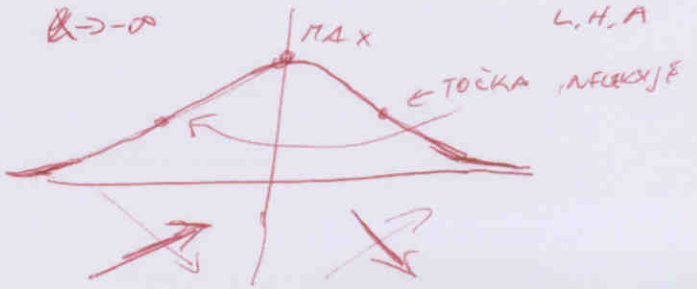
D.H.A $f(x) = \lim_{x \to \infty} 2e^{-x^2} / :x^2 = \lim 2e^{-1} = \frac{2}{e} = 0.735$. D.H.A.

L.H.A $f(x) = \lim_{x \to -\infty} 2e^{-x^2} = |x \to -x| = 2e^{-x^2} = 0.735$. L.H.A.



$\lim_{x \to +\infty} 2e^{-x^2} = 2e^{-\infty} = 0$ D.H.A

$\lim_{x \to -\infty} 2e^{-x^2} = 2e^{-\infty} = 0$ L.H.A



IME I PREZIME: Frane Zenić

BROJ INDEKSA:

5. $h(x) = \sqrt[3]{x^2-x-1}$

$$h'(x) = \frac{1}{3\sqrt[3]{x^2-x-1}} \cdot 2x-1$$

$$h'(x) = \frac{2x-1}{3\sqrt[3]{x^2-x-1}}$$

$x^2 - x - 1 = 0$

$$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2}$$

$$x_{1,2} = \frac{1 \pm \sqrt{5}}{2}$$

$$x_1 = \frac{1+\sqrt{5}}{2} \qquad x_2 = \frac{1-\sqrt{5}}{2}$$

$$\left(\sqrt[3]{x^2-x-1}\right)' = \left((x^2-x-1)^{\frac{1}{3}}\right)'$$

$f(g(x))$

$$= \begin{cases} f = x^{\frac{1}{3}} \\ g = x^2-x-1 \end{cases} \quad \begin{matrix} f'(x) = \frac{1}{3}x^{-\frac{2}{3}} \\ g'(x) = 2x-1 \end{matrix}$$

$$= f'(g(x)) \cdot g'(x)$$

$$= \frac{1}{3}(x^2-x-1)^{-\frac{2}{3}} \cdot (2x-1)$$

$$= \frac{1}{3} \frac{1}{(x^2-x-1)^{\frac{2}{3}}} \cdot (2x-1)$$

$$= \frac{1}{3} \frac{1}{\sqrt[3]{(x^2-x-1)^2}} \cdot (2x-1)$$

EKSTREMI KADA JE

$h'(x) = 0$

$D(f)$ = kada je $x^2-x-1 \geq 0$

$$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$D(f) = \langle -\infty, \underbrace{\frac{1-\sqrt{5}}{2}}_{\approx -0.62}] \cup [\underbrace{\frac{1+\sqrt{5}}{2}}_{\approx 1.61}, +\infty \rangle$$

$$h'(x) = \frac{1}{3} \frac{2x-1}{\sqrt[3]{(x^2-x-1)^2}} = 0$$

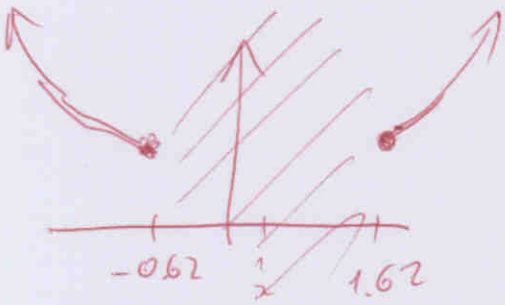
$+$

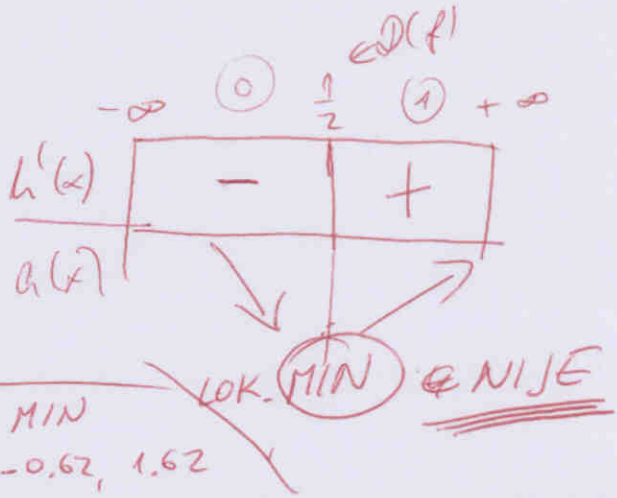
za $2x-1=0$

$$x = \frac{1}{2}$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = +\infty$$





LOK. MIN SU -0.62, 1.62

GENIJE