Popuniti odmah!

IME I PREZIME: Mateja Pečarić BROJ INDEKSA: 17-0032-2010 (37)

DATUM: 31.03.2011. VRIJEME: OD DO

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

xoxo

Broj ↓ bodova

1. Odrediti determinantu matrice $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ — 20

2. Odrediti domenu i sve asimptote funkcije $f(x) = x + \sqrt{x^2 + x + 1}$ — 7

3. Ispitati konvergenciju reda $\sum \left( \frac{n^2 + 4n + 4}{2n + 2n^2} \right)^{2n}$ — 0

4. Ispitati domenu, periodičnost, parnost i prvu derivaciju funkcije $g(x) = \ln(\sin(3x))$. — 10

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2 + 2}{x^2 + 1}$. — 0

1. $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = 1 \begin{vmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{vmatrix} = 7 - 6$

$= 1 \left( 1 \begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} \right) - 1 \left( 2 \begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} \right)$

$-3 \quad -2 \quad -3 \quad 0$

$-1 \qquad 6$

$= 5$ $\det A = 5$

$\begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$ $\begin{vmatrix} 2 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}$ $\begin{vmatrix} 0 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 1$

$-1 \qquad 2$

$= -2$

$= -1 - 2 = -3$

$\begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = -1 - 2 = -3$

IME I PREZIME: Hateja Pečarić BROJ INDEKSA: 17-0032-2010

2. $f(x) = x + \sqrt{x^2+x+1}$

$\infty + \infty = +\infty$

$$\lim_{x\to\infty} x+\sqrt{x^2+x+1} = \lim x+\sqrt{x^2+x+1} \cdot \frac{x-\sqrt{x^2+x+1}}{x-\sqrt{x^2+x+1}}$$

$D = \mathbb{R}$ ✓

$$\lim_{x\to\infty} \frac{x^2+x^2-x-1}{x-\sqrt{x^2+x+1}} = \lim_{x\to\infty} \frac{-x-1}{x-\sqrt{x^2+x+1}} \Big/ :x$$

7

$$\lim_{x\to\infty} \frac{-\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}-\sqrt{\frac{x^2}{x^3}+\frac{x}{x^2}+\frac{1}{x^3}}} = \frac{-1-\frac{1}{x}}{1-\sqrt{\frac{x^2}{x^3}+\frac{x}{x^2}+\frac{1}{x^3}}} = \frac{-1}{0} \quad H.A.$$

$\leq -\frac{1}{2}$ ✓

1

$\lim_{x\to+\infty} f(x) = +\infty$ NEMA D.H.A.

MOŽE LI SE IMATI D.+K.A.?

$\lim_{+\infty} \frac{f(x)}{x}$

K.H.A?

$\lim_{x\to-\infty} f(x) = -\infty + \infty =$ RACIONALIZACIJA

0

IME I PREZIME: Hatija Pečarić

BROJ INDEKSA:

$$\lim_{x\to\infty} \frac{x^2+2}{x^2+1} \Big/ x^2 = \frac{\frac{x^2}{x^2}+\frac{2}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \frac{1+\frac{2}{x^2}}{1+\frac{1}{x^2}} = 1$$ ✓

3. $\sum \left(\frac{n^2+4n+4}{2n+2n^2}\right)^{2n}$ $\lim \frac{n^2+4n+4}{2n^2+2n} \Big/ n^2 = \lim \frac{\frac{n^2}{n^2}+\frac{4n}{n^2}+\frac{4}{n^2}}{\frac{2n^2}{n^2}+\frac{2n}{n^2}} = \frac{1}{2} \neq 0$

- red konvergira

NUŽAN:

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{n^2+4n+4}{2n+2n^2}\right)^{2n} = \left(\frac{1}{2}\right)^{\infty} = 0$$

$\to \frac{1}{2}$

$$\lim_{n\to\infty} q^n = \begin{cases} 0, & |q|<1, \ -1<q<1 \\ +\infty, & q\geq 1 \\ N/P, & q\leq -1 \end{cases}$$

IME I PREZIME: Hatja Pećarić BROJ INDEKSA:

4. $f(x) = \ln(\sin(3x))$

$\sin(3x) \in \mathcal{D}(\ln)$

$\sin(3x) > 0$

$D(f) = \langle 0, +\infty \rangle$ ✗

- funkcija je periodična ✓, KOJI PERIOD?

$f(x) = f(x+P)$

$\ln(\sin(3x)) = \ln(\sin(3x+3P))$

$\Rightarrow \sin(3x) = \sin(3x+3P)$ ←

JA ZNAM

$\sin x = \sin(x+2\pi)$

UVRSTIM $3x$

$\sin(3x) = \sin(3x+2\pi)$ ←

$\sin(3x+3P) = \sin(3x+2\pi)$

~~$3x+3P = 3x+2\pi$~~

$3P = 2\pi$

$P = \frac{2\pi}{3}$

$f(x) = f(-x)$ $\quad f(x) = -f(x)$

$\ln(\sin(3x)) = \ln(\sin(-3x))$ - funkcija nije parna, nije ni neparna ✓

$f'(x) = \ln(\sin(3x))$

$= \frac{1}{\sin(3x)} \cdot \sin'(3x) \cdot (3x)' = \frac{1}{\sin(3x)} \cos 3x \cdot 3$ ✓

$= \frac{3\cos(3x)}{\sin(3x)}$

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IME I PREZIME: Hateja Pečarić

BROJ INDEKSA:

$T_1(0, 2)$

$T_2(1{,}41, 0)$

5. $f(x) = \frac{x^2+2}{x^2+1}$

- funkcija nije periodična
- funkcija je parna

$x=0$
$y=2$

$y=0$
$x+2=0$
$x^2=-2$
$x$

ASI

$D(f)=\mathbb{R}$ ✓

$f'(x) = \frac{x^2+2}{x^2+1}$

$= \frac{(x^2+2)'(x^2+1) - (x^2+2)(x^2+1)'}{(x^2+1)^2} = \frac{2x(x^2+1) - (x^2+2)\cdot 2x}{(x^2+1)^2}$

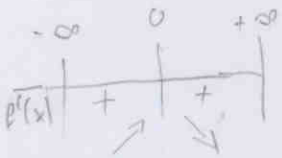
$= \frac{2x^3+2x-2x^3-4x}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$ $\Rightarrow$ funkcija pada

$f'(x)=0$

$\frac{-2x}{(x^2+1)^2} = 0 \;/\cdot (x^2+1)^2$

$-2x=0$

$x = \frac{0}{-2} = 0$ $\Rightarrow$ nema uvjetnih točaka

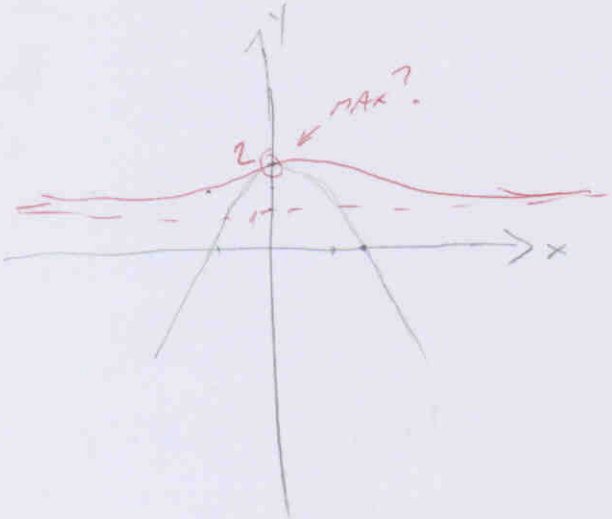



$f''(x) = \frac{2x}{(x^2+1)^2}$

$= \frac{(2x)'(x^2+1)^2 - 2x\left((x^2+1)^2\right)'}{\left((x^2+1)^2\right)^2}$

$= \frac{2\cdot(x^2+1)^2 - 2x(2x)^2}{(x^2+1)^4}$

$= \frac{2(x^4+4x^2+1) - 6x^3}{(x^2+1)^4} = \frac{2x^4+8x^2+2-6x^3}{(x^2+1)^4}$



Popuniti odmah!

IME I PREZIME: MARKO JULELIJA BROJ INDEKSA:

DATUM: 31.3. VRIJEME: OD DO .

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

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2. Odrediti domenu i sve asimptote funkcije $f(x) = x + \sqrt{x^2 + x + 1}$

3. Ispitati konvergenciju reda $\sum \left( \frac{n^2 + 4n + 4}{2n + 2n^2} \right)^{2n}$

4. Ispitati domenu, periodičnost, parnost i prvu derivaciju funkcije $g(x) = \ln(\sin(3x))$.

7

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2 + 2}{x^2 + 1}$.

1. $A = \begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix} = -1 \begin{vmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{vmatrix}$

$= -1 \left( -1 \begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} \right) + 1 \left( -1 \begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} \right)$ ✓ ✓ ✓ ✓ ✗

$\begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{matrix} = 0+0+0-0-0-0 = 0$

$\begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{matrix} = 1+0+0-0-2-2 = -3$

$\begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{matrix} = 2+0+0-0-0-4 = -2$

$\begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{matrix} = 1+0+0-0-2-2 = -3$

$= -1 \left( -1(0) + 2(-3) \right) + 1 \left( -1(-2) + 2(-3) \right)$ ✓

$= -1 \left( 0 + (-6) \right) + 1 \left( 2 + (-6) \right)$ ✓

$= -1(-6) + 1(-4)$

$= 6 - 4$

$= 2$

IME I PREZIME: MARKO VOLEDIA BROJ INDEKSA:

2. $f(x)=x+\sqrt{x^2+x+1}$ domena i asimptote

$x^2+x+1\geq 0$ $a=1\ b=1\ c=1$

$x_{12}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1^2-4\cdot1\cdot1}}{2\cdot1}=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{-3}}{2}\notin\mathbb{R}$

$x_1=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$x_2=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

$\Rightarrow x^2+x+1\geq 0\ \forall x\in\mathbb{R}$

$\lim f(x)=\ldots=+\infty$

$k=\lim_{x\to-\infty}\frac{x+\sqrt{x^2+x+1}}{x}=(x\Rightarrow(-x))$

$\lim_{x\to\infty}\frac{-x+\sqrt{(-x)^2+(-x)+1}}{-x}\ /:x=\frac{-\frac{x}{x}+\sqrt{\frac{x^2}{x^2}+\frac{(-x)}{x^2}+\frac{1}{x^2}}}{-\frac{x}{x}}=\frac{-1+\sqrt{\frac{1}{x}+(-1)+0}}{-1}=\frac{-1+\sqrt{0+(-1)+0}}{-1}=\frac{-1+1}{-1}=\frac{0}{-1}=\infty/k$ ✗

$l=\lim(f(x)-kx)=(x+\sqrt{x^2+x+1}-$

$D(f)=?\quad D(g)=\mathbb{R}$

$\lim_{x\to+\infty}\frac{x+\sqrt{x^2+x+1}}{x}\ /:x$

$=\lim_{x\to+\infty}\frac{\frac{x}{x}+\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}}{\frac{x}{x}}=\frac{?}{1}=2$

3. $\lim$

$(-x)^2=x^2$

$-\frac{x^2}{x}=-x$

$\frac{\sqrt{-x^2-x+1}}{x}=\sqrt{\frac{-x^2-x+1}{x^2}}$

4. $g(x)=\ln(\sin(3x))$ 1. deriv. parnost, period, domena

$D_f=\langle\ ?$

1. der.

$g'(x)=\ln(\sin(3x))$

$=\frac{1}{\sin(3x)}\cdot(\sin(3x))'$

$=\frac{1}{\sin(3x)}\cdot\cos3x\cdot3$ ✓

$=\frac{1}{\sin(3x)}\cdot3\cos3x$

$=\frac{3\cos3x}{\sin3x}$

$g(-x)=\ln(\sin(3x))$

$=\ln(\sin(-3x))$

$=\ln(-\sin(3x))=g(x)$ parno ✗

funkcija je periodična imat će

$P=?$

za domenu:

$\sin(3x)>0$

$\sin x>0$ ?

$x\in\langle0,\pi\rangle$ na osnovnom periodu

$\sin(3x)>0$

$3x\in\langle0,\pi\rangle$ na os. per.

$x\in\langle0,\frac{\pi}{3}\rangle$ na osnovnom

IME I PREZIME: MARKO VOLEVIA

BROJ INDEKSA:

(5)

$h(x) = \frac{x^2+2}{x^2+1}$ $D_f: x \in \mathbb{R}$ ✓ $x^2+1 \neq 0$

$x^2 = -1$

$\langle -\infty, +\infty \rangle$

↑ ↑

$\lim_{x\to\infty} \frac{x^2+2 \,/\, x^2}{x^2+1 \,/\, x^2} = \frac{\frac{x^2}{x^2}+\frac{2}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \frac{1+0}{1+0} = \frac{1}{1} = 1$ (0)

$\lim_{x\to-\infty} \frac{-x^2+2 \,/\, x^2}{-x^2+1 \,/\, x^2} = \frac{-\frac{x^2}{x^2}+\frac{2}{x^2}}{-\frac{x^2}{x^2}+\frac{1}{x^2}} = \frac{-1+0}{-1+0} = \frac{-1}{-1} = 1$ (0)

} H.A ✓

$\lim_{x\to 0} \frac{0^2+2}{0^2+1} = \frac{2}{1} = 2$

$\lim_{x\to -0} \frac{-0^2+2}{-0^2+1} = \frac{2}{1} = 2$

} H.A Nema ji ✗

$h(x)' = \frac{x^2+2}{x^2+1} = \frac{(x^2+2)'\cdot(x^2+1) - (x^2+2)(x^2+1)'}{(x^2+1)^2} = \frac{2x(x^2+1) - (x^2+2)\cdot 2x}{(x+1)^2} = \frac{2x^3+2x-(2x^3+4x)}{(x+1)^2}$

$= \frac{2x^3+2x-2x^3-4x}{(x+1)^2} = \frac{-2x}{(x+1)^2}$ ✓

$-2x > 0 \;/\cdot(-2)$

$x < 0$

rastuća za $x<0$

$\frac{-2x}{(x+1)^2} = 0 \;/\,(x+1)^2$

$-2x = 0 \;/\cdot(-2)$

$x = -\frac{0}{2} = 0$ MIN ✗

$h''(x) \frac{-2x}{(x+1)^2} = \frac{(-2x)'(x+1)^2 - (-2x)(x+1)^2}{(x+1)^4}$

$= \frac{-2(x+1)^2 - (-2x)\,2(x+1)\cdot 1}{(x+1)^3}$ ✗

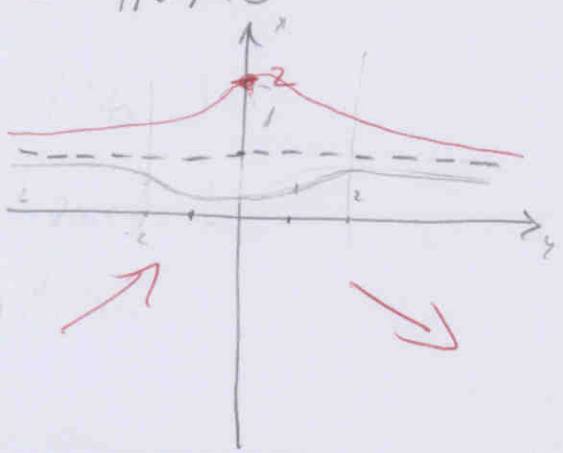
$= \frac{(-2x-2)-(-2x)\cdot(2x+2)}{(x+1)^2}$

$= \frac{(-2x-2)-(-4x^2-4x)}{(x+1)^2}$

$= \frac{-2x-2+4x^2+4x}{(x+1)^2} = \frac{4x^2+2x-2}{(x+1)^2}$

$= \frac{(x+1)(4x-2)}{(x+1)^2} = \frac{4x-2}{x+1}$ $A'' > 0$

$h'(x) > 0$ $h(x) < 0$




↗ ↘