Popuniti odmah!

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DATUM: 31.3 VRIJEME: OD DO .

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

oxxx

Broj ↓ bodova

1. Ovisno od parametra λ odrediti rang matrice $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & \lambda \end{pmatrix}$ i riješiti matrični sustav

$$\mathbf{A}\,\mathbf{X} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

2. Odrediti modul (r) i argument (φ) kompleksnog broja $z = \dfrac{\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^4}{(-1+i)^6}$. $\emptyset$

3. Istražiti konvergenciju reda: $\sum_{n=1}^{\infty}\left(\sqrt{n^2-n}-n\right)$ 20

4. Odrediti drugu derivaciju funkcije: $f(x) = \mathrm{e}^{-x^2}$ 20

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $g(x) = \sqrt{x^2-x} - x$. 12

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2. $z = \dfrac{\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)^4}{(-1+i)^6} \approx -\frac{1}{2} + \frac{\sqrt{3}}{2}$

$\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)^4 = \frac{1}{16} + 4\left(\frac{1}{2}\right)^3 \cdot \left(-\frac{\sqrt{3}}{2} i\right)^1 + 6\left(\frac{1}{2}\right)^2 \left(-\frac{\sqrt{3}}{2} i\right)^2 + 4\left(\frac{1}{2}\right)^1 \left(-\frac{\sqrt{3}}{2} i\right)^3 + \left(-\frac{\sqrt{3}}{2} i\right)^4$

$\sqrt{27} = \sqrt{3 \cdot 9} = 3\sqrt{3}$

$\frac{1}{16} - \frac{4}{8} \cdot \frac{\sqrt{3}}{2} i + \frac{6}{4} \cdot \frac{3}{4} - \frac{4}{2} \frac{\sqrt{3^3}}{8} i + \frac{9}{16}$

$\frac{1}{16} - \frac{\sqrt{3}}{4} i + \frac{9}{8} - \frac{3\sqrt{3}}{4} i + \frac{9}{16} = \frac{1+18+9}{16} - \frac{4\sqrt{3}}{4} i$

$\frac{28}{16} - \sqrt{3} i = \frac{7}{4} - 3i$ ✗

$(-1+i)^6 = (-1)^6 + 6 \cdot (-1)^5 (i)^1 + 15(-1)^4 (i)^2 + 20(-1)^3 (i)^3 + 15(-1)^2 (i)^4 + 6(-i)(i)^5 + i^6$

$= 1 - 6i - 15 + 20i + 15 - 6i - 1$

$= 8i$ ✓

$z = \dfrac{\frac{7}{4} - 3i}{8i} \cdot \dfrac{-i}{-i} = \dfrac{\frac{7}{4}i + 3}{-8}$

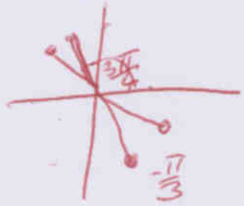
$z = -\dfrac{\frac{7}{4}}{8} i - \dfrac{3}{8} = -\dfrac{7}{32} i - \dfrac{3}{8}$

$r = \sqrt{\left(\frac{7}{32}\right)^2 + \left(\frac{3}{8}\right)^2} = 0.434$

$\varphi = \pi + \operatorname{arctg} \frac{y}{x} = \pi + \operatorname{arctg} \dfrac{\frac{7}{32}}{-\frac{3}{8}}$

$\varphi = \pi + \operatorname{arctg} -\frac{7}{12} = \pi - 30.25°$

$\varphi = 27.708$ ✗



$\frac{3\pi}{4} \cdot \frac{3}{6} = \frac{9\pi}{2} = (4\frac{1}{2})\pi$

$-\frac{\pi}{3} \cdot 4 = -\frac{4\pi}{3} = \frac{2\pi}{3}$

3. $\sum_{n=1}^{\infty} \left(\sqrt{n^2 - n} - n\right)$

NUŽAN

$\lim_{n\to\infty} \left(\sqrt{n^2-n} - n\right) = \lim_{n\to\infty} \left(\sqrt{n^2-n} - n \cdot \dfrac{\sqrt{n^2-n}+n}{\sqrt{n^2-n}+n}\right) = \lim_{n\to\infty} \dfrac{n^2 - n - n^2}{\sqrt{n^2-n}+n}$

$\infty - \infty$

$= \lim_{n\to\infty} \dfrac{-n \;/ :n}{\sqrt{n^2-n}+n \;/ :n} = \lim_{n\to\infty} \dfrac{-1}{\sqrt{\frac{n^2}{n^2} - \frac{n}{n^2}} + 1} = -\dfrac{1}{2}$ ✓ NE KONVERGIRA

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4. $f(x) = e^{-x^2}$

$f'(x) = e^{-x^2} \cdot (-2x) = -2x e^{-x^2}$ ✓

$f''(x) = -2e^{-x^2} - 2x\, e^{-x^2} \cdot (-2x)$ ✓

$f''(x) = -2e^{-x^2} + 4x^2 e^{-x^2}$ ✓

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5. $g(x)=\sqrt{x^2-x}-x$

$x^2-x\geq 0$

$x(x-1)$

$x=0 \quad x=1$

		0		1	
$x-1$	$-$		$-$	0	$+$
x	$-$	0	$+$		$+$
-1	$+$		$-$		$+$

$D_f=\langle-\infty,0]\cup[1,\infty\rangle$

1.H.A. $\lim\limits_{x\to\infty}\left(\sqrt{x^2-x}-x\right)\cdot\frac{\sqrt{x^2-x}+x}{\sqrt{x^2-x}+x}=\lim\limits_{x\to\infty}\frac{\cancel{x^2}-x-\cancel{x^2}\,/:x}{\sqrt{x^2-x}+x\,/:x}$

$\lim\limits_{x\to\infty}\frac{-1}{\sqrt{1}+1}=-\frac{1}{2}$

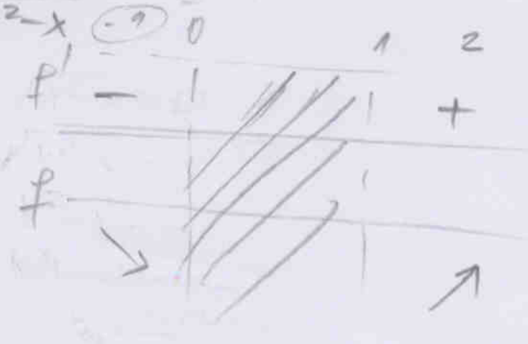
2.H.A. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-x}-x\right)=[x\to-x]=\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}+x\right)\cdot\frac{\sqrt{x^2+x}-x}{\sqrt{x^2+x}-x}$

$\lim\limits_{x\to\infty}\frac{\cancel{x^2}+x-\cancel{x^2}\,/:x}{\sqrt{x^2+x}-x\,/:x}=\lim\limits_{x\to-\infty}\frac{1}{1-1}=\frac{1}{0}=+\infty$

L.K.A. ?

$f(0)=0 \qquad f(1)=-1$

$f'(x)=\frac{1}{2}(x^2-x)^{-\frac{1}{2}}-1=\frac{1}{\sqrt{x^2-x}}-1=\frac{1-\sqrt{x^2-x}}{\sqrt{x^2-x}}$



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