17.3.2011.

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO .

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. oox

70

IME I PREZIME: ČIPIN MARIO BROJ INDEKSA:

1. Odrediti duljinu krivulje s parametrizacijom $x = t$, $y = t^{3/2}$ i $z = t$ između točaka $A(0,0,0)$ i $B(1,1,1)$. 20

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2 \\ x^2 \\ x^2 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(1,1,1)$, a koji je orijentiran vanjskom normalom. 0

15

3. Izračunati volumen tijela omeđenog plohama: $2z = x^2 + y^2$, $z = 1$.

4. Neka je točkama $A(0,3)$, $B(3,0)$ i $C(2,2)$ dan trokut ABC i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 20

$$\oint_C x^2 \mathrm{d}x + y^2 \mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$f'''(t) - 4f'(t) = \sin(2t), \quad f(0) = f'(0) = f''(0) = 0.$$

$x'''(t) + 4x'(t) = 0$, $x(0) = x'(0) = 2$, $x'(0) = 0$ 15

IME I PREZIME: BROJ INDEKSA:

1. $r'(t) = \begin{bmatrix} 1 \\ \frac{3}{2} t^{\frac{1}{2}} \\ 1 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow t = 0$ ✓

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \rightarrow t = 1$ ✓

$L = \int_0^e |r'(t)| dt$

$L = \int_0^1 \sqrt{2 + \frac{9}{4} t} \, dt$

$u = 2 + \frac{9}{4} t$

$du = \frac{9}{4} dt$

$dt = \frac{du}{\frac{9}{4}} = \frac{4}{9} du$

$t = 1$

$u = 2 + \frac{9}{4} \cdot 1 = \frac{8+9}{4} = \frac{17}{4}$

$t = 0$

$u = 2$

$L = \int_2^{\frac{17}{4}} \sqrt{u} \cdot \frac{4}{9} du$ ✓

$L \frac{4}{9} \int_2^{\frac{17}{4}} u^{\frac{1}{2}} du = \frac{4}{9} \int u^{\frac{1}{2}} du$

$L = \frac{4}{9} \left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) \Big|_2^{\frac{17}{4}} du$

$L = \frac{4}{9} \frac{2}{3} u^{\frac{3}{2}} \Big|_2^{\frac{17}{4}} = \frac{8}{27} u^{\frac{3}{2}} \Big|_2^{\frac{17}{4}}$

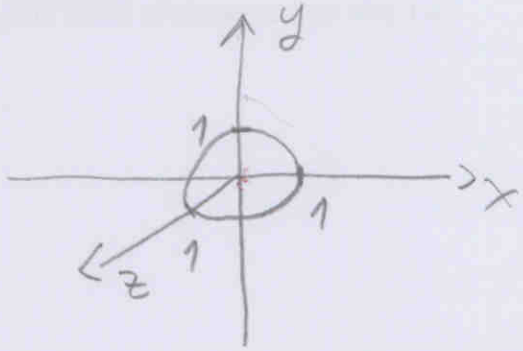
$L = \frac{8}{27} \left(\left(\frac{17}{4}\right)^{\frac{3}{2}} - 2^{\frac{3}{2}} \right) = \frac{8}{27} (8{,}76 - 2{,}82) = 1{,}758$ ✓

20

IME I PREZIME: BROJ INDEKSA:

2.

$$F=\begin{pmatrix} x^2 \\ x^2 \\ x^2 \end{pmatrix} \qquad W=\begin{bmatrix} x^2 \\ x^2 \\ x^2 \end{bmatrix}$$



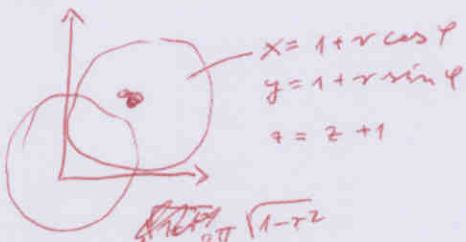
$$\operatorname{div} W = 2x$$

$r=1 \quad T(1,1,1)$

$$\iiint_{\partial K} 2x\,dx\,dy\,dz = \int_0^1\int_0^{2\pi}\int_0^1 r^2\,dr\,d\varphi\,dz$$

$$= \int_0^1\int_0^{2\pi}\int_0^1 2r^2\cos\varphi\,dr\,d\varphi\,dz = \int_0^1 dz \int_0^1 2r^2 \int_0^{2\pi}\cos\varphi$$

$$= z\Big|_0^1 \frac{2r^3}{3}\Big|_0^1 \cdot \sin 2\pi = 1\cdot\frac{2}{3}\cdot 0 = 0$$



$$\iint_{\partial K} F\cdot dS = \iiint_{K(T,1)} \underbrace{\operatorname{div} F}_{=2x}\,dx\,dy\,dz = 2\int_0^1\int_0^{2\pi}\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} (1+r\cos\varphi)\cdot r\,dz\,d\varphi\,dr$$

$$= 2\int_0^1\int_0^{2\pi} (1+r\cos\varphi)\, r\cdot 2\sqrt{1-r^2}\,dr\,d\varphi$$

$$= 2\int_0^1\int_0^{2\pi} r\cdot 2\sqrt{1-r^2}\,d\varphi\,dr + 2\int_0^1\int_0^{2\pi} \cos\varphi\, r^2\cdot 2\sqrt{1-r^2}\,d\varphi\,dr$$

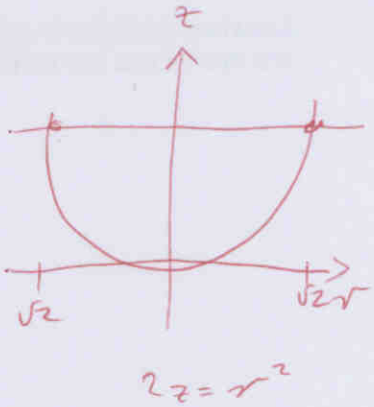
$u = 1-r^2$, $du = -2r$, $\sqrt{u}$

$$\int_0^{2\pi}\cos\varphi\,d\varphi = (\sin\varphi)_0^{2\pi} = 0-0 = 0$$

$2\cdot 2\pi$

IME I PREZIME: BROJ INDEKSA:

3. $x^2+y^2=2z$, $z=1$



$r^2=2z$

$r=\pm\sqrt{2z}$

$z=\frac{r^2}{2}$ =

$r=\pm\sqrt{2\cdot 1}=\pm\sqrt{2}$ ✓

$$V=\int_0^{2\pi}\int_0^{\sqrt{2}}\int_{\frac{r^2}{2}}^{1} r\,dz\,dr\,d\varphi$$ ✓✓✓

15

RADIO: $\frac{r^2}{2}$

$$V=\int_0^{2\pi}d\varphi\int_0^{\sqrt{2}}dr\int_{r^2}^{1} r\,dz = \int_0^{2\pi}d\varphi\int_0^{\sqrt{2}} r\,dr\left(1-\frac{r^2}{2}\right)$$

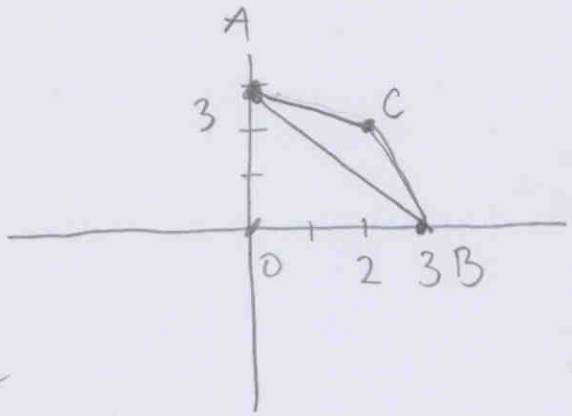
$$V=\int_0^{2\pi}d\varphi\int_0^{\sqrt{2}} r(1-r^2)\,dr = \varphi\Big|_0^{2\pi}\int_0^{\sqrt{2}}\left(r-r^3\right)dr$$

$$V=\varphi\Big|_0^{2\pi}\left(\frac{r^2}{2}-\frac{r^4}{2\cdot 4}\right)\Big|_0^{\sqrt{2}} = 2\pi\left(\frac{2}{2}-\frac{2^2}{2\cdot 4}\right)$$

$$=2\pi\left(1-\frac{4}{2\cdot 4}\right)=2\pi\cdot\frac{1}{2}=\pi$$

IME I PREZIME: BROJ INDEKSA:

4.



A(0|3)
B(3|0)
C(2|2)

$$\oint x^2 dx + y^2 dy$$

$$\oint P dx + Q dy = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

$$\frac{\partial Q}{\partial x} = 0 dx \,, \frac{\partial Q}{\partial y} = 0 dy$$

$$\iint (0 - 0) dx dy = 0 \checkmark$$

20

IME I PREZIME: BROJ INDEKSA:

5. $s^3X(s) - s^2x(0) - sx'(0) - x''(0) + 4(sX(s) - x(0)) = 0$

$s^3X(s) - 2s^2 - 0 - 2 + 4sX(s) - 8 = 0$

$s^3X(s) - 2s^2 - 2 + 4sX(s) - 8 = 0$

$s^3X(s) - 2s^2 + 4sX(s) - 10 = 0$

$s^3X(s) + 4sX(s) = 2s^2 + 10$

$X(s)(s^3 + 4s) = 2s^2 + 10$

$X(s) = \frac{2s^2+10}{s^3+4s} = \frac{2s^2+10}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4}$

$X(s) = \frac{A(s^2+4) + (Bs+C)s}{s(s^2+4)}$

$X(s) = As^2 + 4A + Bs^2 + Cs = 2s^2 + 10$

$s^2 \ldots A + B = 2 \rightarrow B = 2 - A = 2 - \frac{5}{2} = \frac{4-5}{2} = -\frac{1}{2}$

$s \ldots 0 = C$

$s^0 \ldots 10 = 4A$

$A = \frac{10}{4} = \frac{5}{2}$

$-\frac{1}{2}$ $\frac{1}{2}$ $\frac{2}{s^2+4}$

$\mathcal{L}^{-1}\left\{ \frac{\frac{5}{2}}{s} + \frac{-\frac{1}{2}}{s^2+4} \right\} = \frac{5}{2}\cdot\frac{1}{s} + \frac{-\frac{1}{2}\cdot 2}{2}\cdot\frac{2}{s^2+4}$ ✓

NIJE DOBRO VRAĆEN NATRAG U ORIGINALNO PODRUČJE

$\mathcal{L}^{-1}(F) = \frac{5}{2} - \frac{1}{2\cdot 2}\sin 2t$ ✓ ×

15