

Popuniti odmah!

IME I PREZIME: PAULO VUKARIĆ

BROJ INDEKSA: 17-2-0040-2040

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DATUM: 28.04.2011. VRIJEME: OD 15:00 DO

MATEMATIKA 1: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

xox
Broj ↓
bodova

1. Koju relaciju zadovoljava inverz matrice? Provjeriti tu relaciju za inverz matrice (ako postoji)

$$A = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

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2. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3 + 4i| = -\frac{5}{i^{27}}$.

~~0~~

3. Odrediti domenu i sve asimptote funkcije $f(x) = \ln\left(\frac{x+2}{1-x}\right)$.

6

4. Ispitati domenu, periodičnost, (ne)parnost i prvu derivaciju funkcije $g(x) = \cos(\sin(2x))$.

14

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije f iz zadatka 3.

~~0~~

1.

$$A \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \frac{1}{2}R_1 \\ \\ \\ R_4 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 & 0 & -2 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 & 0 & -2 \end{array} \right] R_3 - 2R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 & 0 & -2 \end{array} \right] \xrightarrow{-\frac{1}{3}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 & 0 & -2 \end{array} \right] R_2 - 2R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 & 0 & -2 \end{array} \right] \xrightarrow{-\frac{1}{3}R_4} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 & \frac{2}{3} & 0 \end{array} \right] R_1 - 2R_4$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{2}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 & \frac{2}{3} & 0 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} \frac{2}{3} & 0 & 0 & -\frac{1}{3} \\ 0 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & 0 & 0 & \frac{2}{3} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{3} \\ \\ \\ \end{bmatrix}$$

PROVERA

$$A \cdot A^{-1} = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{3} & 0 & 0 & -\frac{1}{3} \\ 0 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & 0 & 0 & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} I$$

$$\frac{4}{3} - \frac{1}{3} = 1$$

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$$\begin{array}{l} -2 \cdot \frac{-1}{3} = \frac{2}{3} \\ 1 - 2 \cdot \frac{2}{3} = \\ 1 - \frac{4}{3} = \frac{3-4}{3} \\ 1 - \frac{4}{3} = -\frac{1}{3} \\ -2 \cdot (-\frac{1}{3}) = \frac{2}{3} \\ 1 - 2 \cdot (-\frac{2}{3}) = \\ 1 + \frac{4}{3} = \frac{3+4}{3} \\ 1 + \frac{4}{3} = \frac{7}{3} \end{array}$$

$$1 - \frac{4}{3} = -\frac{1}{3}$$

$$-\frac{4}{3} - \frac{1}{3}$$

(2)

$$i^{24} = i^3 = -i$$

$$z^3 + |3+4i| = -\frac{5}{i^{24}}$$

$$z^3 + |3+4i| = -\frac{5}{-i}$$

$$z^3 + |3+4i| = \frac{5}{i} \cdot \frac{-i}{-i}$$

$$z^3 + |3+4i| = \frac{-5i}{-i \cdot 2}$$

$$z^3 + |3+4i| = -5i \quad \checkmark$$

$$z^3 = 3+4i-5i \quad \times$$

$$z^3 = 3-i$$

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} = 5 \end{aligned}$$

$$\begin{aligned} \varphi &= \arctan \frac{y}{x} \\ &= \arctan \frac{-1}{3} \\ &= \arctan -0.33 \\ &= -0.32 \end{aligned}$$

$$|3+4i| = \sqrt{3^2+4^2} = 5$$

$$\Rightarrow z^3 = -5-5i$$

$$z_1 = \sqrt[3]{3.16} \left(\cos \frac{-0.32 + 2 \cdot 0 \cdot \pi}{3} + i \sin \frac{-0.32 + 2 \cdot 0 \cdot \pi}{3} \right) = 1.47 \left(\cos -0.1 + i \sin -0.1 \right) =$$

$$= 1.47 - 0.14i$$

$$z_2 = 1.47 \left(\cos \frac{-0.32 + 2\pi}{3} + i \sin \frac{-0.32 + 2\pi}{3} \right) = 1.47 \left(\cos 1.99 + i \sin 1.99 \right) = -0.42 + 1.61i$$

$$z_3 = 1.47 \left(\cos \frac{-0.32 + 4\pi}{3} + i \sin \frac{-0.32 + 4\pi}{3} \right) = 1.47 \left(\cos 4.08 + i \sin 4.08 \right) =$$

$$= -1.05 - 1.43i$$

3. $f(x) = \ln\left(\frac{x+2}{1-x}\right)$

$D(f) = \langle -2, 1 \rangle$ ✓ 6

ASIMPTOTE?

4. $g(x) = \cos(\sin(2x))$

$D(f) = \mathbb{R}$ ✓

→ periodična Period π ✓

$g'(x) = \begin{cases} g(x) = \sin(2x), & g'(x) = \cos 2x \\ f(x) = \cos x, & f'(x) = -\sin x \end{cases} = -\sin(\sin 2x) \cdot \cos 2x$ ✗

$= -\sin(\sin(2x)) \cdot \cos(2x) \cdot 2$

$f(x) = f(x)$

$f(-x) = \cos(\sin(-2x)) \neq f(x)$ NIJE PARNA NIJE NEPARNA ✓

$-f(x) = -\cos(\sin(2x)) \neq f(-x)$

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~~6~~

$$3.) f(x) = \ln\left(\frac{x+2}{1-x}\right)$$

$$\frac{x+2}{1-x} > 0$$

$$x+2 > 0 \quad 1-x > 0$$

$$x+2 = 0 \quad 1-x = 0$$

$$x = -2 \quad x = 1$$

~~D(f) = (-2, 1)~~

$$D(\ln) = [0, +\infty)$$

$$\boxed{\text{D}(f) = (-2, 1)}$$

$$\text{D}(f) = \langle -2, +\infty \rangle \setminus \{1\}$$

$$\text{D}(f) = \langle -2, 1 \rangle \checkmark$$

V.A.

$$x = 0$$

H.A.

$$\lim_{x \rightarrow \infty} \frac{f(x)}{x} = \ln$$

	$-\infty$	-2	1	$+\infty$
$x+2$	-	+	+	
$1-x$	+	+	-	
$\frac{x+2}{1-x}$	-	(+)	-	

$$\text{D}(f) = \langle -2, 1 \rangle$$

$$1.) \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & -1 & 6 & 0 & -2 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & -1 & 6 & 0 & -2 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & -3 & -1 & 6 & 0 & -2 & 0 \end{array} \right] \cdot (-\frac{1}{3}) \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & -3 & -1 & 6 & 0 & -2 & 0 \end{array} \right] \cdot (-\frac{1}{3})$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \end{array} \right] \sim AA = I \quad \checkmark$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 & 0 & \frac{2}{3} \end{array} \right] \checkmark$$

$$-\frac{4}{3} + \frac{3}{3} = \frac{-4+3}{3} = -\frac{1}{3}$$

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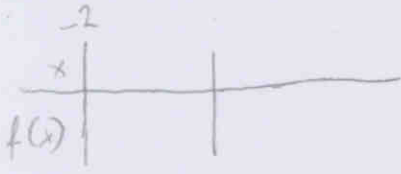
TREBA JOŠ PROVERITI:

$$\begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \frac{2}{3} & 0 & 0 & -\frac{1}{3} \\ 0 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & 0 & 0 & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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5)



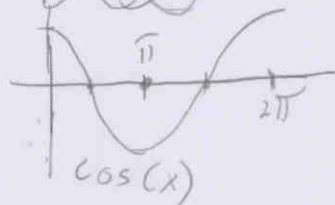
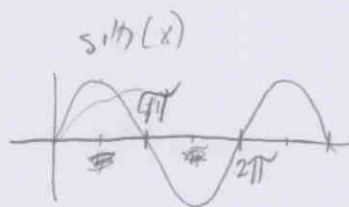
4.) $g(x) = \cos(\sin(2x))$

$D_f = \mathbb{R}$ ✓

$g'(x) = (\cos(\sin(2x)))'$

~~$= -\sin(\sin(2x)) + \cos(\cos(2x)) \cdot 2$~~

~~$= -\sin^2(2x) + 2\cos^2(2x)$~~



Periodičnost

Parnost

$g(x) + P = g(x)$

$g(x+P) = g(x)$

$g(-x) = g(x)$

$P = 2\pi?$

$g(-x) = -g(x)$

PERIODIČNOST:

$\sin(x+2\pi) = \sin x$

$\sin(2x+2\pi) = \sin(2x)$

$\sin(2(x+\pi)) = \sin(2x)$

$g(x+\pi) = g(x)$

$P = \pi$

$g'(x) = -\sin(\sin(2x)) \cdot \cos(2x) \cdot 2$ ✓

$g'(x) = -\sin^2(2x) \cdot 2\cos(2x)$ ✗

$\sin^2(2x) = \sin(2x) \cdot \sin(2x)$

✗ $\sin(\sin(2x))$

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