IME I PREZIME: IVAN VUKIĆ BROJ INDEKSA: 55709 $\emptyset$

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: DATUM OD DO .

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. ooxx

1. X je zadan kao trokut s vrhovima $O(0,0)$, $A(-1,2)$ i $C(2,-1)$. Skicirati taj trokut i izračunati dvostruki integral
$$\iint\limits_X xy\,\mathrm{d}x\mathrm{d}y$$

2. Neka je X dio kugle $x^2+y^2+z^2=16$ za koji vrijedi $z\le 2$. Označimo sa ∂X rub od X. Izračunati plošni integral
$$\iint\limits_{\widehat{\partial K}} x\,\mathrm{d}y\mathrm{d}z + z\,\mathrm{d}x\mathrm{d}z + y\,\mathrm{d}x\mathrm{d}y$$

3. Izračunati: $\int_{\widehat{\Gamma}}(\mathbf{w}|\mathrm{d}\mathbf{r})$, ako je $\mathbf{w}(x,y,z)=(y,z,x)$ i krivulja $\widehat{\Gamma}=\left\{(x,y,z)\,|\,x=\frac{1}{2}\cos t,\ y=\frac{1}{2}\sin t,\ z=\frac{\sqrt{3}}{2},\ t\in[0,\pi]\right\}$.

4. Izračunati
$$\int\limits_{(2,2)}^{(1,1)} (y^2+2xy)\,\mathrm{d}x + (2xy+x^2)\,\mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:
$$y'''(t)-2y''(t)=\mathrm{e}^t,\quad y(0)=y''(0)=1,\ y'(0)=1.$$

IME I PREZIME: IVAN VUKIĆ BROJ INDEKSA: 55709

1. $\iint\limits_{X} xy \, dx dy$

IME I PREZIME: Jure Mažić BROJ INDEKSA: 52915-2005 (10)

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: DATUM OD DO .

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. ooxx

1. X je zadan kao trokut s vrhovima $O(0,0)$, $A(-1,2)$ i $C(2,-1)$. Skicirati taj trokut i izračunati dvostruki integral

$$\iint\limits_X xy\,\mathrm{d}x\mathrm{d}y$$

Ø

2. Neka je X dio kugle $x^2+y^2+z^2=16$ za koji vrijedi $z\le 2$. Označimo sa ∂X rub od X. Izračunati plošni integral

$$\iint\limits_{\widehat{\partial K}} x\,\mathrm{d}y\mathrm{d}z + z\,\mathrm{d}x\mathrm{d}z + y\,\mathrm{d}x\mathrm{d}y$$

3. Izračunati: $\int_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$, ako je $\mathbf{w}(x,y,z)=(y,z,x)$ i krivulja $\widehat{\Gamma}=\left\{(x,y,z)\,|\,x=\frac{1}{2}\cos t,\ y=\frac{1}{2}\sin t,\ z=\frac{\sqrt{3}}{2},\ t\in[0,\pi]\right\}$.

4. Izračunati

$$\int\limits_{(2,2)}^{(1,1)} (y^2+2xy)\,\mathrm{d}x + (2xy+x^2)\,\mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t)-2y''(t)=\mathrm{e}^t,\quad y(0)=y''(0)=1,\ y'(0)=1.$$

10

1) $\iint xy\,dxdy$

$O(0,0)\quad A(-1,2)\quad C(2,-1)$

$$P_1=\int_0^2 y\,dy\int_{-\frac{1}{2}y}^{-y+1} x\,dx=\int_0^2 y\,dy\,(x)\Big|_{-\frac{1}{2}y}^{-y+1}=\int_0^2 y\left(\frac{x^2}{2}\right)\Big|_{-\frac{1}{2}y}^{-y+1}dy$$

$$=\int_0^2 y\left(-y+1+\frac{1}{2}y\right)dy=\int_0^2 y\left(-\frac{1}{2}y+1\right)dy$$

$$=\int_0^2\left(-\frac{1}{2}y^2+y\right)dy$$

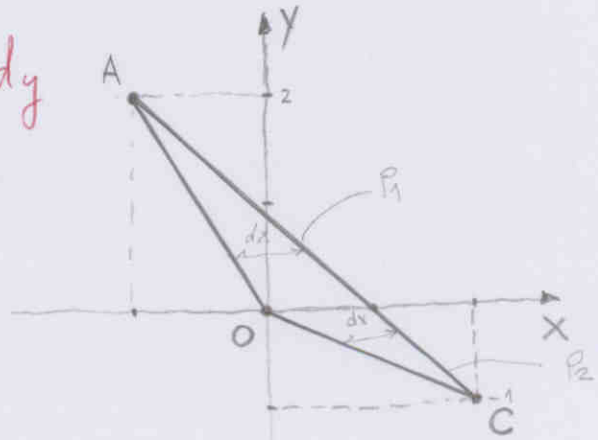
$$=\left(-\frac{1}{2}\cdot 2y+1\right)\Big|_0^2$$

$$=-y+1\Big|_0^2$$

$$=-2$$

$P_2=?$

Ø



$\overline{OA}\Rightarrow y-0=\frac{2-0}{-1}(x-0)\Rightarrow y=-2x\Rightarrow x=-\frac{1}{2}y$

$\overline{AC}\Rightarrow y-2=\frac{-1-2}{2+1}(x+1)\Rightarrow y-2=-x-1\Rightarrow y=-x+1$

$x=-y+1$

$\overline{OC}\Rightarrow y-0=\frac{-1-0}{2-0}(x-0)\Rightarrow y=-\frac{1}{2}x$

$x=-2y$

Ime i prezime: Jure Mazić Broj indeksa: 52915-2005

5) $y'''(t)-2y''(t)=e^t$

$y(0)=1$
$y'(0)=1$
$y''(0)=1$

$$s^3Y(s)-s^2\cdot y(0)-sy'(0)-y''(0)-2\left(s^2Y(s)-s\cdot y(0)-y'(0)\right)=\frac{1}{s-1}$$

$$s^3Y(s)-s^2-s-1-2\left(s^2Y(s)-s-1\right)=\frac{1}{s-1}$$

$$\cancel{s^3Y(s)}-s^2-s-1-\cancel{2s^2Y(s)}+2s+2=\frac{1}{s-1}$$

$$Y(s)\left(s^3-2s^2\right)=\frac{1}{s-1}+s^2-s-1$$

$$Y(s)\left(s^3-2s^2\right)=\frac{1+s^3-s^2-\cancel{s}-s^2+\cancel{s}+1}{s-1}$$

$$Y(s)\left(s^3-2s^2\right)=\frac{s^3-2s^2+2}{s-1}$$

$$Y(s)=\frac{\frac{s^3-2s^2+2}{s-1}}{s^3-2s^2}=\frac{\frac{s^3-2s^2+2}{s-1}}{\frac{s^2(s-2)}{1}}=\frac{s^3-2s^2+2}{s^2(s-1)(s-2)}$$

$$Y(s)=\frac{s^3-2s^2+2}{s^4-2s^3-s^3+2s^2}$$

$$Y(s)=\frac{s^3-2s^2+2}{s^4-3s^3+2s^2}$$

$$Y(s)=\frac{s^3-2s^2+2}{s^2(s^2-3s+2)}$$

$$\frac{s^3-2s^2+2}{s^2(s^2-3s+2)}=\frac{A}{s^2}+\frac{B}{s}+\frac{Cs+D}{s^2-3s+2}$$

$$=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s-1}+\frac{D}{s-2}$$

$A=-2$
$B=0$
$C+D=2 \Rightarrow D=2$
$C=0$

$$y(t)=-2\underbrace{\mathcal{L}^{-1}\left(\frac{1}{s^2}\right)}_{=\,?}+2\mathcal{L}^{-1}\left(\frac{1}{\quad}\right.$$

$$s_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{3\pm\sqrt{9-8}}{2}$$

$$s_{1,2}=\frac{3\pm1}{2}\qquad \boxed{s_1=2}\quad\boxed{s_2=1}$$

$$\Rightarrow s^2-3s+2=(s-1)(s-2)$$

10

55 708 – 2006

IME I PREZIME: MARKO ŠARIN BROJ INDEKSA: 0269016121

$\emptyset$

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: DATUM OD DO .

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. ooxx

1. X je zadan kao trokut s vrhovima $O(0,0)$, $A(-1,2)$ i $C(2,-1)$. Skicirati taj trokut i izračunati dvostruki integral
$$\iint\limits_X xy\,\mathrm{d}x\mathrm{d}y$$

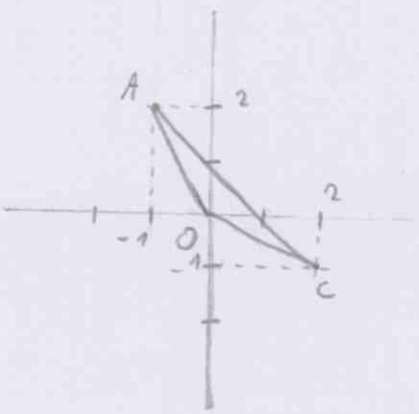
2. Neka je X dio kugle $x^2+y^2+z^2=16$ za koji vrijedi $z\le 2$. Označimo sa ∂X rub od X. Izračunati plošni integral
$$\iint\limits_{\widehat{\partial K}} x\,\mathrm{d}y\mathrm{d}z + z\,\mathrm{d}x\mathrm{d}z + y\,\mathrm{d}x\mathrm{d}y$$

3. Izračunati: $\int_{\widehat{\Gamma}}(\mathbf{w}|\mathrm{d}\mathbf{r})$, ako je $\mathbf{w}(x,y,z)=(y,z,x)$ i krivulja $\widehat{\Gamma}=\left\{(x,y,z)\,|\,x=\frac{1}{2}\cos t,\ y=\frac{1}{2}\sin t,\ z=\frac{\sqrt{3}}{2},\ t\in[0,\pi]\right\}$.

4. Izračunati
$$\int\limits_{(2,2)}^{(1,1)}(y^2+2xy)\,\mathrm{d}x+(2xy+x^2)\,\mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:
$$y'''(t)-2y''(t)=\mathrm{e}^t,\quad y(0)=y''(0)=1,\ y'(0)=1.$$

1



Ime i prezime: NAN KERO **Broj indeksa:**

Obavezno popuniti vrijeme rješavanja ispita: datum OD DO .

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik ooxx o stegovnoj odgovornosti studenata.

$\not{0}$

1. X je zadan kao trokut s vrhovima $O(0,0)$, $A(-1,2)$ i $C(2,-1)$. Skicirati taj trokut i izračunati dvostruki integral

$$\iint\limits_X xy \, \mathrm{d}x\mathrm{d}y$$

$\not{0}$

2. Neka je X dio kugle $x^2+y^2+z^2=16$ za koji vrijedi $z \leq 2$. Označimo sa ∂X rub od X. Izračunati plošni integral

$$\iint\limits_{\widehat{\partial K}} x \, \mathrm{d}y\mathrm{d}z + z \, \mathrm{d}x\mathrm{d}z + y \, \mathrm{d}x\mathrm{d}y$$

3. Izračunati: $\int_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$, ako je $\mathbf{w}(x,y,z)=(y,z,x)$ i krivulja $\widehat{\Gamma}=\left\{(x,y,z) \mid x=\frac{1}{2}\cos t,\ y=\frac{1}{2}\sin t,\ z=\frac{\sqrt{3}}{2},\ t\in[0,\pi]\right\}$.

4. Izračunati

$$\int\limits_{(2,2)}^{(1,1)} \left(y^2+2xy\right) \mathrm{d}x + \left(2xy+x^2\right) \mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t)-2y''(t)=\mathrm{e}^t, \quad y(0)=y''(0)=1,\ y'(0)=1.$$

$\not{0}$

Ime i prezime: IVAN KERO

Broj indeksa: 56434-0269029457

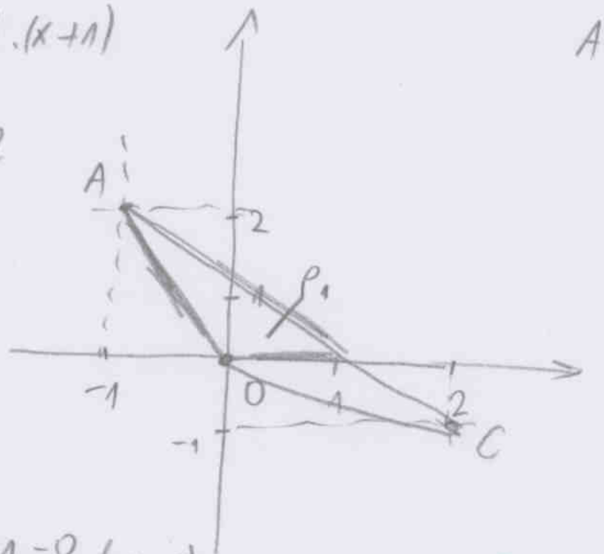
1.

$O(0,0)$, $A(-1,2)$, $C(2,-1)$

$AO = y-2 = \frac{0-2}{0+1}\cdot(x+1)$

$AO = y-2 = -2x-2$

$\boxed{y=-2x}$



$AC = y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-2 = \frac{-1-2}{2+1}\cdot(x+1)$

$y-2 = -\frac{3}{3}(x+1)$

$y-2 = -x-1$

$\boxed{y=-x+1} = AC$

$OC = y-0 = \frac{-1-0}{2-0}(x-0)$

$\boxed{y = -\frac{1}{2}x}$

$P_1 \ldots\ldots \int_{-1}^{0 \to 1}\int_{-2x}^{-x+1} xy\,dx\,dy = \int_{-1}^{1}(-x+1)+2x$

$= \int_{-1}^{0} x\left(\frac{(-x+1)^2}{2} - \frac{(-2x)^2}{2}\right)dx$

$P_1 = \int_{-1}^{1} -x+1 = 2$ //

$P_2 = \int_0^2 \int_{-\frac{1}{2}x}^{-x+1} xy\,dx\,dy = \int_0^2 (-x+1)+\frac{1}{2}x\,dx\,dy = \int_0^2 -\frac{1}{2}x+1$

$= \int_0^2 x\left(\frac{(-x+1)^2}{2} - \frac{(-\frac{1}{2}x)^2}{2}\right)dx$

$P_2 = -\frac{1}{2}\cdot 2+1-\left(-\frac{1}{2}\cdot 0+1\right)$

$P_2 = -1+1-0+1$

$P_2 = 1$

$\boxed{P_1+P_2 = 3}$

~~0~~

IME I PREZIME: IVAN KERO

BROJ INDEKSA: 56434

(5)

$$y'''(t) - 2y''(t) = e^t \qquad y(0) = y''(0) = 1 \qquad y'(0) = 1$$

$$s^3 \cdot y(s) - s^2 \cdot y(0) - s \cdot y'(0) - y''(0) - 2\left(s^2 \cdot y(s) - s \cdot y'(0) - y''(0)\right)$$

$$= e^{\frac{1}{s^2}}$$

$$= s^3 \cdot y(s) - s^2 \cdot 1 - s \cdot 1 - 1 - 2\left(s^2 \cdot y(s) - s \cdot y'(0) - y'''(0)\right)$$

$$= s^3 \cdot y(s) - s^2 - s - 1 - 2 \cdot s^2 y(s) + 2s \cdot 1 + 2 \cdot 1$$

$$= s^3 \cdot y(s) - s^2 - s - 1 - 2s^2 y(s) + 2s + 2$$

$$= s^3 \cdot y(s) - 2s^2 y(s) - s^2 + s + 1 = 0$$

$$s^3 y(s) - 2s^2 y(s) = s^2 - s - 1 \quad / : s^3$$

$$y(s) - \frac{2 \cdot y(s)}{s} = \frac{1}{s} - \frac{1}{s^2} - \frac{1}{s^3}$$

$$y(s) = \frac{2 \cdot y(s)}{s} + 1 - t - \frac{1}{2} t^2$$

$$\frac{2 \cdot y(s)}{s} = 2 \int_0^t f(r) \cdot dr \; ?$$

$$\boxed{y(s) = 2 \cdot \int_0^t f(r)\, dr + 1 - t - \frac{1}{2} t^2}$$

VIDI KARLO PERKOVIĆ
ISPIT OD 2011-02-14

IME I PREZIME: ROKO TANFARA BROJ INDEKSA:

OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: DATUM OD DO .

MATEMATIKA 3: Trajanje 100 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. ooxx

1. X je zadan kao trokut s vrhovima $O(0,0)$, $A(-1,2)$ i $C(2,-1)$. Skicirati taj trokut i izračunati dvostruki integral
$$\iint\limits_X xy\,\mathrm{d}x\mathrm{d}y$$
$\cancel{0}$

2. Neka je X dio kugle $x^2+y^2+z^2=16$ za koji vrijedi $z \leq 2$. Označimo sa ∂X rub od X. Izračunati plošni integral
$$\iint\limits_{\widehat{\partial K}} x\,\mathrm{d}y\mathrm{d}z + z\,\mathrm{d}x\mathrm{d}z + y\,\mathrm{d}x\mathrm{d}y$$
$\cancel{0}$

3. Izračunati: $\int_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$, ako je $\mathbf{w}(x,y,z) = (y,z,x)$ i krivulja $\widehat{\Gamma} = \left\{(x,y,z) \,|\, x = \frac{1}{2}\cos t,\ y = \frac{1}{2}\sin t,\ z = \frac{\sqrt{3}}{2},\ t \in [0,\pi]\right\}$.

4. Izračunati
$$\int\limits_{(2,2)}^{(1,1)} (y^2+2xy)\,\mathrm{d}x + (2xy+x^2)\,\mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:
$$y'''(t) - 2y''(t) = \mathrm{e}^t, \quad y(0) = y''(0) = 1,\ y'(0) = 1.$$
$\cancel{0}$

IME I PREZIME: ROKO TANFARA

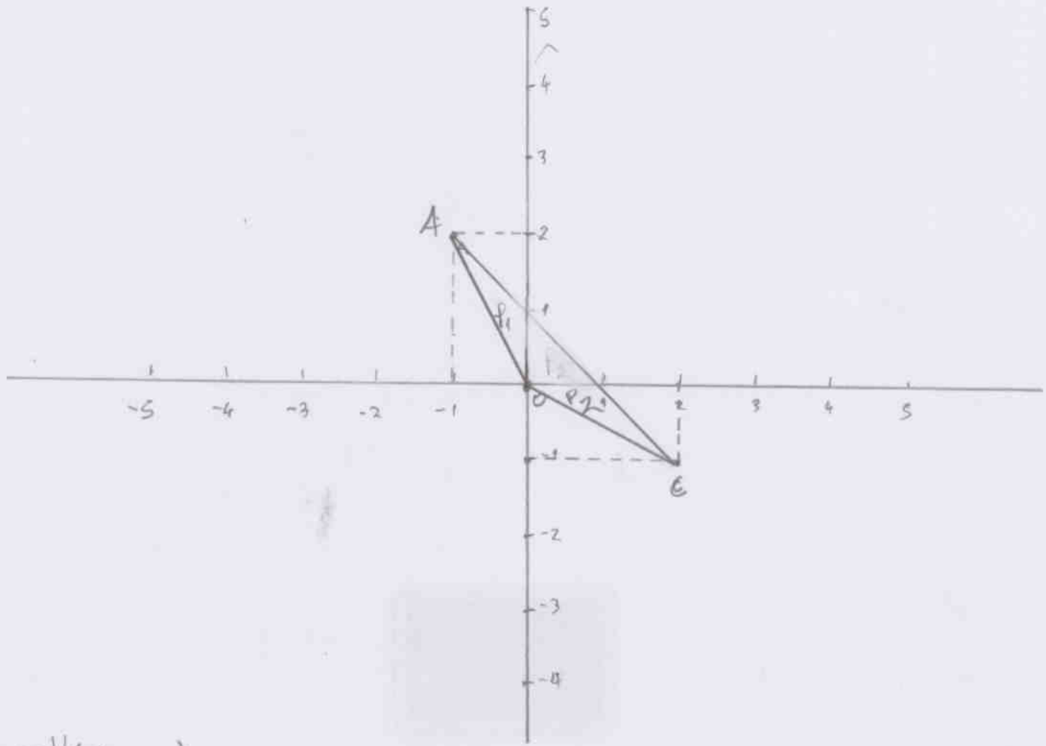
BROJ INDEKSA: 56501-2008

0067414586

$O = (0,0)$

$A = (-1,2)$

$C = (2,-1)$



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

(O,A) $y - 0 = \frac{2-0}{-1-0} \cdot x - 0$

$$y = \frac{2}{-1} x$$

$$y = -2x$$

(O,C) $y - 0 = \frac{-1-0}{2-0}(x - 0)$

$$y = -\frac{1}{2}x$$

(A,C) $y - 2 = \frac{-1-2}{2+1}(x+1)$

$$y - 2 = -\frac{3}{3}(x+1)$$

$$y = -x - 1 + 2$$

$$y = 1 - x$$

$$\int_{-1}^{0}\int_{-2x}^{1-x} xy\,dx\,dy = \int_{-1}^{0} x\left(\frac{y^2}{2}\right)\Bigg|_{-2x}^{1-x} dx$$

$$= \int_{-1}^{0} x\left(\frac{(1-x)^2}{2} - \frac{(-2x)^2}{2}\right) dx$$

$1 - x + 2x$

$1 + x$

$$= \int_0^1 x\big((1-x) + 2x\big)\,dx$$

$$= \int_0^1 x(1+x)\,dx = \int_0^1 (x + x^2)\,dx \Big|_0^1$$

$$= (1 + 1^2) - 0 + 0^2$$

$P_1 = 2$

$(1+1) - (1+0)$

$2 - 1 = 1$

IME I PREZIME: ROKO TANFARA

BROJ INDEKSA: 56501-2008
0067414586

P_2

$$\int_0^1 \int_{-\frac{1}{2}x}^{1-x} xy\,dx\,dy = \int_0^1 x\left((1-x)-\left(-\frac{1}{2}x\right)\right)dx$$

o.K.

$$= \int_0^2 x\left(\frac{(1-x)^2}{2} - \frac{\left(-\frac{1}{2}x\right)^2}{2}\right)dx$$

$$= \int_0^1 x\left(1-x+\frac{1}{2}x\right)dx = \int_0^1 x\left(1-\frac{1}{2}x\right)dx$$

$$= \int_0^1 \left(x-\frac{1}{2}x^2\right)dx \Big|_0^1 = \left(1-\frac{1}{2}\cdot 1\right) - 0 - \frac{1}{2}\cdot 0^2$$

$$= 1-\frac{1}{2} = \frac{1}{2}$$

$$P_1 + P_2 = 2 + \frac{1}{2} = \frac{5}{2}$$

$\varnothing$

(2) $x^2+y^2+z^2=16 \qquad z \leq 2$

?

IME I PREZIME: ROKO TANFARA

BROJ INDEKSA: 56501-2008

0067414586

(5) $y'''(t) - 2y''(t) = e^t$

$y(0) = 1$

$y''(0) = 1$

$y'(0) = 1$

$$\left(s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0)\right) - 2\left(s^2 Y(s) - s y(0) - y'(0)\right) = \frac{1}{s-1}$$

$$Y(s)\left(s^3 - s^2 - s - 1 - 2s^2 + 2s + 2\right) = \frac{1}{s-1}$$

$$Y(s^3 - s^2 - s - 2s^2 + 2s) = \frac{1}{s-1} - 1$$

$$s^3 - 3s^2 + s = \frac{1}{s-1} - 1$$

?