OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO .

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. 000

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IME I PREZIME: ANTE BARIČEVIĆ BROJ INDEKSA: 0035160783

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$x'''(t) + 4x'(t) = 0, \quad x(0) = x''(0) = 2,\ x'(0) = 0.$$

2. Neka je C cilindar zadan sa $C = \{(x, y, z) : (x + 1)^2 + (y - 2)^2 \leq 1,\ 0 \leq z \leq 3\}$. Izračunati plošni integral 20

$$\iint_{\widehat{\partial C}} 2xyz\,\mathrm{d}y\mathrm{d}z + (2y + z)\,\mathrm{d}x\mathrm{d}z - yz^2\,\mathrm{d}x\mathrm{d}y$$

3. Zadana je krivulja s parametrizacijom $x = t^2$, $y = t^3$ i 6. Izračunati duljinu krivulje između točaka $A(1, 1, 6)$ i $B(4, 8, 6)$. 20

4. Zadan je dio stošca (oznaka Y) omeđen plohama $x^2 + y^2 = (2z)^2$, $z = 2$ i $z = 3$. Izračunati $\int_Y x\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$ prijelazom na cilindrične koordinate. 20

5. Izračunati $\int_{\widehat{ABC}} y^2\mathrm{d}y + x^2\mathrm{d}z$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(2, 0, 0)$, $B(0, 2, 0)$, $C(0, 0, 0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

1. $x'''(t) + 4x'(t) = 0 \qquad x(0) = x''(0) = 2, \quad x'(0) = 0$

$x'''(t) \Rightarrow s^3 X(s) - s^2 x(0) - s x'(0) - x''(0) = s^3 X(s) - 2s^2 - 2$

$x'(t) \Rightarrow s X(s) - x(0) = s X(s) - 2$

$s^3 X(s) - 2s^2 - 2 + 4(s X(s) - 2) = \emptyset$

$s^3 X(s) + 4s X(s) - 8 = 2s^2 + 2$

$X(s)(s^3 + 4s) = 2s^2 + 10$

$$X(s) = \frac{2s^2 + 10}{s^3 + 4s} = \frac{2s^2 + 10}{s(s^2 + 4)}$$ ✓

$$\frac{2s^2 + 10}{s(s^2 + 4)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4} \quad / \cdot s(s^2 + 4)$$

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① NASTAVAK

$$2s^2+10 = A(s^2+4) + (Bs+C)\cdot s$$

$$2s^2+10 = As^2+4A + Bs^2 + Cs$$

$$2s^2+10 = (A+B)s^2 + Cs + 4A$$

$$A+B = 10$$

$$C = 0$$

$$4A = 10 \Rightarrow \boxed{A = \frac{5}{2}}$$

$$B = 10 - \frac{5}{2}$$

$$\boxed{B = \frac{15}{2}} \times$$

$$\frac{5}{2}\cdot\frac{1}{s} + \frac{15}{2}\,\frac{s}{s^2+4} = \frac{5s^2+20+15s^2}{2s(s^2+4)} = \frac{20s^2+20}{2s(s^2+4)}$$

$$= \frac{10(s^2+1)}{s(s^2+4)}$$

$$X(s) = \frac{\frac{5}{2}}{s} + \frac{\frac{15}{2}\cdot s}{s^2+4} = \frac{5}{2}\cdot\frac{1}{s} + \frac{15}{2}\cdot\frac{s}{s^2+4}$$

$$=$$

$$\frac{15}{}$$

$$x(t) = \frac{5}{2} + \frac{15}{2}\cdot\cos(2t) = \frac{5}{2}\left(1 + 3\cos(2t)\right)$$

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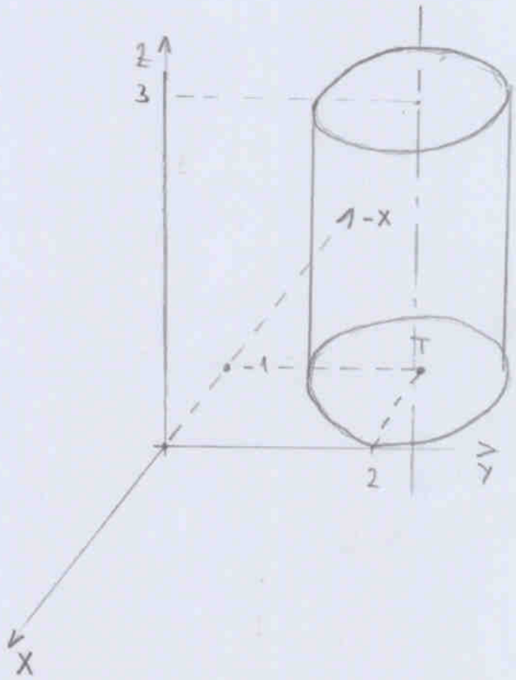
$$C = \left\{ (x,y,z) : \ (x+1)^2 + (y-2)^2 \le 1 \ , \ 0 \le z \le 3 \right\}$$

$$\iint_{\partial C} 2xyz \, dy dz + (2y+z) dx dz - yz^2 dx dy = *$$

$$w = \begin{bmatrix} 2xyz \\ 2y+z \\ -yz^2 \end{bmatrix} \Rightarrow \operatorname{div} w = \cancel{2yz} + 2 - \cancel{2yz} = 2$$



$x_1 = r\cos\varphi$

$y_1 = r\sin\varphi$

$z = z$

$x = x_1 - 1 = r\cos\varphi - 1$

$y = y_1 + 2 = r\sin\varphi + 2$

$$(r\cos\varphi - 1 + 1)^2 + (r\sin\varphi + 2 - 2)^2 \le 1$$

$$r^2(\cos^2\varphi + \sin^2\varphi) = 1$$

$$r = \pm 1$$

$\varphi \in [0, 2\pi]$, $r \in [0,1]$, $z \in [0,3]$

$$* = \int_0^{2\pi}\int_0^3\int_0^1 2r \, dr d\varphi dz = 4\pi \int_0^3 \frac{r^2}{2}\Big|_0^1 dz = 4\pi \cdot \frac{1}{2} z \Big|_0^3 = 2\pi \cdot 3 = 6\pi$$

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(3) $x=t^2,\ y=t^3,\ z=6$

$A(1,1,6)\ ;\ B(4,8,6)$

$$\|r'\| = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2} = \sqrt{4t^2+9t^4} = t\sqrt{4+9t^2}$$

$$L=\int_a^b \|r'\|\,dt$$

$$\begin{bmatrix}1\\1\\6\end{bmatrix}=\begin{bmatrix}t^2\\t^3\\6\end{bmatrix} \Rightarrow t_1=1 \qquad \begin{bmatrix}4\\8\\6\end{bmatrix}=\begin{bmatrix}t^2\\t^3\\6\end{bmatrix} \Rightarrow t_2=2$$

$$L=\int_1^2 t\sqrt{4+9t^2}\,dt = \left\{\begin{matrix} z=4+9t^2 \\ dz=18t\,dt \\ dt=\frac{dz}{18t}\end{matrix}\right\} = \int_1^2 \cancel{t}\cdot\sqrt{z}\cdot\frac{dz}{18\cancel{t}} = \frac{1}{18}\int_1^2\sqrt{z}\,dz =$$

$$=\frac{1}{18}\cdot\int_1^2 z^{\frac{1}{2}}dz = \frac{1}{\cancel{18}}\cdot\frac{\cancel{2}}{3}z\sqrt{z}\Big|_1^2 = \frac{1}{27}\cdot\Big[(4+9t^2)\sqrt{4+9t^2}\Big]_1^2 =$$

$$=\frac{1}{27}\left((4+36)\sqrt{4+36}-(4+9)\sqrt{4+9}\right) = \frac{1}{27}\left(40\sqrt{40}-13\sqrt{13}\right) =$$

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$$=\frac{1}{27}\cdot 40\cdot 2\sqrt{10}-\frac{1}{27}\cdot 13\sqrt{13} = \frac{80}{27}\sqrt{10}-\frac{13}{27}\sqrt{13} \approx 7{,}63$$

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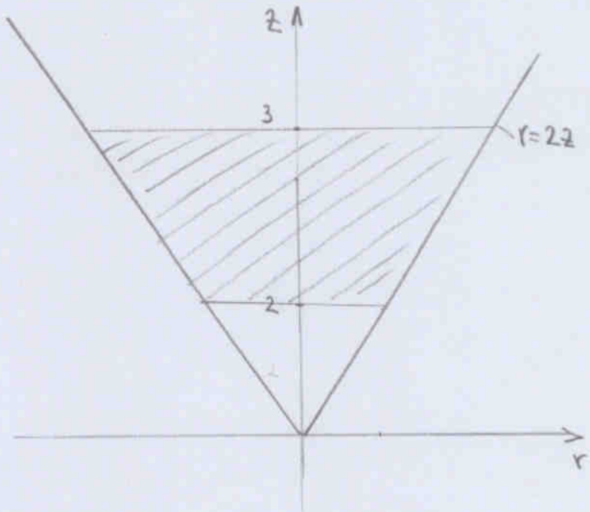
BROJ INDEKSA: 0035160783

④ $x^2+y^2=(2z)^2 \Rightarrow r^2=4z^2$

$z=2$ $\qquad r=\pm\sqrt{4z^2}=\pm 2z$

$z=3$



$$\int_Y x\,dx\,dy\,dz = *$$

$x=r\cos\varphi$ $\qquad \varphi\in[0,2\pi]$

$y=r\sin\varphi$ $\qquad r\in[0,2z]$

$z=z$ $\qquad z\in[2,3]$

$$* = \int_0^{2\pi}\int_2^3\int_0^{2z} r\cos\varphi\, r\,dr\,d\varphi\,dz = \int_0^{2\pi}\cos\varphi\,d\varphi\int_2^3 \frac{r^3}{3}\Big|_0^{2z}dz = \int_0^{2\pi}\cos\varphi\cdot\left(\frac{8}{3}\frac{z^4}{4}\right)\Big|_2^3 d\varphi =$$

$$= \int_0^{2\pi}\cos\varphi\left(\frac{8}{3}\cdot\frac{81}{4}-\frac{8}{3}\cdot\frac{16}{4}\right)d\varphi = \frac{130}{3}\sin\varphi\Big|_0^{2\pi} = \frac{130}{3}(\sin 2\pi - \sin 0) = \phi$$

✓

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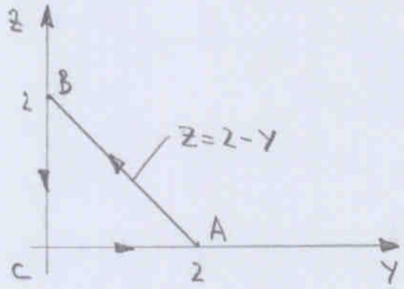
$$\int_{\overrightarrow{ABC}} y^2 dy + x^2 dz = *$$ $A(2,0,0),\ B(0,2,0),\ C(0,0,0)$



$$w = \begin{bmatrix} 0 \\ y^2 \\ x^2 \end{bmatrix} \Rightarrow \operatorname{rot} w = \nabla \times w = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \times \begin{bmatrix} 0 \\ y^2 \\ x^2 \end{bmatrix}$$

$$\operatorname{rot} w = \begin{bmatrix} 0-0 \\ 0-2x \\ 0-0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2x \\ 0 \end{bmatrix}$$ ✓

$$\vec{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ ✓

$$* = \int_0^2 \int_0^{2-y} \begin{bmatrix} 0 \\ -2x \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} dx\,dy = \emptyset$$ ✓

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