OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO .

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE <u>JEDNOSTRANO</u> NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. oox

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IME I PREZIME: IVAN BULOVAC **BROJ INDEKSA:** 54629

1. Odrediti duljinu krivulje s parametrizacijom $x = t$, $y = t^{3/2}$ i $z = t$ između točaka $A(0,0,0)$ i $B(1,1,1)$. 20

2. Izračunati $\iint\limits_{\partial K} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2 \\ x^2 \\ x^2 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(1,1,1)$, a koji je orijentiran vanjskom normalom. $\cancel{0}$

3. Izračunati volumen tijela omeđenog plohama: $2z = x^2 + y^2$, $z = 1$. 20

4. Neka je točkama $A(0,3)$, $B(3,0)$ i $C(2,2)$ dan trokut ABC i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral $\cancel{0}$

$$\oint\limits_C x^2 \mathrm{d}x + y^2 \mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$f'''(t) - 4\,f'(t) = \sin(2t)\,, \quad f(0) = f'(0) = f''(0) = 0.$$

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1. $x=t$, $y=t^{\frac{3}{2}}$, $z=t$ između $A(0,0,0)$ i $B(1,1,1)$

za $A \Rightarrow \boxed{t=0}$ za $B \Rightarrow \boxed{t=1}$

$$L = \int_0^1 \sqrt{\left(\frac{\partial x}{\partial t}\right)^2+\left(\frac{\partial y}{\partial t}\right)^2+\left(\frac{\partial z}{\partial t}\right)^2}\,dt$$

$$= \int_0^1 \sqrt{(1)^2+\left(\frac{3}{2}t^{\frac{1}{2}}\right)^2+(1)^2}\,dt = \int_0^1 \sqrt{1+\frac{9}{4}t+1}\,dt$$ ✓

$$= \int_0^1 \sqrt{2+\frac{9}{4}t}\,dt = \left| \begin{array}{l} 2+\frac{9}{4}t = x \\ \frac{9}{4}dt = dx \end{array} \right| = \int_2^{2+\frac{9}{4}} \sqrt{x}\cdot\frac{4}{9}\,dx$$

$$= \frac{4}{9}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_2^{\frac{17}{4}} = \frac{8}{27}\cdot\left(\frac{17}{4}\right)^{\frac{3}{2}} - \frac{8}{27}\cdot(2)^{\frac{3}{2}}$$

$$\cancel{\frac{8}{27}\cdot\frac{17}{4}\cdot\frac{\sqrt{17}}{2} - \frac{17\sqrt{17}}{27}}$$

$$L = \frac{8}{27}\left[\frac{17\sqrt{17}}{8} - \sqrt{8}\right]$$ ✓ 20

$$L = \frac{17\sqrt{17}-8\sqrt{8}}{27} = \frac{17\sqrt{17}-16\sqrt{2}}{27}$$

2. $\operatorname{div} F = \begin{bmatrix} 2x \\ 0 \\ 0 \end{bmatrix}$ $(x-1)^2+(y-1)^2+(z-1)^2=1$ 0

$$\iint_{\partial K} F\cdot dS = \int_0^2 dx \int_0^2 dy \int_0^2 (2x)\,dz = 4\cdot\int_0^2 2x\,dx = 4\cdot x^2\Big|_0^2 = \underline{16}$$ ×

$$\iint_{\partial K} F\cdot dS = \iiint_K 2x\,dx\,dy\,dz \overset{\boxed{\begin{array}{l} x=x_1+1 \\ y=y_1+1 \\ z=z_1+1 \end{array}}}{=} \iiint_{\{x_1^2+y_1^2+z_1^2=1\}} 2(x_1+1)\,dx_1\,dy_1\,dz_1 \overset{\boxed{\begin{array}{l} x_1=r\cos\varphi \\ y_1=r\sin\varphi \\ z_1=z_1 \end{array}}}{=} \int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} 2(r\cos\varphi+1)\,r\,dz\,dr\,d\varphi$$

$$K = \{(x,y,z) : (x-1)^2+(y-1)^2+(z-1)^2=1\}$$

$$= \underbrace{\int_0^{2\pi}\cos\varphi \int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} r^2\,dz\,dr\,d\varphi}_{=0} + 2\cdot 2\pi\cdot 2\int_0^1 r\sqrt{1-r^2}\,dr$$

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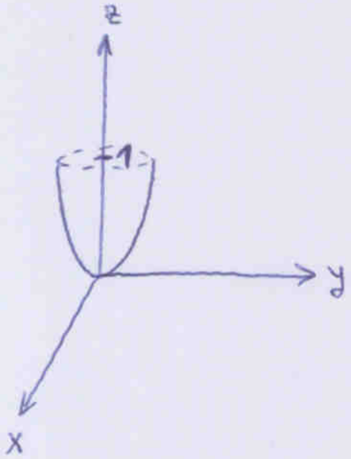
3. $x^2+y^2=2z \quad , \quad z=1$



CILINDRIČNE KOORDINATE:

$$x = r\cos\varphi$$
$$y = r\sin\varphi \qquad dx\,dy\,dz = r\,dr\,d\varphi\,dz$$
$$z = z$$

$$V = \int_0^1 dz \int_0^{2\pi} d\varphi \int_0^{\sqrt{2z}} r\,dr = 2\pi \cdot \int_0^1 dz \int_0^{\sqrt{2z}} r\,dr$$

$$= 2\pi \cdot \int_0^1 \left(\frac{r^2}{2}\right)\Big|_{r=0}^{\sqrt{2z}} dz = 2\pi \cdot \int_0^1 \frac{2z}{2}\,dz = 2\pi\, \frac{z^2}{2}\Big|_0^1$$

$$= \pi$$

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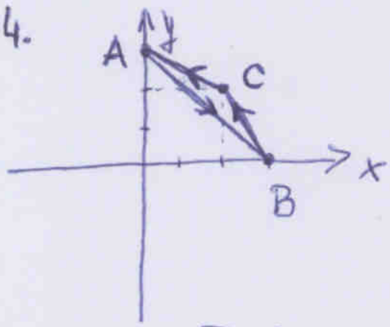
4.



STOKES

$$\oint_C P\,dx + Q\,dy = \iint_\Omega \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$\Rightarrow \iint_{\Delta ABC} \left(\frac{\partial (y^2)}{\partial x} - \frac{\partial (x^2)}{\partial y}\right) dx\,dy = 0$$

$$w = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}, \quad \operatorname{div} w = \begin{bmatrix} 2x \\ 2y \end{bmatrix} = 2x+2y$$

$$\boxed{\begin{array}{ll} AC\ldots & y = -\frac{1}{2}x+3 \\ BC\ldots & y=-2x+6 \\ AB\ldots & y=-x+3 \end{array}}$$

$$\iint_{\Delta ABC} (2x+2y)\,dx\,dy =$$

$$= \int_0^2 dx \int_{-x+3}^{-\frac{1}{2}x+3} (2x+2y)\,dy + \int_2^3 dx \int_{-x+3}^{-2x+6} (2x+2y)\,dy$$

$$= \int_0^2 \left[2x\left(-\tfrac{1}{2}x+\cancel{3}+x\cancel{-3}\right) + \left(3-\tfrac{1}{2}x\right)^2 - (3-x)^2\right]dx + \int_2^3 \left[2x(-2x+6+x-3) + (6-2x)^2 - (3-x)^2\right]dx$$

$$= \int_0^2 \left[\cancel{x^2} + \cancel{9} - 3x + \frac{x^2}{4} - \cancel{9} + 6x - \cancel{x^2}\right]dx + \int_2^3 \left[-2x^2+6x+36-24x+4x^2-9+6x-x^2\right]dx$$

$$= \int_0^2 \left[3x + \frac{x^2}{4}\right]dx + \int_2^3 \left[x^2-12x+27\right]dx = \left(\frac{3x^2}{2} + \frac{x^3}{12}\right)\Big|_0^2 + \left(\frac{x^3}{3} - 6x^2 + 27x\right)\Big|_2^3$$

$$= \frac{20}{3} + (9-54+81) - \left(\frac{8}{3} - 24 + 54\right) = \frac{20}{3} + 36 - \frac{8}{3} - 30 = 6 + \frac{12}{3} = 10$$

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5. $f'''(t) - 4f'(t) = \sin(2t)$, $f(0) = f'(0) = f''(0) = 0$

$f'''(t) \circ\!\!-\!\!\bullet\; s^3F(s) - s^2f(0) - sf'(0) - f''(0) = s^3F(s)$

$f'(t) \circ\!\!-\!\!\bullet\; sF(s) - f(0) = sF(s)$

$\sin(2t) \circ\!\!-\!\!\bullet\; \frac{2}{s^2+4}$

$\Rightarrow \quad s^3F(s) - 4sF(s) = \frac{2}{s^2+4}$

$F(s)\cdot[s^3-4s] = \frac{2}{s^2+4} \Rightarrow \boxed{F(s) = \frac{2}{s(s^2+4)(s^2-4)}}$

$\frac{2}{s(s^2+4)(s^2-4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4} + \frac{Ds+E}{s^2-4} \quad / \cdot s(s^2+4)(s^2-4)$

$2 = A(s^2+4)(s^2-4) + (Bs+C)s(s^2-4) + (Ds+E)s(s^2+4)$

$2 = A(s^4-16) + (Bs+C)(s^3-4s) + (Ds+E)(s^3+4s)$

$-16A = 2 \Rightarrow \boxed{A = -\frac{1}{8}}$, $-4C+4E=0$, $-4B+4D=0$; $C+E=0$

$\boxed{C=E}$ $\boxed{B=D}$ $\Rightarrow \boxed{C=E=0}$

$A+B+D=0$

$\Rightarrow 2B = \frac{1}{8} \Rightarrow \boxed{B = \frac{1}{16} = D}$

RJEŠENJE:

$f(t) = -\frac{1}{8} + \frac{1}{16}\cos(2t) + \frac{1}{16}\cosh(2t)$, za $t \geq 0$

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