OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO .

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. oxo

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IME I PREZIME: JOSIP VLASTELIĆ **BROJ INDEKSA:** 54951-2007

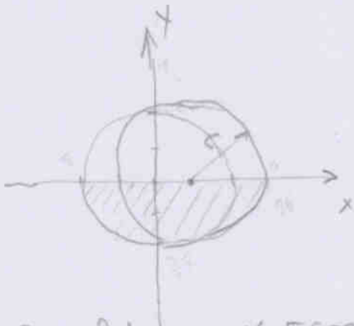
1. Izračunati dvostruki integral:
$$\iint\limits_S y\,\mathrm{d}x\mathrm{d}y\,,$$
gdje je S područje donje poluravnine ($y \leq 0$) omeđeno kružnicom $(x+1)^2 + y^2 = 4$. Ø

2. Izračunati $\int_{\widehat{ABC}} z\mathrm{d}x + y\mathrm{d}y + x\mathrm{d}z$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(1,0,0)$, $B(0,1,0)$, $C(0,0,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

3. Izračunati volumen tijela omeđenog valjkom $x^2 + z^2 = 4$ i ravninama $4 + 2z = y$ i $y = 0$. 20

4. Izračunati
$$\int\limits_{(0,0,0)}^{(1,\pi,\pi)} x\,\mathrm{d}x + z^2\cos y\,\mathrm{d}y + 2z\sin y\,\mathrm{d}z$$
20

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: n
$$x'''(t) - x''(t) = \mathrm{e}^t, \quad x'(0) = x''(0) = 1,\ x(0) = -1.$$



1.
$y \leq 0$

$(x+1)^2 + y^2 = 4$

GLUPA GREŠKA

$$\int_0^{\pi} d\varphi \int_0^{2} (r\sin\varphi)\,dr = \int_0^{2\pi} d\varphi\,(-2\cos\varphi) = \quad \times$$

$x_1 = x = r\cos\varphi + 1$
$y = r\sin\varphi$

$x = r\cos\varphi$
$y = r\sin\varphi$

$$-2\int_0^{\pi}\cos\varphi = -2(\cos\pi - \cos 0) = -2(-1-1) = -2(-2) = 4$$

$\varphi \in [0, \pi]$
$r \in [0, 2]$

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3. $x^2+z^2=4$ $4+2z=y$; $y=0$

$$\int_0^{2\pi} d\varphi \int_0^2 r\,dr \int_0^{4+2r\sin\varphi} dy = \int_0^{2\pi} d\varphi \int_0^2 r\,dr\,(4+2r\sin\varphi) =$$

$$\int_0^{2\pi} d\varphi \left[4\int_0^2 r\,dr + 2\int_0^2 r^2 \sin\varphi\, dr \right] = \int_0^{2\pi} d\varphi \left[8 + \left(-\frac{16}{3}\cos\varphi\right)\right] =$$

$$8\int_0^{2\pi} d\varphi + \left(-\frac{16}{3}\int_0^{2\pi}\cos\varphi\, d\varphi\right) = 16\pi + \left(-\frac{18}{3}\underbrace{(\sin 2\pi - \sin 0)}_{0}\right)$$

$$= 16\pi + \left(-\frac{18}{3}(0)\right) = 16\pi$$ ✓

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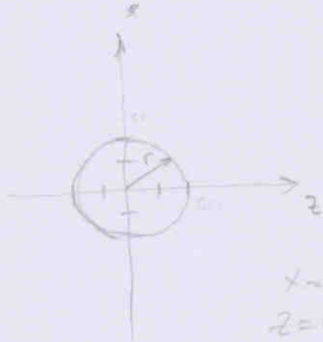
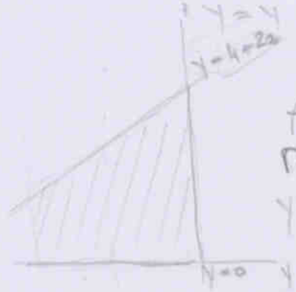


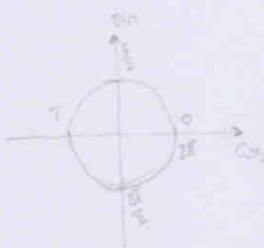
$x = r\cos\varphi$
$z = r\sin\varphi$



$\varphi \in [0, 2\pi]$
$r \in [0, 2]$
$y \in [0, 4+2r\sin\varphi]$

$4+2(r\sin\varphi) = y$

$4+2r\sin\varphi = y$



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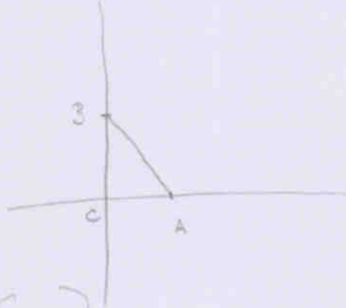
2. $\int\limits_{ABC} z\,dx + y\,dy + x\,dz$ $\qquad A(1,0,0)\ B(0,1,0)\ C(0,0,0)$



$$W = \begin{bmatrix} z \\ y \\ x \end{bmatrix}$$

$$\text{rot } W = \nabla \times W = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \times \begin{bmatrix} z \\ y \\ x \end{bmatrix} = \begin{bmatrix} 0-0 \\ 1-1 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} dx & & z \\ dy & & y \end{matrix}$$

20

$$r(x,y,0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0-0 \\ 0-0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{matrix} 1 & & 0 \\ 0 & & 1 \end{matrix}$$

$$r \cdot \text{rot } W = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \underline{\underline{0}} \checkmark \qquad \iint\limits_{0} 0\,dx\,dy = \underline{\underline{0}} \checkmark$$

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4. $\int_{(0,0,0)}^{(1,\pi,\pi)} x\,dx + z^2\cos y\,dy + 2z\sin y\,dz$

$f(A)-f(B)$

$W\begin{bmatrix} x \\ z\cos y \\ 2z\sin y \end{bmatrix} = -\operatorname{grad} f$

$d_x f = -x \;/\int dx$

$f = -\frac{x^2}{2} + C(y,z)$

$d_y f = -z^2\cos y$

$d_y\left(-\frac{x^2}{2} + C(y,z)\right) = -z^2\cos y$

$\frac{dC(y,z)}{dy} = -z^2\cos y \;/\int dy$

$C(y) = -z^2\sin y + C(z)$

$d_z f = -2z\sin y$

$d_z\left(-\frac{x^2}{2} - z^2\sin y + C(z)\right) = -2z\sin y$

$-2z\sin y + \frac{dC(z)}{dz} = -2z\sin y$

$\frac{dC(z)}{dz} = -2z\sin y + 2z\sin y$

$\frac{dC(z)}{dz} = 0 \;/\int dz$

$C(z) = 0$

$f = -\frac{x^2}{2} - z^2\sin y$ ✓

$f = -\frac{0^2}{2} - 0^2\sin 0 - \left(-\frac{1^2}{2} - \underbrace{\pi^2\sin\pi}_{0}\right)$

$f = 0 - \left(-\frac{1}{2}\right)$

$f = \frac{1}{2}$ ✓

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IME I PREZIME: KARLO PERKOVIĆ **BROJ INDEKSA:** 56175-2008

1. Izračunati dvostruki integral:

$$\iint_S y \,\mathrm{d}x\mathrm{d}y\,,$$

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3. Izračunati volumen tijela omeđenog valjkom $x^2 + z^2 = 4$ i ravninama $4 + 2z = y$ i $y = 0$. Ø

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$$\int_{(0,0,0)}^{(1,\pi,\pi)} x\,\mathrm{d}x + z^2\cos y\,\mathrm{d}y + 2z\sin y\,\mathrm{d}z$$

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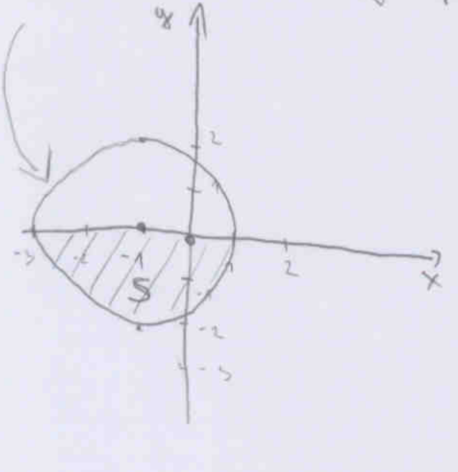
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: n

$$x'''(t) - x''(t) = \mathrm{e}^t, \quad x'(0) = x''(0) = 1,\ x(0) = -1.$$

20

1. $\boxed{(x+1)^2 + y^2 = 4}$ $r^2 = 4$, $r = 2$



$x = r\cos\varphi - 1$ $\quad \varphi \in [\pi, 2\pi]$

$y = r\sin\varphi$ $\quad r \in [0,2]$

$r = 2$

$$\int_\pi^{2\pi}\int_0^2 (r\sin\varphi)\cdot r\,dr\,d\varphi = \int_\pi^{2\pi}\int_0^2 r^2\sin\varphi\,dr\,d\varphi$$

$$= \int_\pi^{2\pi} \sin\varphi \cdot \left(\frac{r^3}{3}\right)_0^2 = \frac{8}{3}\cdot(-\cos\varphi)_\pi^{2\pi}$$

20

$$= -\frac{8}{3}\cdot(1-(-1)) = -\frac{8}{3}\cdot 2 = -\frac{16}{3}$$

IME I PREZIME: KARLO PERKOVIĆ BROJ INDEKSA: 56175-2008

5. $x'''(t) - x''(t) = e^{t} \quad x'(0) = x''(0) = 1 \quad x(0) = -1$

$$x'''(t) = s^3 X(s) - s^2 x(0) - s x'(0) - x''(0) = \boxed{s^3 X(s) + s^2 - s - 1}$$

$$x''(t) = s^2 X(s) - s x(0) - x'(0) = \boxed{s^2 X(s) + s - 1}$$

$$e^{1t} = \frac{1}{s-1}$$

$$s^3 X(s) + s^2 - s \cancel{-1} - \left(s^2 X(s) + s \cancel{-1}\right) = \frac{1}{s-1}$$

$$s^3 X(s) + s^2 - s - s^2 X - s = \frac{1}{s-1}$$

$$s^3 X(s) - s^2 X(s) + s^2 - 2s = \frac{1}{s-1}$$

$$(s^3 - s^2) X(s) = \frac{1}{s-1} - s^2 + 2s$$

$$X(s) = \frac{\frac{1}{s-1} - s^2 + 2s}{s^3 - s^2} = \frac{\frac{1 - s^3 + s^2 + 2s^2 - 2s}{s-1}}{s^2(s-1)} = \frac{-s^3 + 3s^2 - 2s + 1}{s^2(s-1)(s-1)} = \frac{-s^3 + 3s^2 - 2s + 1}{s^2 \cdot (s-1)^2}$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s-1)} + \frac{D}{(s-1)^2} = \frac{A \cdot s \overbrace{(s-1)(s-1)}^{s^2-2s+1} + B \cdot (s-1)(s-1) + C \cdot s^2 (s-1) + D s^2}{s^2 (s-1)(s-1)}$$

$$= \frac{As^3 - 2As^2 + As + Bs^2 - 2Bs + B + Cs^3 - Cs^2 + Ds^2}{s^2(s-1)(s-1)}$$

$s^3 (A + C) \quad s^2 (-2A + B - C + D) \quad s (A - 2B) \quad 1 (B)$

$A + C = -1$

$-2A + B - C + D = 3$

$A - 2B = -2$

$\boxed{B = 1}$

$\boxed{A = 0}$

$\boxed{C = -1}$

$D = 3 - 1 - 1$

$\boxed{D = 1}$

IME I PREZIME: KARLO PERKOVIĆ BROJ INDEKSA: 56125-2008

5.

$$X(s) = \frac{0}{s} + \frac{1}{s^2} + \frac{-1}{(s-1)} + \frac{1}{(s-1)^2}$$

$$x(t) = \mathcal{L}^{-1}\frac{1}{s^2} - \mathcal{L}^{-1}\frac{1}{(s-1)} + \mathcal{L}^{-1}\frac{1}{(s-1)^2}$$

$$\boxed{x(t) = t - e^t + te^t}$$

✓ 20

$$x' = 1 - \cancel{e^t} + \cancel{e^t} + te^t$$

$$x'' = e^t + te^t$$

$$x''' = 2e^t + te^t$$

$$x''' - x'' = 2e^t + \cancel{te^t} - e^t - \cancel{te^t} = e^t$$

IME I PREZIME: KARLO PERKOVIĆ BROJ INDEKSA: 56175-2008

$x = r\cos\varphi$ $\varphi \in [0, 2\pi]$
$z = r\sin\varphi$ $r \in [0, 2]$
$y \in [0, 4 + 2\cancel{z}\sin\varphi)$

3. $x^2 + z^2 = 4 \quad 4 + 2z = y \quad y = 0$

$r^2 = 4$

$r = \pm 2$

$2z = y - 4$

$2z = -4$

$\boxed{z = -2}$

$\boxed{z = \frac{y-4}{2}}$

$\varphi \in [0, 2\pi]$

$r \in [0, 2]$

$z \in \left[-2, \frac{y-4}{2}\right]$

$$V = \int_0^{2\pi}\int_0^{2}\int_{-2}^{\frac{r\sin\varphi - 4}{2}} r \, dz\,dr\,d\varphi \quad \cancel{\varnothing}$$

$$V = \int_0^{2\pi}\int_0^{2}\int_0^{4+2r\sin\varphi} r\,dy\,dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^{2} r \cdot \left(\frac{r\sin\varphi \cancel{-4}}{2} \cancel{+2}\right) dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^{2} \frac{r^2\sin\varphi}{2}\,dr\,d\varphi = \int_0^{2\pi} \frac{\sin\varphi}{2}\cdot\left(\frac{r^3}{3}\right)\Big|_0^2 d\varphi = \frac{1}{2}\int_0^{2\pi}\sin\varphi\cdot\frac{8}{3}\,d\varphi$$

$$= \frac{8}{6}\cdot(-\cos\varphi)\Big|_0^{2\pi} = -\frac{8}{6}\cdot(1-1) = 0 \,//$$

4. $$\int_{(0,0,0)}^{(1,\pi,\pi)} x\,dx + z^2\cos y\,dy + 2z\sin y\,dz$$

$$w = \begin{bmatrix} x \\ z^2\cos y \\ 2z\sin y \end{bmatrix} = -\operatorname{grad} f$$

$d_x f = -x$

$$\boxed{f = -\frac{x^2}{2} + C(y,z)}$$

$d_y f = -z^2\cos y$

$$d_y\left(-\frac{x^2}{2} + C(y,z)\right) = -z^2\cos y$$

$$\frac{dC(y,z)}{dy} = -z^2\cos y$$

$$\boxed{C(y) = -z^2\sin y + C(z)}$$

$d_z f = -2z\sin y$

$$d_z\left(-\frac{x^2}{2} - z^2\sin y + C(z)\right) = -2z\sin y$$

$$-2z\cancel{\sin y} + \frac{dC(z)}{dy} = -2z\cancel{\sin y}$$

$$\boxed{C(z) = 0}$$

$$\boxed{f = -\frac{x^2}{2} - z^2\sin y + 0}$$ ✓

$$f(0,0,0) - f(1,\pi,\pi) = 0 - 0 - \left(-\frac{1}{2} - 0\right) = \frac{1}{2} \,//$$ ✓

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