Popuniti odmah!

IME I PREZIME: BORIS RUDELKO BROJ INDEKSA: 17-2-0039-2010

DATUM: 10.02 VRIJEME: OD 14:00 DO 14:45

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

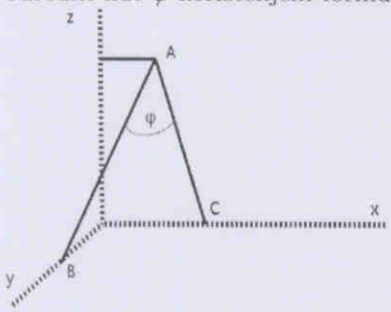
oxxo

Broj ↓ bodova

1. Odrediti sve asimptote funkcije $f(x) = \dfrac{x}{\ln x}$. 14
2. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $f(x)$ iz zadatka 1. ~~0~~
3. Odrediti prvu derivaciju funkcije $g(x) = \dfrac{x-1}{\sqrt{x^2-x-1}}$. 20
4. Gaussovom metodom riješiti matrični sustav: ~~0~~ ~~20~~

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & -1 & -3 & 3 \\ 1 & -8 & -9 & -6 \\ 5 & 5 & 0 & 10 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -8 \\ 14 \end{bmatrix}$$

5. Zadana je konfiguracija nosača kao na slici ispod. Točke su A(2,1,3), B(0,2,0) i C(2,0,0). Potrebno je odrediti kut φ korištenjem formule za kut između vektora. 20



UKUPNO: ~~74~~ 54

1.) $f(x) = \dfrac{x}{\ln x}$ $D(f) = \ln x > 0$ $\langle 0, +\infty \rangle \setminus \{1\}$ ✓ 14

$x \neq 1$

$\lim\limits_{x\to 0} \dfrac{x}{\ln x} = \dfrac{0}{\ln 0} / \ln 0 = \text{N/D}$ ✗ $\lim\limits_{x\to 0} \dfrac{x}{\ln x} = \dfrac{0}{\ln 0} = \dfrac{0}{-\infty} = 0$

ZBOG: $\lim\limits_{x\to 0} \ln x = -\infty$

$\lim\limits_{x\to 1} \dfrac{x}{\ln x} = \dfrac{1}{0} = \infty$ $\boxed{x=1 \text{ U.A.}}$ ✓ LIMES SLIJEVA: $-\infty$ ZDESNA: $+\infty$

$\lim\limits_{x\to\infty} \dfrac{x}{\ln x} = \left(\dfrac{\infty}{\infty}\right) = \text{L'H} \lim\limits_{x\to\infty} \dfrac{(x)'}{(\ln x)'} = \dfrac{1}{\frac{1}{x}} = \dfrac{x}{1} = \infty$ ✓ nema H.A. ✓

$\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{\frac{x}{\ln x}}{x} = \dfrac{x}{\ln x \cdot (x)} = \dfrac{\infty}{\infty} = \text{N/D}$ ✗ nema K.A. ✓

$\lim\limits_{x\to+\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to+\infty} \dfrac{\frac{x}{\ln x}}{\frac{x}{1}} = \lim\limits_{x\to+\infty} \dfrac{x}{x\ln x} = \lim\limits_{x\to+\infty} \dfrac{1}{\ln x} = \dfrac{1}{\ln+\infty} = \dfrac{1}{\infty} = 0$

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2.) $f(x) = \frac{x}{\ln x}$

$f(-x) = \frac{-x}{\ln(-x)} = -\frac{x}{\ln(-x)}$ funkcija nije parna niti neparna

funkcija nije periodična $\Rightarrow$ nema trig. funkc.

$f(x) = 0$

$\frac{x}{\ln x} = 0$

$x = 0$ $S_x(0,y)$

$f(0) = \frac{0}{\ln 0}$ / $S_y -$

$f'(x) = \left(\frac{x}{\ln x}\right)' = \frac{(x)' \cdot \ln x - (\ln x)' \cdot x}{(\ln x)^2} = \frac{1 \cdot \ln x - \frac{1}{x} \cdot x}{(\ln x)^2}$

$= \frac{\ln x - 1}{(\ln x)^2}$

$f'(x) = 0$

$\ln x - 1 = 0$

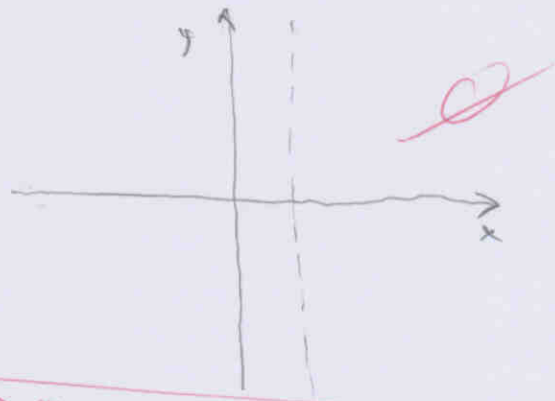
$\ln x = 1 \Rightarrow \underline{x = e}$ KRITIČNA TOČKA

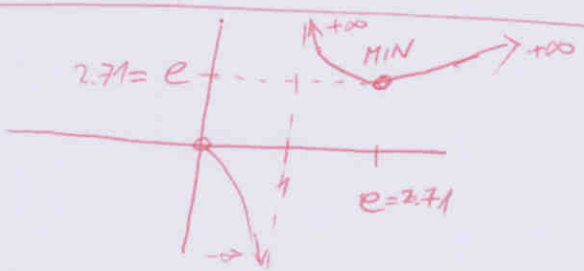
$f(e) = e$

$x \in I_1 \Rightarrow f(x) = \frac{x}{\ln x} = \frac{+}{-} = -$ ↓ PADA

$x \in I_2 \Rightarrow f(x) = \frac{x}{\ln x} = \frac{+}{+} = +$ ↑ RASTE

GRAF?





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3)

$$g(x) = \frac{x-1}{\sqrt{x^2-x-1}} = \left(\frac{x-1}{(x^2-x-1)^{\frac{1}{2}}}\right)' = \frac{(x-1)'\cdot(x^2-x-1)^{\frac{1}{2}} - \left[(x^2-x-1)^{\frac{1}{2}}\right]'\cdot(x-1)}{\left((x^2-x-1)^{\frac{1}{2}}\right)^2}$$

$$= \frac{1\cdot(x^2-x-1)^{\frac{1}{2}} - \frac{1}{2}(x^2-x-1)^{-\frac{1}{2}}\cdot(x^2-x-1)'\cdot(x-1)}{x^2-x-1}$$

20

$$= \frac{(x^2-x-1)^{\frac{1}{2}} - \frac{1}{2}(x^2-x-1)^{-\frac{1}{2}}\cdot(2x-1)\cdot(x-1)}{x^2-x-1} = \frac{(x^2-x-1)^{\frac{1}{2}} - \frac{2x^2-2x-x+1}{2\sqrt{x^2-x-1}}}{x^2-x-1}$$

$$= \frac{\sqrt{x^2-x-1} - \frac{2x^2-x+1}{2\sqrt{x^2-x-1}}}{x^2-x-1}$$

4)

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 4 & 4 \\ 2 & -1 & -3 & 3 & 2 \\ 1 & -8 & -9 & -6 & -8 \\ 5 & 5 & 0 & 10 & 14 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-R_1 \\ R_4-5R_1 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & 4 & 4 \\ 0 & -5 & -5 & -5 & -6 \\ 0 & -10 & -10 & -10 & -12 \\ 0 & -5 & -5 & -10 & -6 \end{array}\right] (-\tfrac{1}{5})R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & 4 & 4 \\ 0 & 1 & 1 & 1 & 6/5 \\ 0 & -10 & -10 & -10 & -12 \\ 0 & -5 & -5 & -10 & -6 \end{array}\right] \begin{array}{l} R_1-2R_2 \\ \\ R_3+10R_2 \\ R_4+5R_2 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 8/5 \\ 0 & 1 & 1 & 1 & 6/5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 \end{array}\right]$$

sustav ima jedinstveno rješenje

$$4 - \frac{12}{5} = \frac{20-12}{5} = \frac{8}{5}$$

$$\left.\begin{array}{l} a = \frac{8}{5} + \lambda \\ b = \frac{6}{5} - \lambda \\ c = \lambda \\ d = 0 \end{array}\right\} \quad \forall \lambda \in \mathbb{R}$$

0

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1.)

$A(2,1,3)$

$B(0,2,0)$

$C(2,0,0)$

$\overline{AB} = v_1 = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$\overline{AC} = v_2 = \begin{bmatrix} 0 \\ -1 \\ -3 \end{bmatrix}$

$v_1 \cdot v_2 = -2 \cdot 0 + 1 \cdot (-1) - 3 \cdot (-3)$

$= 0 - 1 + 9 = 8$

$\|v_1\| = \sqrt{(-2)^2 + (1)^2 + (-3)^2}$

$= \sqrt{4+1+9} = \sqrt{14} \approx 3,74$

$\|v_2\| = \sqrt{(0)^2 + (-1)^2 + (-3)^2}$

$= \sqrt{0+1+9} = \sqrt{10} \approx 3,16$

$v_1 \cdot v_2 = \|v_1\| \|v_2\| \cos \sphericalangle (v_1, v_2)$

$\cos \sphericalangle (v_1, v_2) = \frac{v_1 \cdot v_2}{\|v_1\| \|v_2\|}$

$\cos \sphericalangle (v_1, v_2) = \frac{8}{3,74 \cdot 3,16}$

$\cos \sphericalangle (v_1, v_2) = \frac{8}{11,81}$

$\cos \sphericalangle (v_1, v_2) = 0,67739$

$\sphericalangle (v_1, v_2) = \arccos (0,67739)$

$\sphericalangle (v_1, v_2) = 0,826$

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