Popuniti odmah!

IME I PREZIME: TIBOR MANDARIĆ BROJ INDEKSA: 57661

DATUM: VRIJEME: OD 13:30 DO 14:30 .

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

ooox

Broj ↓ bodova

1. Odrediti determinantu matrice $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}$ — 20

2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2 + x + 1}$ — 5

3. Ispitati konvergenciju reda $\sum \left(\frac{2n^2 + 3n + 4}{\frac{1}{n} + 2n + 3n^2} \right)^n$ — 0

4. Ispitati domenu, periodičnost, parnost i prvu derivaciju funkcije $g(x) = \ln(\cos(2x))$. — 15

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2 + 1}{x^2 + 2}$. — 15

VKUPNO 55

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1. $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}$ $\det A = \begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{vmatrix}$

$$= -2 \begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{vmatrix} =$$

$$= -2 \left(1 \cdot \begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 2 \end{vmatrix} \right) + 1 \left(1 \cdot \begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} \right) =$$

$$= -2 \cdot \left(1 \cdot \left(2 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \right) - 2 \left(2 \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} \right) \right) + 1 \cdot \left(1 \cdot \left(1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} \right) - 2 \left(2 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \right) \right) =$$

$$= -2 \left(1 \cdot (-6) - 2 \cdot (4) \right) + 1 \cdot \left(1 \cdot \underset{-7}{\left(1 \cdot (-3) - 2 \cdot 2 \right)} \right)$$

$$-2 \underset{+12}{(-6)} = 28 + 5 = \underline{33}$$

DETA = 33 ✓ 20

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2. $f(x) = x - \sqrt{x^2+x+1} \cdot \frac{x+\sqrt{x^2+x+1}}{x+\sqrt{x^2+x+1}} =$

$= \frac{x^2-(x^2+x+1)}{x+\sqrt{x^2+x+1}} = \frac{-x-1}{x+\sqrt{x^2+x+1}}$

$x+\sqrt{x^2+x+1} = 0$

$x = -\sqrt{x^2+x+1} \,/^2$

$x^2 = x^2+x+1$

$x = -1$

1. $D(f) = R \setminus \{-1\}$ ✗

2. ASIMPTOTA

VERTIKALNA

$x = -1$ ✗

HORIZONTALNA

LIJEVA

$\lim_{x\to-\infty} \left(\frac{-x-1}{x+\sqrt{x^2+x+1}}\right) = \lim_{x\to+\infty} \left(\frac{x-1}{-x+\sqrt{x^2-x+1}}\right) = \lim_{x\to+\infty} \left(\frac{x(1-\frac{1}{x})}{x(-1+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}})}\right)$

1 // 0

$= \frac{1}{0} = +\infty$ NEMA JE

DESNA

$\lim_{x\to+\infty} \left(\frac{-x-1}{x+\sqrt{x^2+x+1}}\right) = \lim_{x\to+\infty} \left(\frac{x(-1-\frac{1}{x})}{x(1+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}})}\right) = \frac{-1}{2}$ ✓

5

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2. KOSA ASIMPTOTA

$$k = \lim_{x\to\pm\infty}\left(\frac{f(x)}{x}\right) = \lim_{x\to\pm\infty}\frac{\frac{-x-1}{x+\sqrt{x^2+x+1}}}{\frac{x}{1}} = \lim_{x\to\pm\infty}\left(\frac{-x-1}{x^2+x\sqrt{x^2+x+1}}\right)^{I}$$

$$= \lim_{x\to\pm\infty}\frac{-1}{2x+2x+1+\frac{1}{2}x^{-\frac{1}{2}}} = \lim_{x\to+\infty}\frac{-1}{4x+1+\frac{1}{2\sqrt{x}}} =$$

$$= \lim\frac{-1}{\frac{8x^2+2x+\sqrt{x}}{2x}} \quad \lim\left(\frac{-2x}{8x^2+2x+\sqrt{x}}\right)^{II} = \frac{-2}{16x+2+\frac{1}{2}x^{-\frac{1}{2}}} = 0$$

NEMA KOSIH ASIMPTOTA

VIDI CVAR

$$x\sqrt{x^2+x+1} = \sqrt{x^4+x^2+x}$$
$$= x^{\frac{4}{2}} + x^{\frac{2}{2}} + x^{\frac{1}{2}}$$
$$= x^2 + x + \sqrt{x}$$

$$\frac{1}{2\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{2x}$$

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4. $g(x) = \ln(\cos(2x))$

1. $Df = \left\langle -\frac{\pi}{4} + 2k\pi, +\frac{\pi}{4} + 2k\pi \right\rangle$ ✓ 15

2. $g(x)$ je periodična jer je trigonometrijska funkcija ✓ PERIOD?

$g(-x) = \ln(\cos(-2x)) \Rightarrow$ ✗ $\begin{cases} g(-x) \neq -g(x) \text{ nije parna} \\ g(-x) \neq g(x) \text{ ni neparna} \end{cases}$

$g'(x) = \frac{1}{\cos(2x)} \cdot (-\sin(2x)) \cdot 2 = \frac{-2\sin(2x)}{\cos(2x)}$ ✓

5. $h(x) = \frac{x^2+1}{x^2+2}$

1. $Df = R \setminus \{-\sqrt{2}, +\sqrt{2}\}$

$x^2 + 2 = 0$

$x^2 = -2$

$x_{1/2} = \pm\sqrt{2}$ ✗

2. nije periodična

$h(-x) = \frac{x^2+1}{x^2+2}$

$h(-x) = h(x)$ parna je

3. nultočke

$x^2 + 1 = 0$

$x^2 = -1$

$x = \sqrt{-1}$

UPOTRIJEBITI KALKULATOR:

$(\sqrt{2})^2 \neq -2$

$(-\sqrt{2})^2 \neq -2$

4. ekstremi

$h'(x) = \frac{2x(x^2+2) - 2x(x^2+1)}{(x^2+2)^2}$

$h'(x) = \frac{2x^3 + 4x - 2x^3 - 2x}{x^4 + 4x^2 + 4}$

$h'(x) = \frac{2x}{x^4 + 4x^2 + 4}$

$h'(x) = 0$ $T(0, \frac{1}{2})$ MIN

$2x = 0$

$x = 0$ $y = \frac{1}{2}$

$h''(x) = \frac{2(x^4+4x^2+4) - 2x(4x^3+8x)}{(x^2+2)^4}$

$h''(x) = \frac{2x^4 + 8x^2 + 8 - 8x^4 - 16x^2}{(x^2+2)^4}$

$h''(x) = \frac{-6x^4 - 8x^2 + 8}{(x^2+2)^4}$ $h''(0) > 0$

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5. ASIMPTOTE

VERTIKALNA

NEMA IH

HORIZONTALNA

lijeva

$$\lim_{x\to-\infty}\frac{x^2+1}{x^2+2}=\lim_{x\to-\infty}\frac{x^2+1}{x^2+2}=\lim_{x\to-\infty}\frac{x^2\left(1+\frac{1}{x^2}\right)}{x^2\left(1+\frac{2}{x^2}\right)}=1$$ ✓

desna

$$\lim_{x\to+\infty}\frac{x^2\left(1+\frac{1}{x^2}\right)}{x^2\left(1+\frac{1}{x^2}\right)}=1$$ ✓

KOSA

$$\lim_{x\to\pm\infty}\left(\frac{x^2+1}{x^3+2x}\right)'=\lim_{x\to\pm\infty}\left(\frac{2x}{3x^2+2}\right)'=\lim_{x\to\pm\infty}\frac{2}{3x}=0$$

NEMA KOSE ASIMPTOTE.

	$-\infty$		0		$+\infty$
$f'(x)$		$-$		$+$	
$f(x)$		↘		↗	

TOČKE PREKIDA:

$$x=\begin{cases}-\sqrt{2}\\-1\\+1\\\sqrt{2}\end{cases}$$ ✗

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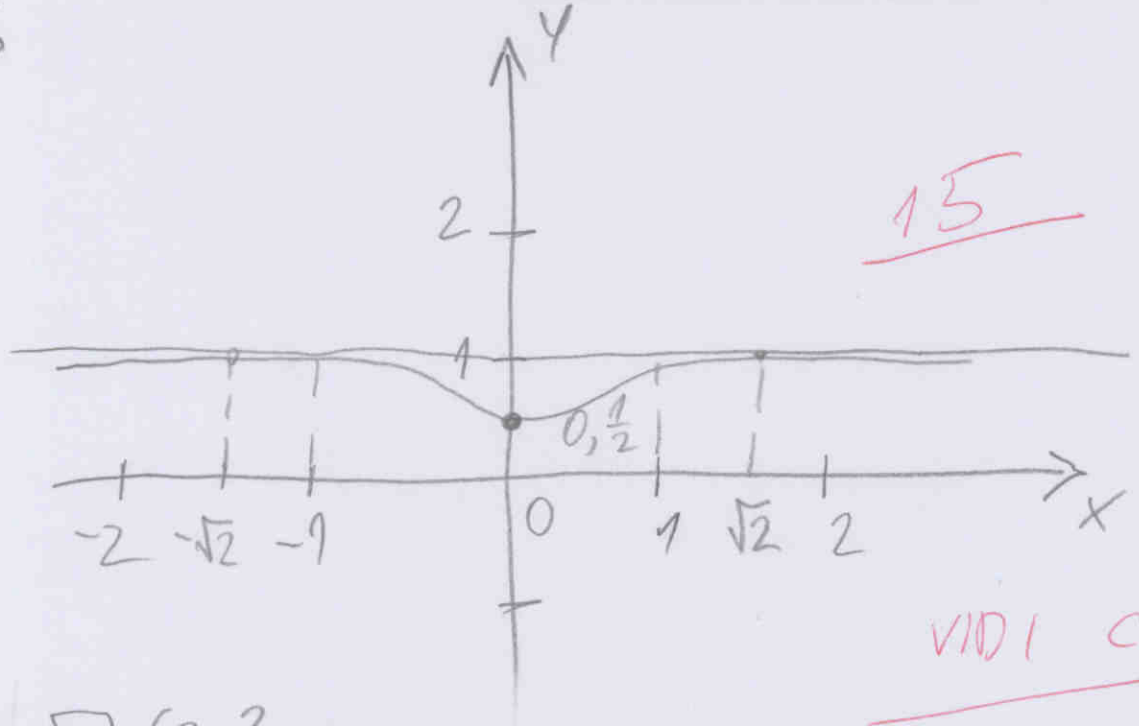
5.



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VIDI CVAR

6. $\sum \left(\frac{2n^2+3n+4}{\frac{1}{n}+2n+3n^2} \right)$

$$\lim_{n\to\infty} \frac{1}{1-q} = \frac{1}{n^{-1}+2n+3n^2-2n^2-3n-4}$$

X