Popuniti odmah! (33)

Ime i prezime: ANDREA SAVIĆ **Broj indeksa:**

Datum: Vrijeme: od 14:15 do 15:45 .

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE <u>JEDNOSTRANO</u> NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

oxox Broj ↓ bodova

1. Odrediti determinantu matrice $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 2 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 2 & 0 & 0 & 2 & 1 \end{bmatrix}$ Ø

2. Odrediti domenu i sve asimptote funkcije $f(x) = \dfrac{x^2-2x-3}{x^2+3x+2}$ 10

3. Istražiti konvergenciju reda: $\displaystyle\sum_{n=1}^{\infty}\left(\frac{5+2n}{5n+3}\right)^{n^2}$

4. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije $g(x) = \arctan(x^2)$. 8

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \ln(x^2+1)$. 15

1. $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 2 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 2 & 0 & 0 & 2 & 1 \end{bmatrix}$
$\quad 1 \;\; 2 \;\; 0 \;\; 0 \;\; 2$

VIDI TOMA MEDIĆ

$D_A = 1\cdot\begin{vmatrix} 1&2&0&0\\2&0&2&0\\0&2&1&2\\0&0&2&1\end{vmatrix} - 2\cdot\begin{vmatrix} 2&0&2&0\\0&2&1&2\\0&0&2&1\\2&0&0&2\end{vmatrix} + 2\cdot\begin{vmatrix} 2&0&0&2\\1&2&0&0\\2&0&2&0\\0&2&1&2\end{vmatrix}$

$= 1\cdot\begin{vmatrix} 0&2&0\\2&1&2\\0&2&1\end{vmatrix} - 2\cdot\begin{vmatrix} 2&1&2\\0&2&1\\2&0&0\end{vmatrix} - 2\left(2\cdot\begin{vmatrix} 2&1&2\\0&2&1\\0&0&2\end{vmatrix} - 2\cdot\begin{vmatrix} 0&2&0\\2&2&1\\0&2&1\end{vmatrix}\right) + 2\cdot\left(2\cdot\begin{vmatrix} 2&0&0\\0&2&0\\2&1&2\end{vmatrix}\right.$

$\left. - 1\cdot\begin{vmatrix} 0&2&0\\2&1&2\\0&0&2\end{vmatrix} + 2\cdot\begin{vmatrix} 2&1&2\\0&0&2\\2&0&0\end{vmatrix}\right)$

$= -2\cdot\begin{vmatrix}2&1\\2&0\end{vmatrix} - 2\cdot\left(2\cdot\begin{vmatrix}2&1\\0&2\end{vmatrix}\right) - 2\cdot\left[2\cdot\begin{vmatrix}2&1\\0&2\end{vmatrix} - 2\left(-1\cdot\begin{vmatrix}2&1\\2&0\end{vmatrix}\right)\right] +$

$2\cdot\left[2\cdot\begin{vmatrix}2&0\\1&2\end{vmatrix} + 2\cdot\begin{vmatrix}0&0\\2&0\end{vmatrix} - 1\left(-2\begin{vmatrix}0&2\\2&0\end{vmatrix}\right) + 2\cdot\left(2\begin{vmatrix}0&2\\0&0\end{vmatrix} + 2\begin{vmatrix}1&2\\0&2\end{vmatrix}\right)\right]$

$= -2\cdot(2\cdot0-1\cdot2) - 4(2\cdot2-0\cdot1) - 2(2\cdot(4-0) - 2(0-2))$

$+ 2\left[2(4-0) + 2(0-0) - 1(-2(0-4)) + 2\cdot(2\cdot(0-2) + 2(2-0))\right]$

$= 4 - 16 - 16 + 4 + 2(8 - 8 + 8 - 8) = \underline{-22}$ ✗ $\underline{\det(A) = 120}$

IME I PREZIME: ANDREA SAVIĆ BROJ INDEKSA:

1. $D_A = 1\cdot\left[1\cdot\begin{vmatrix}0&2&0\\2&1&2\\0&2&1\end{vmatrix} - 2\begin{vmatrix}2&1&2\\0&2&1\\2&0&0\end{vmatrix}\right] - 2\left[2\cdot\begin{vmatrix}2&1&2\\0&2&1\\0&0&2\end{vmatrix} - 2\begin{vmatrix}0&2&0\\2&1&2\\0&2&1\end{vmatrix}\right] + 2\cdot\left[2\cdot\begin{vmatrix}2&0&0\\0&2&0\\2&1&2\end{vmatrix} - 1\begin{vmatrix}0&2&0\\2&1&2\\0&0&2\end{vmatrix}\right.$

$\left. + 2\begin{vmatrix}2&1&2\\0&0&2\\2&0&0\end{vmatrix}\right] = 1\cdot\left[1\cdot\left(-2\begin{vmatrix}2&1\\2&0\end{vmatrix}\right) - 2\cdot\left(2\begin{vmatrix}2&1\\0&0\end{vmatrix} + 2\begin{vmatrix}1&2\\2&1\end{vmatrix}\right)\right]$

$-2\left[2\cdot\left(2\begin{vmatrix}2&1\\0&2\end{vmatrix}\right) - 2\left(-2\begin{vmatrix}2&1\\2&0\end{vmatrix}\right)\right]$

$+2\left[2\cdot\left(2\begin{vmatrix}2&0\\1&2\end{vmatrix} + 2\cdot\begin{vmatrix}0&0\\2&0\end{vmatrix}\right) - 1\cdot\left(-2\begin{vmatrix}0&2\\2&0\end{vmatrix}\right)\right]$

$D_A = -2\cdot(\overset{-2}{0-2}) - 2[2(\overset{0}{0-0}) + 2(\overset{-3}{1-4})] - 2[4(\overset{4}{4-0}) + 4(\overset{-2}{0-2})]$

$+2[4(\overset{4}{4-0}) + 2(\overset{0}{0-0}) + 2(\overset{-4}{0-4})]$

$= +4 + 12 - 32 + \cancel{16} + 8 - \cancel{16} = \underline{\underline{-8}}$

2. $f(x) = \dfrac{x^2-2x-3}{x^2+3x+2}$

$x^2+3x+2 \neq 0$

$D_f = \mathbb{R} \setminus \{-1,-2\}$ ✓

$x_{1,2} = \dfrac{-3\pm\sqrt{9-8}}{2} = \dfrac{-3\pm 1}{2}$

VERTIKALNE ASIMPTOTE?
L'HOPITAL, VIDI PEČARIĆ

$x_1 = -1$

$x_2 = -2$

10

D.H.A $\lim\limits_{x\to\infty} \dfrac{x^2-2x-3}{x^2+3x+2} \begin{matrix} :x^2 \\ :x^2 \end{matrix} = \lim\limits_{x\to\infty} \dfrac{1-\frac{2}{x}-\frac{3}{x^2}}{1-\frac{3}{x}+\frac{2}{x^3}} = \underline{\underline{1}}$ ✓

L.H.A $\lim\limits_{x\to-\infty} [x\to -x] = \lim\limits_{x\to\infty} \dfrac{x^2+2x-3}{x^2-3x+2} \begin{matrix} :x^2 \\ :x^2 \end{matrix} = \underline{\underline{1}}$ ✓

K.A $\lim\limits_{x\to\infty} \dfrac{\frac{x^2-2x-3}{x^2+3x+2}}{x} = \lim\limits_{x\to\infty} \dfrac{x^2-2x-3}{x^3+3x^2+2x} \begin{matrix} :x^3 \\ :x^3 \end{matrix} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}-\frac{2}{x^2}-\frac{3}{x^3}}{1+\frac{3}{x}+\frac{2}{x^2}} = \dfrac{0}{1} = 0$

TAMO GDJE POSTOJI L.H.A. NEMA L.K.A
— // — D.H.A. — // — D.K.A.

IME I PREZIME: ANDREA SAVIĆ

BROJ INDEKSA:

3. $\sum_{n=1}^{\infty}\left(\frac{5+2n}{5n+3}\right)^{n^2} = \lim \frac{5/:n}{5n+3/:n} + \lim \frac{2n/:n}{5n+3/:n} = 0+0=0$? konvergira

$\cancel{\lim_{x\to\infty}\left(\frac{5+2n}{5n+3}\right)^{n^2} = \lim_{n\to\infty}\left(1+\frac{1}{\frac{5n+3}{2-3n}}\right)^{n^2} = \lim_{n\to\infty}\left(1+\frac{1}{\frac{5n+3}{2-3n}}\right)^{\frac{5n+3}{2-3n}+n^2-n^2} = e^{\dots}}$

~~KONVERGIRA~~

$\cancel{\frac{5+2n-2+2+3n-3n}{5n+3}}$

$\cancel{\frac{5n+3}{5n+3}+\frac{2-3n}{5n+3} = 1+\frac{1}{\frac{5n+3}{2-3n}}}$

$\emptyset$

TREBALO JE PRIMIJENITI CAUCHY KRITERIJ

$\lim_{n\to\infty}\sqrt[n]{a_n} = \dots$

4. $\arctan(x^2)$

Parna $f(x)=f(-x)$

$\arctan(x^2)=\arctan((-x)^2)$

$\arctan(x^2)=\arctan(x^2)$

8

$D_f=\mathbb{R}$

Nije periodična

$f'(x)=(\arctan(x^2))' = 2x\cdot(\mathrm{arctg}(x^2))' = 2x\cdot$

$$f'(x)=\frac{1}{1+(x^2)^2}\cdot 2x = \frac{2x}{1+x^4}$$

$$f''(x)=\frac{2(1+x^4)-2x\cdot 4x^3}{(1+x^4)^2} = \frac{2+2x^4-8x^4}{(1+x^4)^2} = \frac{2-6x^4}{(1+x^4)^2}$$

ŠTETA. OVO NIJE TEŠKO.

JOŠ MALO VJEŽBE. VJEŽBATI STARE PISMENE ISPITE.

IME I PREZIME: ANDREA SAVIĆ

BROJ INDEKSA:

SOLIDAN GRAF:

15

5. $f(x) = \ln(x^2+1)$

$D_f \quad x^2+1>0$

$x^2 > -1$

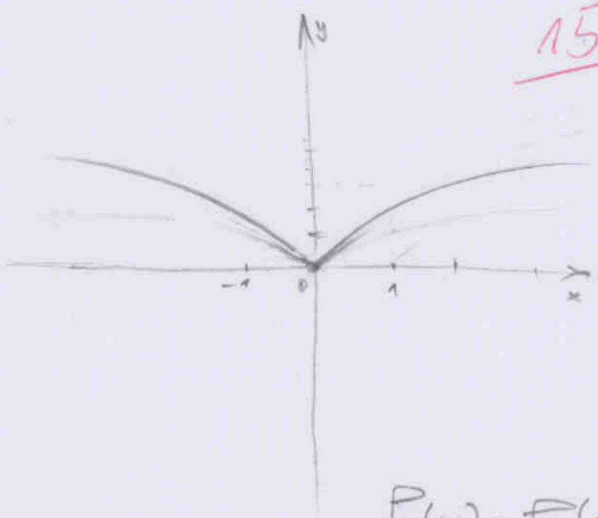
$D_f = \mathbb{R}$

$f(0) = \ln 1 = 0$

$f'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

$2x=0$

$x=0$



$f(x) = f(-x)$

$\lim (x^2+1) = \ln((-x)^2+1)$

$f \nearrow$

$f''(x) = \frac{2(x^2+1)-2x(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x}{(x^2+1)^2} = 0$

$2x^2-4x+2=0$

$x^2-2x+1=0$

$x_{1,2} = \frac{2 \pm \sqrt{4-4}}{2}$

$x_1 = 1 \qquad f(x) = 0.69$

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \ln(x^2+1)$

$= \ln\left(\lim\limits_{x \to +\infty} (x^2+1)\right)$

$= \ln(+\infty)$

$= +\infty$

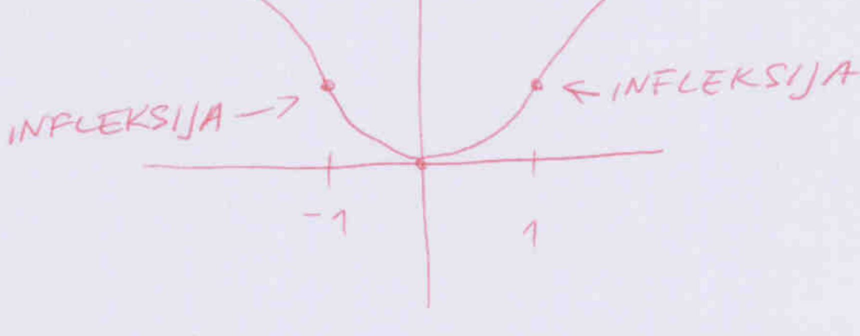
	$-\infty$	-1	0	1	$+\infty$
f'		$\searrow$		$\nearrow$	

MINIMUM ZA $x=0$

$f''(x) = \frac{2-2x^2}{(x^2+1)^2}$

$2-2x^2=0$ za $x=\pm 1$

	$-\infty$	-1	1	$+\infty$
f''	$-$	$+$	$-$	
f	$\frown$	$\smile$	$\frown$	



Popuniti odmah!

IME I PREZIME: Mateja Pećarić BROJ INDEKSA: 17-0032-2010 13

DATUM: 10.2.2010. VRIJEME: OD 14:00 DO 15:30.

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

oxox Broj ↓ bodova

1. Odrediti determinantu matrice $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 2 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 2 & 0 & 0 & 2 & 1 \end{bmatrix}$ — $\cancel{0}$

2. Odrediti domenu i sve asimptote funkcije $f(x) = \dfrac{x^2-2x-3}{x^2+3x+2}$ — 13

3. Istražiti konvergenciju reda: $\displaystyle\sum_{n=1}^{\infty}\left(\frac{5+2n}{5n+3}\right)^{n^2}$ — $\cancel{0}$

4. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije $g(x) = \arctan\left(x^2\right)$. — $\cancel{0}$

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \ln\left(x^2+1\right)$. — $\cancel{0}$

2. $f(x) = \dfrac{x^2-2x-3}{x^2+3x+2}$

$\mathcal{D}(f) = \mathbb{R}\setminus\{-2,-1\}$ ✓

13

$x^2+3x+2=0$

$x_{1,2} = \dfrac{-3\pm\sqrt{3^2-4\cdot1\cdot2}}{2} = \dfrac{-3\pm\sqrt{9-8}}{2} = \dfrac{-3\pm1}{2}$

$x_1 = -2$

$x_2 = -1$

NA STRAN GDJE POSTOJI H.A. NE MOŽE BITI K.A.

H.A.

$\lim\limits_{x\to\infty} \dfrac{x^2-2x-3}{x^2+3x+2} \Big/ \cdot x^2$

$\lim\limits_{x\to\infty} \dfrac{\frac{x^2}{x^2}-\frac{2x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{3x}{x^2}+\frac{2}{x^2}} = \dfrac{1}{1} = 1$ ✓

V.A.

$\lim\limits_{x\to-2_+} \dfrac{(-2_+)^2-2\cdot(-2_+)-3}{(-2_+)^2+3\cdot(-2_+)+2} =$

$\lim\limits_{x\to-2_+} \dfrac{4_++4_+-3}{4_+-6_++2} = +\infty$

$\lim\limits_{x\to-2_-} \dfrac{(-2_-)^2-2\cdot(-2_-)-3}{(-2_-)^2+3\cdot(-2_-)+2} = \dfrac{4_-+4_--3}{4_--6_-+2} = -\infty$

$\lim\limits_{x\to-1_+} \dfrac{(-1_+)^2-2(-1_+)-3}{(-1_+)^2+3(-1_+)+2} = \dfrac{1_++2_+-3}{1_+-3_++2} = +\infty$ $\frac{0}{0}$ X

$\lim\limits_{x\to-1_-} \dfrac{(-1_-)^2-2(-1_-)-3}{(-1_-)^2+3(-1_-)+2} = \dfrac{1_-+2_-\mp3}{1_--3_-+2} = -\infty$ $\frac{0}{0}$ X

- kose asimtote? PRIMIJENITI L'HOPITALOVO PRAVILO!

?

$f(x) = \dfrac{\frac{x^2-2x-3}{x^2+3x+2}}{\frac{x}{1}} = \dfrac{x^2-2x-3}{x(x^2+3x+2)} = \dfrac{x^2-2x-3}{x^3+3x^2+2x}$

$f(x) = \dfrac{x^2-2x-3}{x^2+3x+2} - \underset{kx}{\underset{\Uparrow}{x}} = \dfrac{x^2-2x-3-x(x^2+3x+2)}{x^2+3x+2} = \dfrac{x^2-2x-3-x^3-3x^2-2x}{x^2+3x+2}$

$= \dfrac{-x^3-2x^2-4x-3}{x^2+3x+2}$?

IME I PREZIME: Mateja Pećarić BROJ INDEKSA: 17-0032-2010

3. $\sum_{n=1}^{\infty}\left(\frac{5+2n}{5n+3}\right)^{n^2}$

$$=\left(\frac{1}{1-\frac{5+2n}{5n+3}}\right)=\left(\frac{1}{\frac{5n+3-5+2n}{5n+3}}\right)^{n^2}=\left(\frac{1}{\frac{7n-2}{5n+3}}\right)^{n^2}=\left(\frac{5n+3}{7n-2}\right)^{n^2}$$

$\emptyset$

VIDI RIJEŠENI SEMINAR 9.

4. $g(x)=\text{arc}\tan(x^2)$

- funkcija je periodična jer ima trigonometrijsku funkciju.

$\mathcal{D}(\text{arctan})=\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ X

$\mathcal{D}(x^2)=\mathbb{R}$ $\mathcal{D}(x)=\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ X

arctan nije trigonometrijska već ciklometrijska.
Trigonometrijske su sin, cos, tan, cot.
Nije niti svaka funkcija koja ima trigonometrijsku periodična.

Primjer: $f(x)=\cos(\ln(x))$

$g'(x)=\text{arctg}(x^2)'$

$$=\frac{1}{\sqrt{1-x^2}}\cdot 2=\frac{2}{\sqrt{1-x^2}}$$ X

$$g''(x)=\frac{2'\cdot(\sqrt{1-x^2})-2\cdot(\sqrt{1-x^2})'}{(\sqrt{1-x^2})^2}=\frac{-2\cdot(\sqrt{1-x^2})'}{1-x^2}=$$

$$=\frac{-2\cdot\left(\frac{1}{2\sqrt{1-x^2}}\right)}{1-x^2}=\frac{-\frac{2}{2\sqrt{1-x^2}}}{1-x^2}$$

$\mathcal{D}(\text{arctan})=\mathbb{R}$

VIDI SEMINAR 3

$(\text{arctan}\,x)'=\frac{1}{1+x^2}$

$$=-\frac{2}{(2\sqrt{1-x^2})(1-x^2)}$$

$\emptyset$

Ime i prezime: Mateja Pećarić **Broj indeksa:** 17-0032-2010

1.

$$A=\begin{array}{c} + \\ - \\ + \\ - \\ + \end{array}\begin{bmatrix} 1 & 2 & 0 & 0 & 2 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 2 & 0 & 0 & 2 & 1 \end{bmatrix}$$

$$1\begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{vmatrix} -2\begin{vmatrix} 2 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{vmatrix} +0\begin{vmatrix} 2 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{vmatrix} -0\begin{vmatrix} 2 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 1 \end{vmatrix}$$

$$+2\begin{vmatrix} 2 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 1 & 2 \end{vmatrix} = 1\cdot\left(1\begin{vmatrix} 0 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} -2\begin{vmatrix} 2 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} +0\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix} -0\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 1 & 2 \end{vmatrix}\right)$$

$$-2\left(2\begin{vmatrix} 0 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} -2\begin{vmatrix} 0 & 0 & 2 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} +0\begin{vmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix} -0\begin{vmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 1 & 2 \end{vmatrix}\right) =$$

izračunato na sljedećoj strani (ZABORAVILA)

$$= \left(1\left(0\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} -2\begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} +0\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix}\right) -2\left(2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} -2\begin{vmatrix} 0 & 0 \\ 2 & 1 \end{vmatrix} +0\begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix}\right)\right)$$

$$-2\left(2\left(0\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} -2\begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} +0\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix}\right) -2\left(0\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} -2\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} +0\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix}\right)\right)$$

$$= \left(\overbrace{-2\left((2\cdot1)-(0\cdot2)\right)}^{-4}\right) \underbrace{-2\left(2\left((1\cdot1)-(2\cdot2)\right) -2\left((0\cdot1)-(0\cdot2)\right) +0\left((0\cdot2)-(0\cdot1)\right)\right)}_{-4}$$

$$\underbrace{-2\left(2\left(-2\left((2\cdot1)-(0\cdot2)\right)\right) -2\left(-2\left((0\cdot1)-(2\cdot2)\right)\right)\right)}_{-8}$$

$$= -4-4-8+32=16$$

IME I PREZIME: Mateja Pećarić

BROJ INDEKSA: 17-0032-2010

$$=2\left(2\begin{vmatrix} +2&0&0\\ -0&2&0\\ +2&1&2\end{vmatrix} -1\begin{vmatrix}+0&0&2\\-0&2&0\\+2&1&2\end{vmatrix} +2\begin{vmatrix}+0&0&2\\-2&0&0\\+2&1&2\end{vmatrix} -0\begin{vmatrix}+0&0&2\\-2&0&0\\+0&2&0\end{vmatrix}\right)$$

$$\underbrace{=2\left(\underbrace{\left(2\left(2\begin{vmatrix}2&0\\1&2\end{vmatrix}-0\begin{vmatrix}0&0\\1&2\end{vmatrix}+2\begin{vmatrix}0&0\\2&0\end{vmatrix}\right)\right)}_{16}\underbrace{-1\left(2\begin{vmatrix}0&2\\2&0\end{vmatrix}\right)}_{4}+2\underbrace{\left(-2\begin{vmatrix}0&2\\1&2\end{vmatrix}+2\begin{vmatrix}0&2\\0&0\end{vmatrix}\right)}_{-4}\right)}$$

$$2(16+4-4)=32 \quad \times$$

VIDI TOMA MEDIĆ

5. $h(x)=\ln(x^2+1)$

$\mathcal{D}(\ln)=\langle 0,+\infty\rangle$

$\mathcal{D}(h)=\langle 0,+\infty\rangle \times$

nul točke

$y=\ln(x^2+1)$

$y=0$

$0=\ln(x^2+1)$

$\ln=0$

$x^2+1=0$

$x^2=1$

$x=\pm 1$

$x=0$

$y=\ln(0+1)$

$y=\ln$

$T_1(0,\ln)$

$T_2(1,0)$

$\emptyset$

UVRSTITI U KALKULATOR: $h(-1)=\ldots$

BODUJE SE GRAF.

NA DOBROM STE PUTU. NE ODUSTATI. JOŠ VJEŽBATI.

Popuniti odmah!

IME I PREZIME: Toma Medić **BROJ INDEKSA:** 17-2-0052 15

DATUM: 10.02.2011. **VRIJEME:** OD 13:15 DO 14:45 .

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE <u>JEDNOSTRANO</u> NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

oxox
Broj ↓
bodova

1. Odrediti determinantu matrice $A = \begin{array}{c} \begin{matrix} + & - & + & - & + \end{matrix} \\ \begin{bmatrix} 1 & 2 & 0 & 0 & 2 \\ -2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 2 & 0 & 0 & 2 & 1 \end{bmatrix} \end{array}$ Ø

2. Odrediti domenu i sve asimptote funkcije $f(x) = \dfrac{x^2 - 2x - 3}{x^2 + 3x + 2}$ 10

3. Istražiti konvergenciju reda: $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5 + 2n}{5n + 3} \right)^{n^2}$

4. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije $g(x) = \arctan(x^2)$. 5

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \ln(x^2 + 1)$. Ø

1.)

$$A = \begin{bmatrix} \cancel{1} & \cancel{2} & \cancel{0} & \cancel{0} & \cancel{2} \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 2 & 0 & 0 & 2 & 1 \end{bmatrix} = 1 \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix} - 2 \begin{bmatrix} 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 2 & 0 & 2 & 1 \end{bmatrix} + 2 \begin{bmatrix} 2 & 1 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 2 & 0 & 0 & 2 \end{bmatrix} =$$

$$= 1\left[1\begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix} - 2\begin{pmatrix} 2 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}\right] - 2\left[2\begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix} - 2\begin{pmatrix} 0 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}\right]$$

$$+ 2\left[-2\begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix} + 2\begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\right] =$$

$$= 1\left[1\left(-2\begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}\right) - 2\left(2\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\right)\right] - 2\left[2\left(-2\begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}\right) - 2\left(-2\begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix}\right)\right]$$

$$+ 2\left[-2\left(1\underbrace{\begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix}}_{-4} - 2\begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}\right) + 2\left(2\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}\right)\right] =$$

$$= 1\left[1(-4) - 2(-6)\right] - 2\left[2(-4) - 2(8)\right] + 2\Big[\underbrace{-2\big(\underbrace{\overset{\times}{(-3)} - 2(2)}_{-4-4=-8}\big)}_{16} + \underbrace{2(8)}_{16}\Big]_{32} =$$

$$= 1[8] - 2[-24] + 2\underbrace{[30]}_{2 \cdot 32} = 8 + 48 + \underbrace{60}_{64} = 116 \quad \underline{120} \quad det(A) = 120$$

$$D(4) = 116$$

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~~4,0401 - 8,0802 - 3~~

~~4,0401 + 12,1203 + 2~~

5.) $h(x) = \ln(x^2+1)$ $\mathcal{D}(f) = \mathbb{R}$

1° DOMENA FUNKCIJE

$x^2+1>0$ OVO JE UVIJEK VEĆE OD NULE ✓

$D(f) = x \in \mathbb{R}$ ✓

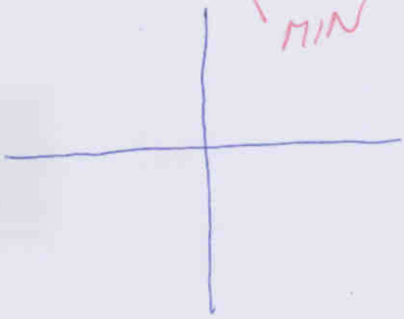
INTERVALI MONOTONOSTI

	$-\infty$	0	∞
$f'(x)$	$-$		$+$
$f(x)$	↘	Max ✗ MIN	↗

2° NULTOČKE I SJECIŠTE S Y OSI

NEMA NULTOČAKA ✗, A ŠTO JE SA $f(0)=0$

$f(0) = \ln 1 = 0$ $A(0,0)$



3° (NE)PARNOST

$f(-x) = \ln((-x)^2+1)$

$f(-x) = \ln(x^2+1)$ PARNA ✓

4° NIJE PERIODIČNA

U OVOM ZADATKU BODUJE SE GRAF.

5° ASIMPTOTE

NEMA VERTIKALNIH ASIMPTOTA

HORIZONTALNE ASIMPTOTE

$\lim_{x\to\pm\infty} \ln(x^2+1) =$

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~~$\lim_{x\to\pm\infty} \frac{x^2}{x^2-1}$~~ =

6° $h'(x) = \frac{1}{x^2+1} \cdot 2x =$ ✓

$h'(x) = \frac{2x}{x^2+1} \Rightarrow h'(0) = 0$

$\frac{2x}{x^2+1} = 0$

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2. $f(x)=\frac{x^2-2x-3}{x^2+3x+2}$

1° DOMENA FUNKCIJE

$x^2+3x+2\neq 0$

$x_{1,2}\neq\frac{-3\pm\sqrt{1}}{2}$

$x_{1,2}\neq\frac{-3\pm 1}{2}$ $x_1\neq -2$ $x_2\neq -1$

$D(f) = x\in\mathbb{R}\setminus\{-2,-1\}$ ✓

2° ASIMPTOTE

VERTIKALNE ASIMPTOTE

$\lim_{x\to -2^-}\frac{x^2-2x-3}{x^2+3x+2}$

$\lim_{x\to -2^-}\frac{4+4-3}{4-12+2}=-\frac{5}{6}$ ✗

2.1° $\lim_{x\to -1}\frac{x^2-2x-3}{x^2+3x+2}=\lim_{x\to -1}\frac{1+2-3}{1+3+2}=\frac{0}{6}=0$ $=\frac{0}{0}$

$V.A_1=-\frac{5}{6}$ ✗

$V.A_2=0$ ✗

2.2° HORIZONTALNE ASIMPTOTE

$\lim_{x\to\pm\infty}\frac{x^2-2x-3\ (:x^2)}{x^2+3x+2\ (:x^2)}=\frac{\infty}{\infty}=$

$\lim_{x\to\pm\infty}\frac{1-\frac{2}{x}-\frac{3}{x^2}}{1+\frac{3}{x}+\frac{2}{x^2}}=\frac{1}{1}=1$

HORIZONTALNA ASIMPTOTA JE 1.

$H.A=1$ ✓

10

Ime i prezime: Toma Medić **Broj indeksa:** 17-2-0052

2.3° KOSE ASIMPTOTE

$y = kx + l$

$k = \frac{f(x)}{x}$

5.) $g(x) = \arctan(x^2)$

DOMENA

$D(f) = x \in R$

ZAPIS : $\mathcal{D}(f) = \mathbb{R}$

5

FUNKCIJA JE PARNA ✓

$g'(x) = \frac{1}{x^4 - 1} \cdot 2x$ ✗

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PERIODIČNOST ?

$g'(x) = \frac{2x}{x^4 - 1}$

$g''(x) = \frac{2(\cancel{x^4} - 1) - 2x(3x^2)}{(x^4 - 1)^2}$ ✗

KAKVO SKRAĆIVANJE ?

Popuniti odmah!

IME I PREZIME: DANIJEL MILOVAC BROJ INDEKSA: 17-2-0003-2010

DATUM: VRIJEME: OD 13.45 DO 14:40 .

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

OXOX

Broj ↓ bodova

4

1. Odrediti determinantu matrice $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 2 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 2 & 0 & 0 & 2 & 1 \end{bmatrix}$ ~~2~~

2. Odrediti domenu i sve asimptote funkcije $f(x) = \dfrac{x^2-2x-3}{x^2+3x+2}$ 3

3. Istražiti konvergenciju reda: $\displaystyle\sum_{n=1}^{\infty}\left(\frac{5+2n}{5n+3}\right)^{n^2}$

4. Ispitati domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije $g(x) = \arctan(x^2)$. 1

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \ln(x^2+1)$.

$$A = \begin{bmatrix} 1&2&0&0&2\\ 2&1&2&0&0\\ 0&2&0&2&0\\ 0&0&2&1&2\\ 2&0&0&2&1 \end{bmatrix} = \begin{bmatrix} 1&2&0&0&2\\ 0&-3&2&0&-4\\ 0&2&0&2&0\\ 0&0&2&1&2\\ 0&-4&0&2&-3 \end{bmatrix} = 1^{1+1}\cdot 1 \begin{bmatrix} -3&2&0&-4\\ 2&0&2&0\\ 0&2&1&2\\ -4&0&2&-3 \end{bmatrix} = \begin{bmatrix} 3&-2&0&4\\ 2&0&2&0\\ 0&2&1&2\\ -4&0&2&-3 \end{bmatrix} = \begin{bmatrix} 3&-2&0&4\\ 2&0&2&0\\ 3&0&1&6\\ -4&0&2&-3 \end{bmatrix} =$$

$1r\cdot(-2)+2r$

$1r\cdot(-2)+5r$

$1r\cdot(-1)$

$1r+3r$

$$= 1\cdot(-1)^{1+2}\cdot(-2)\begin{bmatrix} 2&2&0\\ 3&1&6\\ -4&2&-3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2&2&0\\ 3&1&6\\ -8&4&-6 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2&2&0\\ -5&5&0\\ -8&4&-6 \end{bmatrix} = (-2)\cdot\frac{1}{2}\,1^{3+3}\cdot(-6)\underbrace{\begin{bmatrix} 2&2\\ -5&5 \end{bmatrix}}_{2\cdot 5-2\cdot(-5)=20}$$

$3r\cdot 2$

$3r+2r$

KOD OVE OPERACIJE DETERMINANTA SE DIJELI S 2 } UVODI SE FAKTOR $\frac{1}{2}$

$= -12(10-10) = -12\cdot 0 = 0$ ×

$\det A = 0$

$\det(A) = 120$

~~0~~

(2) $f(x) = \dfrac{x^2-2x-3}{x^2+3x+2}$

$D(f) = \mathbb{R} \setminus \{-2,-1\}$ ✓ 3

$x^2-2-3 \neq 0$? x^2-2x-3

$x_{1,2} = \dfrac{-3 \pm \sqrt{1}}{2}$

VIDI PEČARIĆ

~~0~~

$x_1 = \dfrac{-3-1}{2} = \dfrac{-4}{2} = -2$ $x_1 \neq -2$

$x_2 = \dfrac{-3+1}{2} = \dfrac{-2}{2} = -1$ $x_2 \neq -1$

IME I PREZIME: DANJEL MILOVAC

BROJ INDEKSA:

$g(x) = \arctan(x^2)$

$g'(x) = \frac{1}{1+(x^2)^2} \cdot 2x$

$g'(x) = \frac{1}{1+x^4} \cdot 2x$

$\boxed{\text{NAUČITI DERIVIRATI KOMPOZICIJU FUNKCIJE!}}$

$g''(x) = \frac{1}{1+x^4}$

$g''(x) = \frac{(1)' \cdot (1+x^4) - (1+x^4)' \cdot 1}{(1+x^4)^2}$

$g''(x) = \frac{\cancel{1+x^4} - 4x \cdot 1}{(1+x^4)^{\cancel{2}}}$

$g''(x) = \frac{-4x}{1+x^4}$

Ø

funkcija nije periodična jer nije trigonometrijska ✓

1