Popuniti odmah!

IME I PREZIME: LOVRE KERES BROJ INDEKSA: 57933 (26)

DATUM: 10.02.2011. VRIJEME: OD 11^{30} DO 13:15.

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. oxoo

Broj ↓ bodova

1. Koju relaciju zadovoljava inverz matrice? Provjeriti tu relaciju za inverz matrice (ako postoji)

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{bmatrix}$$

20

2. Među kompleksnim brojevima odrediti $\sqrt[4]{\dfrac{3+2i}{2-3i}}$.

0

3. Zadana je funkcija $f(x) = e^x$ i funkcija $g(x) = \dfrac{-1}{x^2}$. Odrediti domenu i sve asimptote funkcije $h(x) = f(g(x))$.

0

4. Ispitati domenu i drugu derivaciju funkcije $S(x) = \ln\left(\dfrac{x+1}{x-1}\right)$.

6

5. Na temelju ispitivanja toka napraviti skicu grafa funkcije $S(x)$ iz zadatka 4.

0

1. $A = \left[\begin{array}{cccc|cccc} 0&1&0&2&1&0&0&0 \\ 1&0&2&0&0&1&0&0 \\ 0&2&0&1&0&0&1&0 \\ 2&0&1&0&0&0&0&1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0 \\ 0&1&0&2&1&0&0&0 \\ 0&2&0&1&0&0&1&0 \\ 2&0&1&0&0&0&0&1 \end{array}\right] \cdot(-2)$

$\left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0 \\ 0&1&0&2&1&0&0&0 \\ 0&2&0&1&0&0&1&0 \\ 0&0&-3&0&0&-2&0&1 \end{array}\right] \cdot(-2) \sim \left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0 \\ 0&1&0&2&1&0&0&0 \\ 0&0&0&-3&-2&0&1&0 \\ 0&0&-3&0&0&-2&0&1 \end{array}\right] \cdot\left(\frac{1}{-3}\right) \sim \left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0 \\ 0&1&0&2&1&0&0&0 \\ 0&0&-3&0&0&-2&0&1 \\ 0&0&0&-3&-2&0&1&0 \end{array}\right]$

$\left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0 \\ 0&1&0&2&1&0&0&0 \\ 0&0&1&0&0&\frac{2}{3}&0&-\frac{1}{3} \\ 0&0&0&-3&-2&0&1&0 \end{array}\right] \cdot(+2) \sim \left[\begin{array}{cccc|cccc} 1&0&0&0&0&-\frac{1}{3}&0&\frac{2}{3} \\ 0&1&0&2&1&0&0&0 \\ 0&0&1&0&0&\frac{2}{3}&0&-\frac{1}{3} \\ 0&0&0&-3&-2&0&1&0 \end{array}\right] \cdot\left(-\frac{1}{3}\right)$

$\left[\begin{array}{cccc|cccc} 1&0&0&0&0&-\frac{1}{3}&0&\frac{2}{3} \\ 0&1&0&2&1&0&0&0 \\ 0&0&1&0&0&\frac{2}{3}&0&-\frac{1}{3} \\ 0&0&0&1&\frac{2}{3}&0&-\frac{1}{3}&0 \end{array}\right] \cdot(-2) \sim \left[\begin{array}{cccc|cccc} 1&0&0&0&0&-\frac{1}{3}&0&\frac{2}{3} \\ 0&1&0&0&-\frac{1}{3}&0&\frac{2}{3}&0 \\ 0&0&1&0&0&\frac{2}{3}&0&-\frac{1}{3} \\ 0&0&0&1&\frac{2}{3}&0&-\frac{1}{3}&0 \end{array}\right]$

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$$A^{-1} = \begin{bmatrix} 0 & -1/3 & 0 & 2/3 \\ -1/3 & 0 & 2/3 & 0 \\ 0 & 2/3 & 0 & -1/3 \\ 2/3 & 0 & -1/3 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0-1/3+0+4/3, & 0+0+0+0, & 0-2/3+0+2/3, & 0+0+0+0 \\ 0+0+0+0, & -1/3+0+4/3+0, & 0+0+0+0, & -2/3+0+2/3+0 \\ 0+2/3+0-2/3, & 0+0+0+0, & 0+4/3+0-1/3, & 0+0+0+0 \\ 0+0+0+0, & 2/3+0-2/3+0, & 0+0+0+0, & 4/3+0-1/3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$AA^{-1} = I$ ✓

Matrica A ima inverz !!! ✓

POGREŠNO PREPISAN ZADATAK!

2. $z^4 = \sqrt[4]{\frac{2+2i}{2-3i}}$

$\frac{2+2i}{2-3i}$ RACIONALIZIRATI...

$\frac{2+2i}{2-3i} = r^4(\cos(\varphi) - i\sin(\varphi))$

$r = \sqrt{x^2+y^2} = \sqrt{\left(\frac{2}{2}\right)^2 + \left(\frac{2}{3}\right)^2}$

$\frac{2+2i}{2-3i} = 1.202^4\left(\cos\frac{2.554+k\cdot 2\pi}{4} + i\sin\right) = \sqrt{1+\frac{4}{9}} = \sqrt{\frac{13}{9}}$

$= 1.202$

$z^4 = 1.202\left(\cos\frac{2.554+k\cdot 2\pi}{4} + i\sin\right)$

$\varphi = \pi + \arctan\left(\frac{\operatorname{Im}\frac{2+2i}{2-3i}}{\operatorname{Re}\frac{2+2i}{2-3i}}\right)$

$= \pi + \arctan\left(\frac{-\frac{2}{3}}{\frac{2}{2}}\right) = \left(-\frac{4}{6}\right) = \left(-\frac{2}{3}\right)$

$= \pi + \arctan\left(-\frac{2}{3}\right) = 2.554^0$

$k = 0, 1, 2, 3$

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$k=0$

$$z_1 = 1.202\left(\cos\frac{2.554+0\cdot 2\pi}{4} + i\sin\frac{2.554+0\cdot 2\pi}{4}\right)$$

$$z_1 = 0.965 + 0.716i \text{ //}$$

$k=1$

$$z_2 = 1.202\left(\cos\frac{2.554+1\cdot 2\pi}{4} + i\sin\frac{2.554+1\cdot 2\pi}{4}\right)$$

$$z_2 = 1.202\left(\cos\frac{8.834}{4} + i\sin\frac{8.834}{4}\right)$$

$$z_2 = -0.716 + 0.966i \text{ //}$$

$k=2$

$$z_3 = 1.202\left(\cos\frac{2.554+2\cdot 2\pi}{4} + i\sin\frac{2.554+2\cdot 2\pi}{4}\right)$$

$$z_3 = 1.202\left(\cos\frac{15.114}{4} + i\sin\frac{15.114}{4}\right)$$

$$z_3 = -0.966 - 0.715i \text{ //}$$

$k=3$

$$z_4 = 1.202\left(\cos\frac{2.554+3\cdot 2\pi}{4} + i\sin\frac{2.554+3\cdot 2\pi}{4}\right)$$

$$z_4 = 1.202\left(\cos\frac{21.394}{4} + i\sin\frac{21.394}{4}\right)$$

$$z_4 = 0.714 - 0.967i \text{ //}$$

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3.

$f(x) = e^x$ $\quad D(f) = \mathbb{R}$ $\quad \langle -\infty, \infty \rangle$

Funkcija nije periodična zato što nema trigonometrijskih funkcija!!!

V. i A. Nema

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{e^x}{x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{e^x \,/:x}{x \,/:x}$$

$$\lim_{x\to\infty} \frac{\frac{e^x}{x}}{1} = \frac{0}{1} = 0$$

TRAŽI SE DOMENA I ASIMPTOTE FUNKCIJE $h(x)$, A NE $f(x)$ i $g(x)$!

$$L = \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} (e^x) = \infty$$

Nema kosih asimptota (lijeve i desne) i nema horizontalnih ni vertikalnih asimpt.

$g(x) = \frac{-1}{x^2}$ $\quad D(g) = \mathbb{R} \setminus \{0\}$

Funkcija nije periodična zato što nema trigonometrijskih funkcija!!!

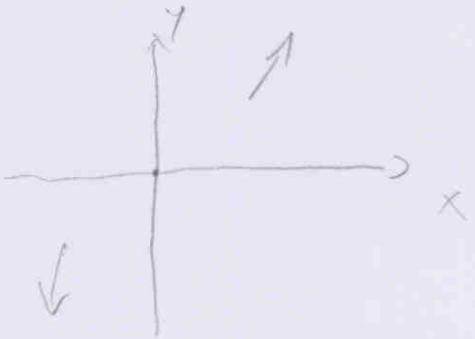
$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{-1}{x^2}}{\frac{x}{1}} = \frac{-1}{x^3} \,/:x^3 = \frac{0}{1} = 0$$

nema kosih asimptota

$$L = \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} \frac{-1}{x^2} \,/:x^2 = \frac{0}{1} = 0$$

$$\lim_{x\to 0_+} \frac{-1}{x^2} = \frac{-1}{0_+} = -\infty$$

$$\lim_{x\to 0_-} \frac{-1}{x^2} = \frac{-1}{0_-} = +\infty$$



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4. $S(x) = \ln\left(\frac{x+1}{x-1}\right)$ $D(S) = \mathbb{R}\setminus\{1\}$ X

$x - 1 \neq 0$

$x \neq 1$

$D(\ln) = \langle 0, +\infty \rangle$

$\Rightarrow \frac{x+1}{x-1} > 0 \Rightarrow \ldots$ VIDI ČULINA MARKO

$S'(x) = \frac{1}{\frac{x+1}{x-1}} \cdot \frac{1\cdot(x-1) - (x+1)\cdot 1}{(x-1)^2} = \frac{x-1}{x+1} \cdot \frac{(x-1)-(x+1)}{(x-1)^2}$ ✓ X

6

$S''(x) = ?$ VIDI PENJALOV

5. $S(x) = \ln\left(\frac{x+1}{x-1}\right)$ $D(S) = \mathbb{R}\setminus\{1\}$

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\ln\left(\frac{x+1}{x-1}\right)/:x}{x/:x} = \frac{\ln\left(\frac{1}{1}\right)}{1} = 0$

$L = \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} \left(\ln\left(\frac{x+1}{x-1}\right)\right)\Big|_{:x} = \ln\left(\frac{1}{1}\right) = 0$

$\lim_{x\to 1_+} \left(\ln \frac{x+1}{x-1}\right) = \lim_{x\to 1_+} \left(\ln \frac{2}{0_-}\right) = -\infty$

$\lim_{x\to 1_-} \left(\ln \frac{x+1}{x-1}\right) = \lim_{x\to 1_-} \left(\ln \frac{2}{0_+}\right) = +\infty$

Funkcija nije omeđena ni odozgo ni odozdo

V. A = $x = 1$

Popuniti odmah!

IME I PREZIME: MATE BALJAK **BROJ INDEKSA:** 57115

DATUM: VRIJEME: OD 11:30 DO 12:50

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

oxoo

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26

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6

5. Na temelju ispitivanja toka napraviti skicu grafa funkcije $S(x)$ iz zadatka 4.

1. $A \cdot A^{-1} = I$

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \leftrightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ \\ \cdot(-2) \\ (-2) \end{matrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & -2 & 0 & 1 \end{array}\right] \cdot\left(-\tfrac{1}{3}\right) \qquad \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & -3 & -2 & 0 & 1 & 0 \end{array}\right] \cdot(-2)$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -\frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 2 & 1 & -\frac{4}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & -3 & -2 & 0 & 1 & 0 \end{array}\right] \cdot\left(-\tfrac{1}{3}\right) \qquad \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -\frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 2 & 1 & -\frac{4}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} & 0 \end{array}\right] \cdot(-2)$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -\frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & -\frac{4}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} & 0 \end{array}\right]$$

IME I PREZIME: MATE BALJAK BROJ INDEKSA: 57115

$$\left[\begin{array}{cccc|cccc} 0&1&0&2&1&0&0&0\\ 1&0&2&0&0&1&0&0\\ 0&2&0&1&0&0&1&0\\ 2&0&1&0&0&0&0&1 \end{array}\right] \hookleftarrow \left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0\\ 0&1&0&2&1&0&0&0\\ 0&2&0&1&0&0&1&0\\ 2&0&1&0&0&0&0&1 \end{array}\right) (-2)$$

$$\left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0\\ 0&1&0&2&1&0&0&0\\ 0&2&0&1&0&0&1&0\\ 0&0&-3&0&0&-2&0&1 \end{array}\right) (-2) \quad \left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0\\ 0&1&0&2&1&0&0&0\\ 0&0&0&-3&-2&0&1&0\\ 0&0&-3&0&0&-2&0&1 \end{array}\right] \hookleftarrow$$

$$\left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0\\ 0&1&0&2&1&0&0&0\\ 0&0&-3&0&0&-2&0&1\\ 0&0&0&-3&-2&0&1&0 \end{array}\right] \cdot\left(-\tfrac{1}{3}\right) \quad \left[\begin{array}{cccc|cccc} 1&0&2&0&0&1&0&0\\ 0&1&0&2&1&0&0&0\\ 0&0&1&0&0&\tfrac{2}{3}&0&-\tfrac{1}{3}\\ 0&0&0&-3&-2&0&1&0 \end{array}\right) (-2)$$

$$\left[\begin{array}{cccc|cccc} 1&0&0&0&0&-\tfrac{1}{3}&0&\tfrac{2}{3}\\ 0&1&0&2&1&0&0&0\\ 0&0&1&0&0&\tfrac{2}{3}&0&-\tfrac{1}{3}\\ 0&0&0&-3&-2&0&1&0 \end{array}\right] \cdot\left(-\tfrac{1}{3}\right) \quad \left[\begin{array}{cccc|cccc} 1&0&0&0&0&-\tfrac{1}{3}&0&\tfrac{2}{3}\\ 0&1&0&2&1&0&0&0\\ 0&0&1&0&0&\tfrac{2}{3}&0&-\tfrac{1}{3}\\ 0&0&0&1&\tfrac{2}{3}&0&-\tfrac{1}{3}&0 \end{array}\right) (-2)$$

$$\left[\begin{array}{cccc|cccc} 1&0&0&0&0&-\tfrac{1}{3}&0&\tfrac{2}{3}\\ 0&1&0&0&-\tfrac{1}{3}&0&\tfrac{2}{3}&0\\ 0&0&1&0&0&\tfrac{2}{3}&0&-\tfrac{1}{3}\\ 0&0&0&1&\tfrac{2}{3}&0&-\tfrac{1}{3}&0 \end{array}\right]$$

$$\left[\begin{array}{cccc} 0&1&0&2\\ 1&0&2&0\\ 0&2&0&1\\ 2&0&1&0 \end{array}\right] \times \left[\begin{array}{cccc} 0&-\tfrac{1}{3}&0&\tfrac{2}{3}\\ -\tfrac{1}{3}&0&\tfrac{2}{3}&0\\ 0&\tfrac{2}{3}&0&-\tfrac{1}{3}\\ \tfrac{2}{3}&0&-\tfrac{1}{3}&0 \end{array}\right] = \left[\begin{array}{cccc} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{array}\right] \quad \checkmark \quad 20$$

IME I PREZIME: MATE BALJAK

BROJ INDEKSA: 57115

(4) $S(x) = \ln\left(\frac{x+1}{x-1}\right)$ $\ln(x)' = \frac{1}{x}$ $\left(\frac{x+1}{x-1}\right)' = \frac{(x+1)'\cdot(x-1) - (x+1)\cdot(x-1)'}{(x-1)^2}$

$S'(x) = \frac{-2}{(x-1)^2}$

$= \frac{1\cdot(x-1) - (x+1)\cdot 1}{x^2 + 2x\cdot -1 + (-1)^2}$

$S'(x) = \frac{1}{\frac{-2}{(x-1)^2}}$ VIDI KOTLAR PENJALOV

$= \frac{(x-1)-(x+1)}{x^2-2x+1} = \frac{x-1-x-1}{x^2-2x+1}$

$= \frac{-2}{(x-1)^2} \cdot \frac{(x+1)}{(x+1)}$ X

$S''(x) =$

$= \frac{-2x-2}{x-1}$

DOMENA ?

VIDI ČULINA

$\left(\frac{-2}{(x-1)^2}\right)' = \frac{(-2)'\cdot(x-1)^2 - (-2)\cdot((x-1)^2)'}{((x-1)^2)^2}$

$\frac{0\cdot(x-1)^2 + 2\cdot 2x}{2}$ X

$-\frac{n}{x^{n-1}}$

$(-2)' = 0$

$\frac{1}{x}' = -\frac{1}{x^2}$