Popuniti odmah!

IME I PREZIME: TOMISCAI TUTA BROJ INDEKSA: 0437

DATUM: VRIJEME: OD 14:45 DO 16:00

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

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Broj ↓ bodova

1. Pravac p prolazi točkama A i B, a pravac q točkama A i C. Koliko iznosi kut između pravaca $\measuredangle(p, q)$ ako je $A(2, -3, 1)$, $B(-1, 2, -3)$ i $C(1, -1, -2)$?

2. Među kompleksnim brojevima riješiti jednadžbu: $z^3 - \left(\overline{i+1}\right)^5 = 0$.

3. Odrediti sve asimptote funkcije $f(x) = \arctan\left(\mathrm{e}^x\right)$.

4. Odrediti drugu derivaciju funkcije $f(x) = \ln\left(x - \frac{1}{x}\right)$.

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $f(x) = \frac{x^2 - 1}{x + 2}$.

1. $A\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ $B\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$

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4. $g(x) = \ln\left(x - \frac{1}{x}\right)$

$g'(x) = \ln \frac{1}{x - \frac{1}{x}} \cdot \left(x - \frac{1}{x}\right)'$ ×

$g'(x) = \ln \frac{1}{\frac{x^2}{x} - \frac{1}{x}} \cdot \left(1 - \frac{1' \cdot x - 1 \cdot x'}{x^2}\right)$

$g'(x) = \ln \frac{1}{\frac{x^2 - 1}{x}} \cdot \left(1 - \frac{-1}{x^2}\right)$

$g'(x) = \ln \frac{x}{x^2 - 1} \cdot \left(\frac{x^2}{x^2} - \frac{-1}{x^2}\right)$

$g'(x) = \ln \frac{x^{-1}}{x^2 - 1} \cdot \left(\frac{x^2 + 1}{x^2}\right)$

$g'(x) = \ln - x$

$g''(x) = \ln - \frac{1}{x}$

$g(x) = \ln\left(x - \frac{1}{x}\right)$

$= g_1(g_2(x))$

$g'(x) = \left[g_1(g_2(x))\right]'$

$= g_1'(g_2(x)) \cdot g_2(x)$

$\begin{cases} g_1(x) = \ln x & g_1'(x) = \frac{1}{x} \\ g_2(x) = x - \frac{1}{x} & g_2'(x) = 1 + \frac{1}{x^2} \end{cases}$

$g_1'(x) = \frac{1}{x}$

$g_1'(g_2(x)) = \frac{1}{g_2(x)} = \frac{1}{x - \frac{1}{x}}$

$\Rightarrow g_1'(g_2(x)) \cdot g_2'(x) = \frac{1}{x - \frac{1}{x}} \cdot \left(1 + \frac{1}{x^2}\right)$

$= \frac{x}{x^2 - 1} \cdot \frac{x^2 + 1}{x^2}$

$= \frac{x^2 + 1}{x(x-1)(x+1)}$

$= \frac{x^2 + 1}{x^3 - x}$

5.) $f(x) = \frac{x^2 - 1}{x + 2}$

1.) DOMENA

$x + 2 \neq 0$ $D_f(x)$ $x \in \mathbb{R} \setminus \{-2\}$ ✓

$x \neq -2$

2.) NUL TOČKE

$x^2 - 1 = 0$

$x_1 = -1$

$x_2 = 1$

SJECIŠTE U OSI Y

$f(0) = \frac{x^2 - 1}{x + 2}$

$f(0) = \frac{0 - 1}{0 + 2}$

$f(0) = \frac{-1}{2}$

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3. ASIMPTOTE

- VERTIKALNA

$\lim_{x \to -2} = \frac{x^2-1}{x+2}$

$\lim_{x \to -2} = \frac{(-2)^2-1}{-2+2}$

$\lim_{x \to -2} = \frac{4-1}{0}$

$\lim_{x \to -2} = \frac{3}{0} = \infty$ $x = -2$ ✓

- HORIZONTALNA

$\lim_{x \to \infty} = \frac{x^2-1 / \cdot x^2}{x+2 / \cdot x^2}$

$\lim_{x \to \infty} = \frac{\frac{x^2}{x^2} - \frac{1}{x^2} \to 0}{\frac{x}{x^2} + \frac{2}{x^2} \to 0}$

$\lim_{x \to \infty} = \frac{1}{0} = \infty$

- KOSA

$k = \lim \frac{f(x)}{x}$, $l = \lim f(x) - k \cdot x$, $y = kx + l$

$k = \lim_{x \to \infty} \left(\frac{\frac{x^2-1}{x+2}}{\frac{x}{1}} \right)$ $k = \lim_{x \to \infty} \frac{x^2-1 / \cdot x^2}{x^2+2x / \cdot x^2}$ $k = \frac{\frac{x^2}{x^2} - \frac{1}{x^2} \to 0}{\frac{x^2}{x^2} + \frac{2x}{x^2} \to 0}$ $k = \frac{1}{1} = 1$

$l = \lim_{x \to \infty} \frac{x^2-1}{x+2} - x$ $l = \lim_{x \to \infty} \frac{x^2-1-(x(x+2)}{x+2}$

$l = \lim_{x \to \infty} \frac{x^2-1-x^2-2x}{x+2}$ $l = \lim_{x \to \infty} \frac{-1-2x / \cdot x}{x+2 / \cdot x}$ $l = \lim_{x \to \infty} \frac{-\frac{1}{x} \to 0 - \frac{2x}{x} \to 2}{\frac{x}{x} \to 1 + \frac{2}{x} \to 0} = \frac{x}{1} = x$

$l = x$ ✗

$y = kx + l$

$y = 1 \cdot x + x$

$y = x + x$

$y = 2x$ ✗

x	-2	-1	0	1	2
y	-4	-2	0	2	4

$l = -2$

~~$y = 2x - 2$~~

$y = x - 2$

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- KRITIČNE TOČKE

$$f(x)=\frac{x^2-1}{x+2}$$

$$f'(x)=\frac{(x^2-1)'\cdot(x+2)-(x^2-1)\cdot(x+2)'}{(x+2)^2}$$

$$f'(x)=\frac{2x\cdot(x+2)-(x^2-1)}{(x+2)^2}$$

$$f'(x)=\frac{2x^2+4x-x^2+1}{(x+2)^2}$$

$$f'(x)=\frac{x^2+4x+1}{(x+2)^2}$$

- MIN ILI MAX

$a=1$
$b=4$
$c=1$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x_{1,2}=\frac{-4\pm\sqrt{4^2-4\cdot1\cdot1}}{2\cdot1} \qquad x_{1,2}=\frac{-4\pm\sqrt{16-4}}{2} \qquad x_{1,2}=\frac{-4\pm\sqrt{12}}{2} \qquad x_{1,2}=\frac{-4\pm2\sqrt{3}}{2}$$

$$x_1=\frac{-6\sqrt{3}}{2} \qquad x_1=-3\sqrt{3} \qquad x_2=\frac{-2\sqrt{3}}{2} \qquad x_2=-\sqrt{3}$$

≈ -5.2 ≈ -1.7

6. EKSTREMI MONOTONOST

x	$-\infty$		$-3\sqrt{3}$		$-\sqrt{3}$		$+\infty$
		-5		-2		2	
$f'(x)$		$+$		$-$		$+$	
$f(x)$		↗		↘		↗	

$$f'(x)=\frac{(-5)^2+4\cdot(-5)+1}{(-5+2)^2}$$

$$f'(x)=\frac{25-20+1}{9}=\frac{6}{9}+$$

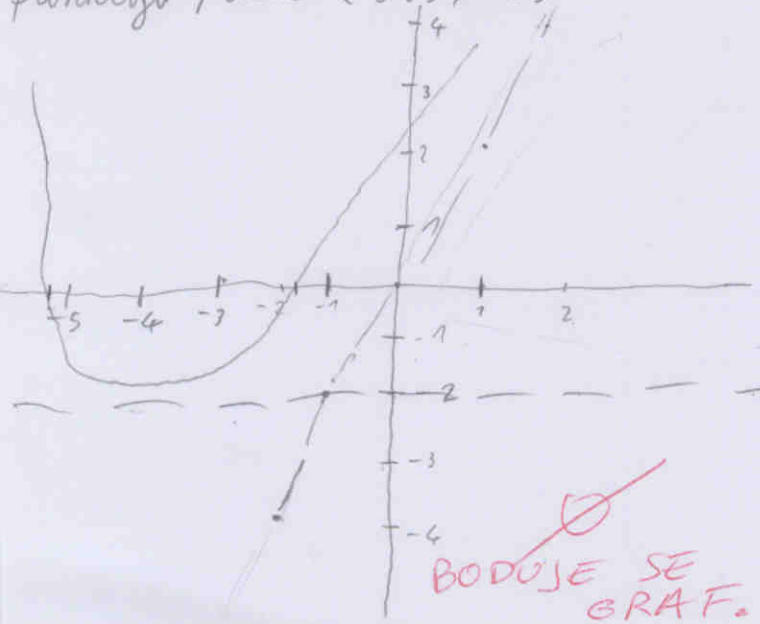
$$f'(x)=\frac{(-2)^2+4\cdot(-2)+1}{(-4+2)^2}$$

$$f'(x)=\frac{-4-8+1}{4}=\frac{-11}{4}-$$

$$f'(x)=\frac{(2)^2+4\cdot2+1}{(2+2)^2}+$$

funkcija raste $\langle-\infty,-3\sqrt{3}\rangle\cup\langle-\sqrt{3},+\infty\rangle$

funkcija pada $\langle-3\sqrt{3},-\sqrt{3}\rangle$



← NACRTALI STE HORIZONTALNU ASIMPTOTU, A NE VERTIKALNU

BODUJE SE GRAF.