

Popuniti odmah!

IME I PREZIME:

VULELIJA

BRJ INDEKSA:

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DATUM: 10.02.2011.

VRIJEME: OD 12:30 DO 14:30

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

ooxo
Broj ↓
bodova

1. Pravac p prolazi točkama A i B , a pravac q točkama A i C . Koliko iznosi kut između pravaca $\angle(p, q)$ ako je $A(2, -3, 1)$, $B(-1, 2, -3)$ i $C(1, -1, -2)$?
2. Među kompleksnim brojevima riješiti jednadžbu: $z^3 - (\overline{i+1})^5 = 0$.
3. Odrediti sve asimptote funkcije $f(x) = \arctan(e^x)$.
4. Odrediti drugu derivaciju funkcije $g(x) = \ln\left(x - \frac{1}{x}\right)$.
5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $f(x) = \frac{x^2 - 1}{x + 2}$.

~~0~~
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$$\angle(\vec{AB}, \vec{AC}) = \angle(p, q)$$

1. $A(2, -3, 1)$
 $B(-1, 2, -3)$
 $C(1, -1, -2)$

$$\vec{AB} = (-1-2, 2-(-3), -3-1) = (-3, 5, -4)$$

$$\vec{AC} = (1-2, -1-(-3), -2-1) = (-1, 2, -3)$$

$$Q = \vec{AC} = (-1, -4, -3) \quad \text{ŠTETA}$$

$$p \cdot q = \|p\| \cdot \|q\| \cos \angle(p, q)$$

$$\cos \angle = \frac{p \cdot q}{\|p\| \cdot \|q\|}$$

$$= \frac{-5 + (-4) + 12}{\sqrt{50} \cdot \sqrt{26}} = \frac{-4}{36,05} = -0,11$$

$$\angle = 1,68 \quad (\arccos -0,11)$$

$$\|AB\| = \sqrt{(-3)^2 + 5^2 + (-4)^2} = \sqrt{9+25+16} = \sqrt{50}$$

$$\|AC\| = \sqrt{(-1)^2 + (-4)^2 + (-3)^2} = \sqrt{1+16+9} = \sqrt{26}$$

$$p \cdot q = \frac{-5}{\sqrt{50} \cdot \sqrt{26}} = \frac{-5}{171}$$

3. $f(x) = \arctan(e^x)$

$$\lim_{x \rightarrow \infty} \arctan(e^x)$$

$$\begin{pmatrix} -3 & | & -1 \\ 5 & | & -4 \\ -4 & | & -3 \end{pmatrix} = \begin{matrix} 4 - 20 + 12 \\ -4 \end{matrix}$$

~~0~~

$$2. \quad 2^3 - (i+1)^5 = 0 \quad (i+1)^5 = (-i+1)^5$$

VIDI BUTERIN

$$4. \quad g(x) = \ln\left(x - \frac{1}{x}\right)$$

$$= \ln'\left(x - \frac{1}{x}\right)$$

$$= \frac{1}{\left(x - \frac{1}{x}\right)} \left(1 - \frac{1 \cdot x - 1 \cdot x}{x^2}\right) \checkmark$$

$$= \frac{1}{\left(x - \frac{1}{x}\right)} \left(1 - \frac{1}{x}\right) \times$$

$$= \frac{1}{\left(x - \frac{1}{x}\right)} \cdot (-1)$$

$$= -\frac{1}{\left(x - \frac{1}{x}\right)}$$

$$\frac{x \cdot 1 \cdot 1}{x^2} = -\frac{x}{x^2} = -\frac{1}{x}$$

$$\left(\frac{x}{1} - \frac{1}{x}\right)'$$

$$\left(\frac{1}{x}\right)' = \frac{1 \cdot x - (1 \cdot x)}{x^2} = \frac{x - (1 \cdot x)}{x^2}$$

$$= \frac{x - 1}{x^2} = -\frac{1}{x}$$

$$g(x)'' = \left(-\frac{1}{\left(x - \frac{1}{x}\right)}\right)'$$

$$= \frac{1 \cdot \left(x - \frac{1}{x}\right) - 1 \cdot \left(x - \frac{1}{x}\right)'}{\left(x - \frac{1}{x}\right)^2}$$

$$\frac{1}{1} + \frac{1}{x} = \frac{x+1}{x}$$

$$= \frac{\left(x - \frac{1}{x}\right) - 1 \cdot \left(1 - \left(-\frac{1}{x}\right)\right)}{\left(x - \frac{1}{x}\right)^2} = \frac{\left(x - \frac{1}{x}\right) \cdot 1}{\left(x - \frac{1}{x}\right)^2} = \frac{1}{\left(x - \frac{1}{x}\right)} \times$$

$$1' = 0$$

$$x' = 1$$

$$f(x) = \frac{x^2 - 1}{x + 2}$$

1. KORAK DOMENA FUNKCIJE

$$f(x) = \frac{x^2 - 1}{x + 2} \quad x + 2 \neq 0 \quad x = -2 \quad Df = \mathbb{R} \setminus \{-2\} \quad \checkmark$$

2. ASIMPTOTE

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{x + 2} \cdot \frac{1}{x^2} = \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x}{x^2} + \frac{2}{x^2}} = \frac{1 - \frac{1}{x^2}}{\frac{1}{x} + \frac{2}{x^2}} = \frac{1}{0} = \infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 1}{x + 2} = \frac{1}{0} = -\infty$$

} H.A.

$$\lim_{x \rightarrow -2^+} \frac{-2^2 - 1}{-2^2 + 2} = \frac{3}{-2} = -\frac{3}{2}$$

} V.A

$$\lim_{x \rightarrow -2^-} \frac{1}{2} (-)$$

$$k = \lim_{x \rightarrow \infty} \frac{x^2 - 1}{x^2 + 2} \cdot \frac{1}{x^2} = \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{2}{x^2}} = \frac{1 - \frac{1}{x^2}}{1 + \frac{2}{x^2}} = \frac{1}{1} = 1$$

$$c = y - kx$$

$$= \frac{x^2 - 1}{x + 2} - 1 \cdot x$$

$$= \frac{x^2 - 1}{x + 2} - \frac{x(x + 2)}{x + 2}$$

3. PARNOST NEPARNOST

$f(-x) = \text{PARNOST}$

$f(x) = \text{NEPARNOST}$

$f(-x) = \frac{-x^2-1}{-x+2} \neq \frac{x^2-1}{-x+2}$ NIJE PARNA

$f(x) = -\left(\frac{x^2-1}{x+2}\right) = \frac{-x^2+1}{-x-2} \neq \text{NIJE NEPARNA}$

4. SJECIŠTA NA KORD. OSIMA

x-osi (y=0) y-osi (x=0) $\frac{x^2-1}{x+2} = \frac{0-1}{0-2} = \frac{1}{2} = 0,5$
 $f(x)=0$

5. 1. i 2. DERIVACIJA

$f'(x) = \frac{x^2-1}{x+2} = \frac{(x^2-1)' \cdot (x+2) - (x^2-1) \cdot (x+2)'}{(x+2)^2} = \frac{2x(x+2) - (x^2-1) \cdot 1}{(x+2)^2} = \frac{2x^2+4x-x^2+1}{(x+2)^2}$

$= \frac{x^2+4x+1}{(x+2)^2}$ ✓

$f''(x) = \left(\frac{x^2+4x+1}{(x+2)^2}\right)' = \frac{(x^2+4x+1)' \cdot (x+2)^2 - (x^2+4x+1) \cdot ((x+2)^2)'}{(x+2)^4}$ $((x+2)^2)' = 2(x+2) \cdot 1$

$= \frac{(2x+4) \cdot (x^2+4x+4) - (x^2+4x+1) \cdot (2x+4)}{(x+2)^4} = \frac{2x^3+8x^2+8x+4x^2+16x+16 - (2x^3+4x^2+8x^2+2x+4)}{(x+2)^4}$

$= \frac{\cancel{2x^3+8x^2+8x+4x^2+16x+16} - \cancel{2x^3+4x^2+8x^2+2x+4}}{(x+2)^4} = \frac{6x-12}{(x+2)^4}$ ✗

$(x+2)(3)$

6. KRITIČNE TOČKE $f'(x)$ -nije definiran ili je $f'(x) = 0$

$$f'(x) = 0$$

$$\frac{x^2 + 4x + 1}{(x+2)^2} = 0 \quad / \cdot (x+2)^2$$

$$x^2 + 4x + 1 = 0 \quad / \sqrt{\quad} \Rightarrow$$

$$x + 2x + \sqrt{1} = 0 \quad \downarrow \times$$

$$x + 2x = 1 \quad \downarrow$$

$$3x = 1$$

$$x = \frac{1}{3}$$

NULTOČKE POLINOMA STUPNJA DVA:

$$x_{1,2} = \frac{-4 \pm \sqrt{16-4}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$$

$$x_1 = -2 - \sqrt{3} = \dots$$

$$x_2 = -2 + \sqrt{3} = \dots$$

7. MONOTONOST FUNKCIJA RAST ILI PAD.

$$f'(x) > 0 \quad f'(x) < 0$$

$$f'(x) = \frac{x^2 + 4x + 1}{(x+2)^2} > 0 \quad \text{Rast jer je kvadratna funkcija veća od 0}$$

IZNAD STE NAŠLI NULTOČKU A OVDJE TVRDITE DA JE SVUGDJE POZITIVNA: $x^2 + 4x + 1$

8. GLOBALNI EKSTREMI

$$y' = 0$$

$$\frac{x^2 + 4x + 1}{(x+2)^2} = 0 \quad / \cdot (x+2)^2$$

$$x^2 + 4x + 1 = 0$$

$$x + 5 = 0$$

$$x = 0$$

$$x(x+4+1) = 0$$

$$x(x+5) = 0$$

OVO ISTO STE GORE RIJEŠILI NA SASVIM DRUGI NAČIN UŽ RAZLIČITA RJEŠENJA

TREBATE NAVESTI TRAJEĆE NULTOČKA POLINOMA DRUGOG STUPNJA I CRTANJE GRAFA TOG ISTOG POLINOMA,

9. GLOBALNI EKSTREMI

IME I PREZIME: MARKO VOLELISA

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10. KONVEKSNOST I KONKAVNOST

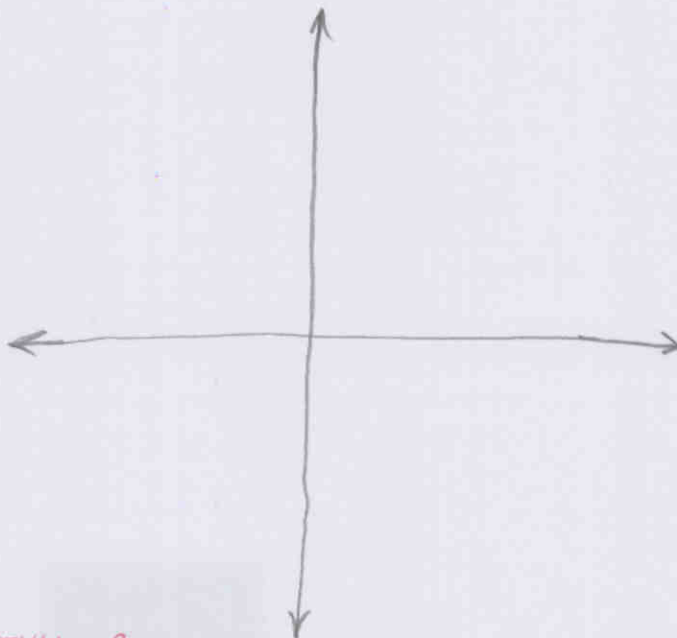
$$f'(x) = 0$$

$$\frac{6x-4}{(x+2)^4} = 0 \quad | \cdot (x+2)^4$$

$$6x-4 = 0$$

$$6x = 4 \quad | :6$$

$$x = \frac{4}{6} > 0 \quad \text{KONVEKSNOST}$$



U OVOM ZADATKU BODUJE SE SE GRAF.

