Popuniti odmah!

IME I PREZIME: MARKO GAMBIRAŽA BROJ INDEKSA: 57827

DATUM: VRIJEME: OD 12:00 DO 14:00

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

ooxo
Broj ↓ bodova

1. Pravac p prolazi točkama A i B, a pravac q točkama A i C. Koliko iznosi kut između pravaca $\measuredangle(p,q)$ ako je $A(2,-3,1)$, $B(-1,2,-3)$ i $C(1,-1,-2)$?

2. Među kompleksnim brojevima riješiti jednadžbu: $z^3 - \left(\overline{i+1}\right)^5 = 0$.

3. Odrediti sve asimptote funkcije $f(x) = \arctan(e^x)$.

4. Odrediti drugu derivaciju funkcije $g(x) = \ln\left(x - \frac{1}{x}\right)$.

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $f(x) = \frac{x^2-1}{x+2}$.

1.

$$p \measuredangle (\overrightarrow{AB}, \overrightarrow{AC})$$

$$\overrightarrow{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}$$

$$\overrightarrow{AB} = (-1-2)\vec{i} + (2-(-3))\vec{j} + (-3-1)\vec{k}$$

$$\overrightarrow{AB} = -3\vec{i} + 5\vec{j} - 4\vec{k}$$

$$\overrightarrow{AC} = (x_C - x_B)\vec{i} + (y_C - y_A)\vec{j} + (z_C - z_A)\vec{k}$$

$$\overrightarrow{AC} = (1-2)\vec{i} + (-1-3)\vec{j} + (-2-1)\vec{k}$$

$$\overrightarrow{AC} = -1\vec{i} - 4\vec{j} - 3\vec{k}$$

$$\cos\varphi = \frac{a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} + \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

$$\cos\varphi = \frac{-3\cdot(-1) + 5\cdot(-4) + (-4)\cdot(-3)}{\sqrt{(-3)^2 + 5^2 \cdot (4)^2} + \sqrt{(-1)^2 + (4)^2 + (-3)^2}}$$

$$\cos\varphi = \frac{3 - 20 + 12}{\sqrt{9+25+16} + \sqrt{1+16+9}}$$

$$\cos\varphi = \frac{-5}{\sqrt{50} + \sqrt{26}}$$ MINUS U PLUS

$$\cos\varphi = \frac{5}{\sqrt{76}} = \frac{-5}{8.717797887}$$

$$\cos\varphi = 0.57354$$

$$\cos\varphi = 55^\circ 0' 9''$$

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(4.) $g(x) = \ln\left(x - \frac{1}{x}\right)$

$$g(x)' = \frac{1}{\left(x - \frac{1}{x}\right)} \cdot \left(1 - \frac{x - 1 \cdot 1}{x^2}\right)$$

$$g(x)' = \frac{1 - \frac{x-1}{x^2}}{x - \frac{1}{x}} = \frac{\frac{2x}{x^2}}{\frac{x-1}{x}}$$

$$g(x)' = \frac{2x^2}{x^3 - x^2} :$$

$$g(x)' = \frac{2}{x^3} //$$

$$g(x)'' = \frac{x^3 - 2 \cdot 3x^2}{(x^3)^2}$$

$$g(x)'' = \frac{-6x^2 + x^3}{x^6}$$

$$g(x)'' = \frac{-6}{x} //$$

(3.) $f(x) = \arctan(e^x)$

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(5.) $f(x) = \frac{x^2-1}{x+2}$

1. DOMENA: $f(x) = \quad x+2 \neq 0$
$x = -2$
$Df = R\{-2\}$

2. ASIMPTOTE

$\lim_{x \to +\infty} \frac{x^2-1 /:x^2}{x+2 /:x^2} = \frac{\frac{x^2}{x^2} \cdot \frac{1}{x^2}}{\frac{x}{x^2} + \frac{2}{x^2}} = \frac{1-\frac{1}{x^2}}{\frac{1}{x} + \frac{2}{x^2}} = \frac{1}{0} = 0 = +\infty$

$\lim_{x \to (+\infty)} = \frac{x^2-1}{x^2+2} = \frac{1}{0} = 0 = \infty$ } H.A.

$\lim_{x \to (-2^+)} = \frac{-2^2-1}{-2^2+2} = \frac{3}{6} = \frac{1}{2}$

$\lim_{x \to -2(-)} = \frac{1}{2}(-)$ } V.A

$k \lim \frac{x^2-1 /:x^2}{x^2+2 /:x^2} = \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x^2}} = 0 = \infty$

$1 \quad 0$

$l = y - kx$

$= \frac{x^2-1}{x+2} - 0 \cdot x = \frac{x^2-1}{x+2}$

3. PARNOST/NEPARNOST

$f(-y)$

$-f(x)$ $f(-x) = \frac{-x^2-1}{-x+2} \neq \frac{x^2-1}{-x+2}$ NIJE PARNA

$-f(x) = -\left(\frac{x^2-1}{x+2}\right) = \frac{-x^2+1}{-x-2}$ NIJE NEPARNA

→

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(5) NASTAVAK

4. SJECIŠTA NA KORD. OSIMA

x-osi ($y=0$) $f(x)=0$

y-osi ($x=0$) $\frac{x^2-1}{x+2} = \frac{0-1}{0-2} = \frac{1}{2} = 0.5$

5. DERIVACIJE

$$f(x)' = \frac{x^2-1}{x+2} = \frac{(x^2-1)'\cdot(x+2) - (x^2-1)\cdot(x+2)'}{(x+2)^2} = \frac{2x(x+2)-(x^2-1)\cdot 1}{(x+2)^2}$$

$$= \frac{2x^2+4x-x^2+1}{(x+2)^2} = \frac{x^2+4x+1}{(x+2)^2}$$

$$f(x)'' = \frac{(x^2+4x+1)\cdot(x+2)^2 - (x^2+4x+1)\cdot((x+2)^2)}{(x+2)^4} = \frac{2x+4\cdot(x^2+4x+4)-(x^2-4x+1)\cdot(2x+4)}{(x+2)^4}$$

$$= \frac{2x^3+8x^2+8x+4x^2+16x+16-(2x^3+4x^2+8x^2+2x+4)}{(x+2)^2} =$$

$$= \frac{2x^3+8x^2+8x+4x^2+16x+16-2x^3-4x^2-8x^2-2x-4}{(x+2)^4} = \frac{6x-4}{(x+2)^4}$$

6. KRITIČNE TOČKE $f'(x)$ - nije definiran ili je $f'(x)=0$

$f'(x)=0 \quad \frac{x^2+4x+1}{(x+2)^2} = 0 \,/\, (x+2)^2$

$x^2+4x+1=0 \,/\sqrt{}$

$x+2x+\sqrt{1}=0$

$x+2x=1$

$3x=1$

$x=\frac{1}{3}$ //

(5) NASTAVAK

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7. MONOTONOST (RAST ILI PAD)

$f'(x) = \frac{x^2+4x+1}{(x+2)^2} > 0$ RAST

8. LOKALNI EKSTREMI

$y' = 0$

$\frac{x^2+4x+1}{(x+2)^2} = 0 \,/\cdot (x+2)^2$

$x^2+4x+1=0$

$x(x+4+1=0$

$x(x+5)=0$

$x+5=0$

$x=-5$

$x=0$

9. KONVEKSNOST I KONKAVNOST

$f'(x)=0$

$\frac{6x-4}{(x+2)^4} = 0 \,/\, (x+2)^4$

$6x-4=0$

$6x=4/:6$

$x=\frac{4}{6} > 0$ KONVEKSNOST