

Popuniti odmah!

IME I PREZIME:

DATUM:

VRIJEME: OD

11:20

DO

13:30

BROJ INDEKSA:

SIME BUTERIN

40

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

ooxo
Broj ↓
bodova

- ✓ 1. Pravac p prolazi točkama A i B , a pravac q točkama A i C . Koliko iznosi kut između pravaca $\angle(p, q)$ ako je $A(2, -3, 1)$, $B(-1, 2, -3)$ i $C(1, -1, -2)$? 20
- ✓ 2. Među kompleksnim brojevima riješiti jednadžbu: $z^3 - (i+1)^5 = 0$. 20
3. Odrediti sve asimptote funkcije $f(x) = \arctan(e^x)$.
4. Odrediti drugu derivaciju funkcije $g(x) = \ln\left(x - \frac{1}{x}\right)$.
- ✓ 5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $f(x) = \frac{x^2 - 1}{x + 2}$. ~~0~~

NISTE NAUČILI DERIVIRATI
KOMPZICIJU FUNKCIJE.
OBAVEZNO NAUČITI ZA
USMENI DIO ISPITA,

$$129 = 14 \cdot 2 \cdot 2 \\ = 1$$

$$\textcircled{1} \quad \begin{aligned} A(2, -3, 1) \\ B(-1, 2, -3) \\ C(1, -1, -2) \end{aligned}$$

$$AB = \begin{bmatrix} -1-2 \\ 2-(-3) \\ -3-1 \end{bmatrix} \quad \vec{AB} = \begin{bmatrix} -3 \\ 5 \\ -4 \end{bmatrix} \quad \vec{v}_1$$

$$AC = \begin{bmatrix} 1-2 \\ -1-(-3) \\ -2-1 \end{bmatrix} \quad \vec{AC} = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} \quad \vec{v}_2$$

$$v_1 \cdot v_2 = \|v_1\| \cdot \|v_2\| \cos \alpha$$

$$v_1 \cdot v_2 = -3 \cdot (-1) + 5 \cdot 2 + (-4) \cdot (-3)$$

$$v_1 \cdot v_2 = 25 //$$

$$\|v_2\| = \sqrt{a^2 + b^2 + c^2}$$

$$\|v_1\| = \sqrt{a^2 + b^2 + c^2}$$

$$\|v_2\| = \sqrt{(-1)^2 + 2^2 + (-3)^2}$$

$$= \sqrt{(-3)^2 + 5^2 + (-4)^2}$$

$$= \sqrt{1+4+9}$$

$$= \sqrt{9+25+16}$$

$$= \sqrt{14}$$

$$= \sqrt{50}$$

$$= 3.74 //$$

$$= 7.07 //$$

$$25 = 7.07 \cdot 3.74 \cos \alpha$$

$$25 = 26.44 \cos \alpha / 26.44$$

$$\cos \alpha = \frac{25}{26.44}$$

$$\cos \alpha = 0.94$$

$$\alpha = 0.33 //$$

20

$$② \quad z^3 - (\overline{i+1})^5 = 0$$

$$z^3 - (i-1)^5 = 0$$

$$z^3 - (-4i+4) = 0$$

$$z^3 + 4i - 4 = 0$$

$$z^3 = 4 - 4i \quad \checkmark$$

$$|z| = \sqrt{x^2 + y^2}$$

$$|z| = \sqrt{4^2 + (-4)^2}$$

$$|z| = \sqrt{16 + 16}$$

$$|z| = \sqrt{32}$$

$$|z| = 5.65 \quad \checkmark$$

$$|z| = 1.78 \quad \checkmark$$

$$\rho = \frac{y}{x} = \frac{-4}{4} = -1 + 2\pi$$

$$= -0.78 + 6.28$$

$$(i-1)^5 = (i-1)^2 \cdot (i-1)^2 \cdot (i-1)^1$$

$$(i-1)^2 = i^2 + 2 \cdot i \cdot (-1) + (-1)^2$$

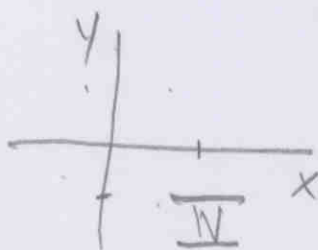
$$= -1 - 2i + 1$$

$$= -2i$$

$$(-2i) \cdot (-2i) = 4i^2 = -4$$

$$-4 \cdot (i-1)$$

$$= -4i + 4$$



$$\text{arctan } \varphi = \frac{y}{x}$$

$$\varphi = \text{arctan } \frac{y}{x}$$

$$k = 0, 1, 2$$

$$z_1 = 1.78 \left(\cos \frac{5.5 + 2 \cdot 0 \cdot \pi}{3} + i \sin \frac{5.5 + 2 \cdot 0 \cdot \pi}{3} \right) = 5.5$$

$$z_2 = 1.78 \left(\cos \frac{5.5 + 2 \cdot 1 \cdot \pi}{3} + i \sin \frac{5.5 + 2 \cdot 1 \cdot \pi}{3} \right)$$

$$z_3 = 1.78 \left(\cos \frac{5.5 + 2 \cdot 2 \cdot \pi}{3} + i \sin \frac{5.5 + 2 \cdot 2 \cdot \pi}{3} \right)$$

$$z_1 = 1.78(\cos 1.83 + i \sin 1.83)$$

$$z_2 = 1.78(\cos 4.03 + i \sin 4.03)$$

$$z_3 = 1.78(\cos 6.02 + i \sin 6.02)$$

$$z_1 = 1.78(-0.256 + i 0.966)$$

$$z_2 = 1.78(-0.630 + i(-0.776))$$

$$z_3 = 1.78(0.965 + i(-0.260))$$

$$z_1 = -0.455 + i 1.719$$

$$z_2 = -1.121 + i(-1.381)$$

$$z_3 = 1.717 + i(-4.628)$$

20

$$f(x) = \frac{x^2 - 1}{x + 2}$$

$$x + 2 \neq 0$$

$$x = -2$$

$$Df = \mathbb{R} \setminus \{-2\} \checkmark$$

② Kuloče

$$y = \frac{x^2 - 1}{x + 2} = 0$$

$$\frac{0^2 - 1}{0 + 2} = -\frac{1}{2} = -0.5$$

$$T_1 = \{-0.5, 0\}$$

$$x = 0$$

$$\frac{x^2 - 1}{x + 2} \quad | : x + 2$$

$$x^2 - 1 = 0$$

$$x_1^2 = 0 \quad T_2 = \{0, 0\}$$

$$x_2 = 1 \quad T_3 = \{1, 0\}$$

Asimptote ① VERTIKALNE

$$\lim_{x \rightarrow -2^-} \frac{-2^2 - 1}{-2^- + 2} = \frac{4^- - 1}{0^-} = \frac{3^-}{0^-} = -\infty$$

$$\lim_{x \rightarrow -2^+} \frac{-2^2 - 1}{-2^+ + 2} = \frac{4^+ - 1}{0^+} = \frac{3^+}{0^+} = +\infty$$

$x = -2 \checkmark$

HORIZONTALNE

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{x + 2} \quad | : x^2 = \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x}{x^2} + \frac{2}{x^2}} = \frac{1 - \frac{1}{x^2}}{\frac{1}{x} + \frac{2}{x^2}} = \frac{1}{0}$$

nema
horizontalnih \checkmark

③ KOSE ASIMPTOTE

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{x + 2} = \frac{x^2 - 1}{x^2 + 2x} \quad / : x^2 = \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2}} = \frac{1 - \frac{1}{x^2}}{1 + \frac{2}{x}} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{x + 2} - bx = \lim_{x \rightarrow \infty} \frac{x^2 - 1 - bx}{x + 2} = \lim_{x \rightarrow \infty} \frac{x^2 - 1 - x^2 - 2x}{x + 2} \quad k=1$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1 - x^2 - 2x}{x + 2} = \frac{-1 - 2x}{x + 2} \quad / : x = \frac{-\frac{1}{x} - \frac{2x}{x}}{\frac{x}{x} + \frac{2}{x}} = \frac{-\frac{1}{x} - 2}{1 + \frac{2}{x}}$$

$$\lim_{x \rightarrow \infty} = \frac{-2}{1} = -2 \quad l = -2$$

$$= y = k \cdot x + l$$

$$y = 1 \cdot x + (-2)$$

$$y = x - 2 \quad \checkmark$$

Derivaciju prvca

$$\left(\frac{x^2 - 1}{x + 2} \right)' = \frac{(x^2 - 1)' \cdot (x + 2) - (x^2 - 1) \cdot (x + 2)'}{(x + 2)^2}$$

$$= \frac{2x(x + 2) - 1(x^2 - 1)}{(x + 2)^2}$$

$$= \frac{3x^2 + 4x - x^2 + 1}{(x + 2)^2} = \frac{2x^2 + 4x + 1}{(x + 2)^2}$$

$$\frac{2x^2 + 4x + 1}{(x + 2)^2} = 0 \quad / : x^2 = \frac{\frac{2x^2}{x^2} + \frac{4x}{x^2} + \frac{1}{x^2}}{\left(\frac{x}{x} + \frac{2}{x}\right)^2} = \frac{2 + \frac{4}{x} + \frac{1}{x^2}}{\left(1 + \frac{2}{x}\right)^2} = \frac{2}{1} = 2$$

	$-\infty$	-2	-0.5	1	$+\infty$	
$2x^2+4x+1$	-	-	+	+		
$(x+2)^2$	+	+	+	+		
$\frac{2x^2+4x+1}{(x+2)^2}$	-	-	+	+		

\leftarrow OVDJE JE OŽ TOČKE PREKIDA TREBALO POSTAVITI JOŠ NULTOČKE BROJNIKA PRVE DERIVACIJE.

\downarrow \downarrow \uparrow \uparrow

minimum funkcije

m

Druge derivacije

$$\left(\frac{2x^2+4x+1}{(x+2)^2}\right)' = \frac{(2x^2+4x+1)'(x+2)^2 - (2x^2+4x+1)((x+2)^2)'}{(x+2)^2}^2$$

$$= \frac{2(2x+4)(x+2)^2 - 2(2x^2+4x+1)(x+2)}{(x+2)^4} = \frac{4x+4(x+2) - 4x^2-8x-2}{(x+2)^2}$$

$$= \frac{5x^2+4x+8-4x^2-8x-2}{(x+2)^2}$$

$$= \frac{x^2-4x+6}{(x+2)^2}$$

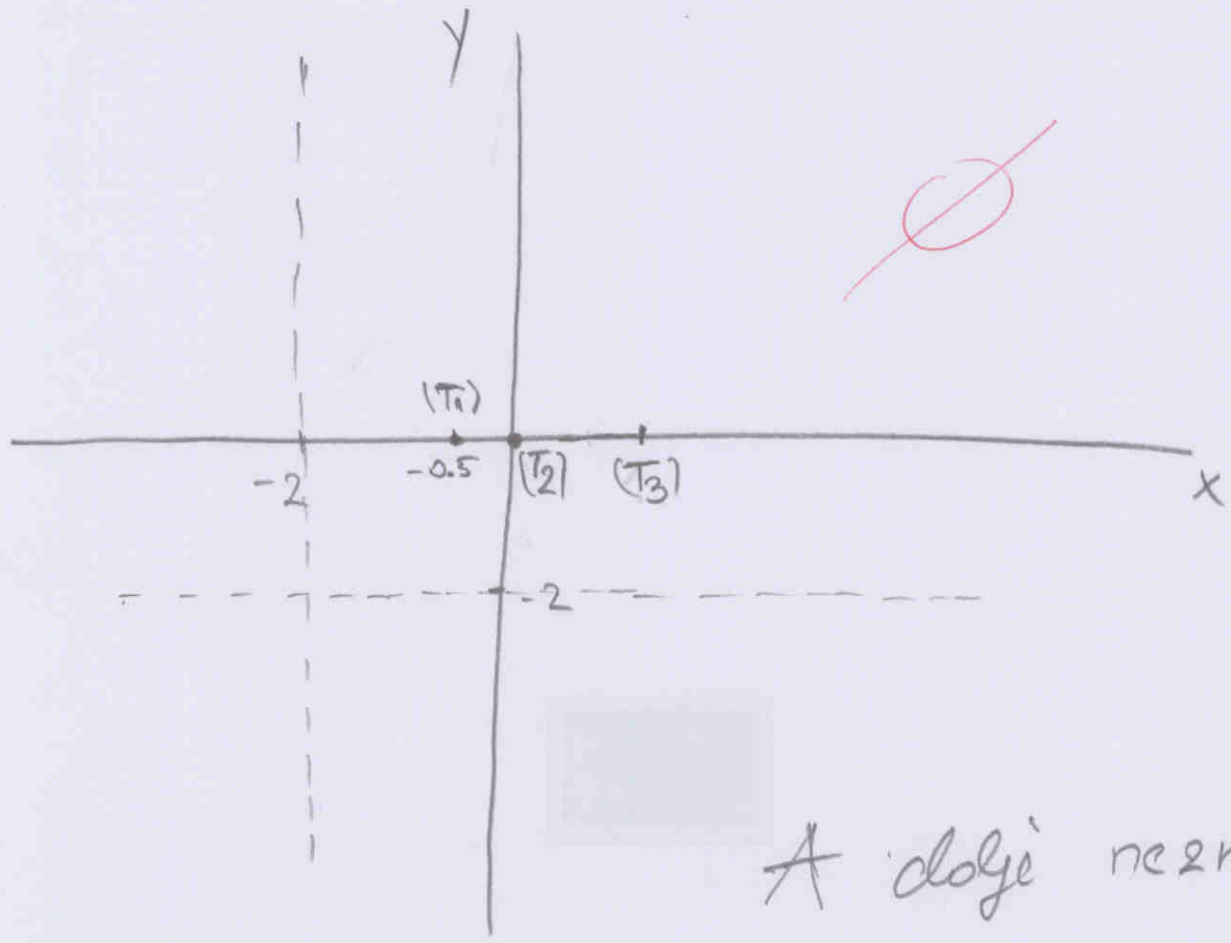
$$\frac{x^2-4x+6}{(x+2)^2} = 0 \quad / : x^2$$

$$\frac{\frac{x^2}{x^2} - \frac{4x}{x^2} + \frac{6}{x^2}}{\left(\frac{x}{x^2} + \frac{2}{x^2}\right)^2} = \frac{1 - \frac{4}{x} + \frac{6}{x^2}}{\frac{1}{x^2} + \frac{2}{x^2}} = \frac{1}{0.5}$$

ne postoji

IME I PREZIME: ŠIME BUTEVIN

BROJ INDEKSA: 17-2-0049-2010



A dolye neznam :)

U OVOM ZADATKU BODUJE SE GRAF.