Popuniti odmah!

IME I PREZIME: SILVIJAN ĆVRK BROJ INDEKSA: 17-2-0066-2010

DATUM: 10.02.2011. VRIJEME: OD 11:45 DO 13:05.

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

ooox

Broj ↓ bodova

1. Odrediti determinantu matrice $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}$ — 20

2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2 + x + 1}$ — 15

3. Ispitati konvergenciju reda $\sum \left(\frac{2n^2 + 3n + 4}{\frac{1}{n} + 2n + 3n^2} \right)^n$ — 0

4. Ispitati domenu, periodičnost, parnost i prvu derivaciju funkcije $g(x) = \ln(\cos(2x))$. — 20

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2 + 1}{x^2 + 2}$. — 20

UKUPNO 75

① DET

$A = \begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{vmatrix}$

$= 1 \cdot \left(1 \cdot \begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} \right) - 2 \cdot \left(2 \cdot \begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{vmatrix} \right)$

$= 1 \cdot 1 \cdot 1 + 2 \cdot 2 \cdot 0 + 0 \cdot 2 \cdot 2 - 0 \cdot 1 \cdot 0 - 1 \cdot 2 \cdot 2 - 1 \cdot 2 \cdot 2 = 1 + 0 + 0 - 0 - 4 - 4 = 1 - 4 - 4 = -7$

$= 2 \cdot 1 \cdot 1 + 2 \cdot 2 \cdot 0 + 2 \cdot 0 \cdot 2 - 0 \cdot 1 \cdot 0 - 2 \cdot 2 \cdot 2 - 1 \cdot 0 \cdot 2 = 2 + 0 + 0 - 0 - 8 - 0 = 2 - 8 = -6$

$= 1 \cdot 1 \cdot 1 + 2 \cdot 2 \cdot 0 + 0 \cdot 2 \cdot 2 - 0 \cdot 1 \cdot 0 - 2 \cdot 2 \cdot 1 - 1 \cdot 2 \cdot 2 = 1 + 0 + 0 - 0 - 4 - 4 = 1 - 4 - 4 = -7$

$= 1 \cdot \left(1 \cdot (-7) - 2 \cdot (-6) \right) - 2 \cdot \left(2 \cdot (-7) \right)$

$= 1 \left(-7 + 12 \right) - 2 \cdot (-14)$

$= 1 \cdot (5) - 2 \cdot (-14)$

$= 5 + 28 = 33$ ✓ 20

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② DOMENA I SVE ASIMPTOTE

$f(x) = x - \sqrt{x^2+x+1}$

POD KORIJENOM NE SMIJE BITI NEGATIVNO

$x^2+x+1 \geq 0$ ✓

$a=1$, $b=1$, $c=1$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1-4\cdot 1}}{2} = \frac{-1 \pm \sqrt{-3}}{2} \to \sqrt{3\cdot(-1)} = \sqrt{3}\cdot\sqrt{-1} = \pm\sqrt{3}\,i$

$= \frac{-1 \pm \sqrt{3}\,i}{2}$

$x_1 = \frac{-1+\sqrt{3}\,i}{2}$ $\quad x_2 = \frac{-1-\sqrt{3}\,i}{2}$

$x_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}\,i$, $\quad x_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}\,i$,

SKICA PARABOLE x^2+x+1

DAKLE, NEMA REALNIH NULTOČAKA.

$\Rightarrow x^2+x+1 \geq 0 \quad \forall x \in \mathbb{R}$

$\Rightarrow D(f) = \mathbb{R}$

$ax^2+bx+c = a(x-x_1)(x-x_2)$

$= \left(x + \frac{1}{2} - \frac{\sqrt{3}}{2}\,i\right)\cdot\left(x + \frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right)$

$x + \frac{1}{2} - \frac{\sqrt{3}}{2}\,i \geq 0$ $\quad x + \frac{1}{2} + \frac{\sqrt{3}}{2}\,i \geq 0$

$x = -\frac{1}{2} + \frac{\sqrt{3}}{2}\,i$ $\quad x = -\frac{1}{2} - \frac{\sqrt{3}}{2}\,i$

	$-\infty$		$-\frac{1}{2}-\frac{\sqrt{3}}{2}i$		$-\frac{1}{2}+\frac{\sqrt{3}}{2}i$		$+\infty$
$x+\frac{1}{2}-\frac{\sqrt{3}}{2}i$		−		−	0	+	
$x+\frac{1}{2}+\frac{\sqrt{3}}{2}i$		−	0	+		+	
x^2+x+1		(+)		−		(+)	

$D_f: x \in \langle -\infty, -\frac{1}{2} - \frac{\sqrt{3}}{2}\,i] \cup [-\frac{1}{2} + \frac{\sqrt{3}}{2}\,i, +\infty\rangle$ ✗

ASIMPTOTE

H.A $\lim_{x\to\infty} x - \sqrt{x^2+x+1} = \infty - \infty = \lim_{x\to\infty}\left(x - \sqrt{x^2+x+1}\right)\cdot\frac{x+\sqrt{x^2+x+1}}{x+\sqrt{x^2+x+1}}$

D: H.A

$\lim_{x\to\infty} \frac{x^2 - x^2 - x - 1}{x+\sqrt{x^2+x+1}} = \lim_{x\to\infty} \frac{(-x-1)}{\left(x+\sqrt{x^2+x+1}\right)} \,/:x = \lim_{x\to\infty} \frac{-1-\frac{1}{x}}{1+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}} = \frac{-1}{2} = -\frac{1}{2}$ ✓

$\lim_{x\to-\infty} x - \sqrt{x^2+x+1} = |x \to (-x)| = \lim_{x\to\infty} -x - \sqrt{(-x)^2+(-x)+1} = \lim_{x\to\infty} -x - \sqrt{x^2-x+1}$

$\lim_{x\to\infty}\left(-x-\sqrt{x^2-x+1}\right)\cdot\frac{-x+\sqrt{x^2-x+1}}{-x+\sqrt{x^2-x+1}} = \lim_{x\to\infty} \frac{x^2 - x^2 + x - 1}{-x+\sqrt{x^2-x+1}} = \lim_{x\to\infty} \frac{x-1}{-x+\sqrt{x^2-x+1}} \,/:x$

$\lim_{x\to\infty} \frac{1-\frac{1}{x}}{-1+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{1}{-1+1} = \frac{1}{0} = +\infty$, N.L.H.A mogući da postoj

Ljeva kosa asim

✓

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L.K.A.

$$\lim_{x\to-\infty} = \frac{A(1)}{x} = \lim_{x\to-\infty} \frac{x-\sqrt{x^2+x+1}}{x} = |x\to(-x)| = \lim_{x\to\infty} \frac{-x-\sqrt{(-x)^2+(-x)+1}}{-x}$$

$$\lim_{x\to\infty} \frac{-x-\sqrt{x^2-x+1}}{-x} \Big/ x = \lim_{x\to\infty} \frac{-1-\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}{-1} = \frac{-1-1}{-1} = \frac{-2}{-1} = 2 \Rightarrow k_2$$

$$l_2 = \lim_{x\to-\infty} (f(x)-k_2x) = \lim_{x\to-\infty} \left(x-\sqrt{x^2+x+1}-2x\right)$$

$$\lim_{x\to-\infty} \left(-x-\sqrt{x^2+x+1}\right) = |x\to(-x)| = \lim_{x\to\infty} \left(x-\sqrt{(-x)^2+(-x)+1}\right)$$

$$\lim_{x\to\infty} \left(x-\sqrt{x^2-x+1}\right)\cdot \frac{x+\sqrt{x^2-x+1}}{x+\sqrt{x^2-x+1}} = \lim_{x\to\infty} \frac{x^2-x^2+x-1}{x+\sqrt{x^2-x+1}}$$

$$\lim_{x\to\infty} \frac{x-1}{x+\sqrt{x^2-x+1}} \Big/ x = \lim_{x\to\infty} \frac{1-\frac{1}{x}}{1+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}} = \frac{1}{1+1} = \frac{1}{2} \Rightarrow l_2$$

$$y_2 = k_2x + l_2 \Rightarrow \boxed{y = 2x + \frac{1}{2}} \quad \text{L.K.A} \quad \checkmark$$

N.D.K.A.

V.A (rasovi docnene)

$$\lim_{x\to-\frac{1}{2}-\frac{\sqrt{3}}{2}i} x-\sqrt{x^2+x+1}$$

N.V.A ✓

$$= -1.56 - \sqrt{1.64-1.36i}$$

$$= 1.36 - 1.24 = 0.12$$

$$\frac{-1-\sqrt{3}i}{2} = \frac{-1-1.73}{2} = \frac{-2.73}{2} = -1.36i$$

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4) DOMENA / PERIODIČNOST / PARNOST i I.D.

$g(x) = \ln(\cos(2x))$

$\frac{\frac{\pi}{2}}{\frac{2}{n}} = \frac{\pi}{4}$

$D_f = ?$

$x \in \left\langle -\frac{\pi}{2} + k2\pi, \frac{\pi}{2} + k2\pi \right\rangle$ ✓

$2x \in \left\langle -\frac{\pi}{4} + k\pi, \frac{\pi}{4} + k\pi \right\rangle$ ✓

$D_f: x \in \left\langle -\frac{\pi}{4} + k\pi, \frac{\pi}{4} + k\pi \right\rangle$ ✓

P/N

$g(-x) = \ln(\cos(2 \cdot (-x)))$ $\quad \cos(-x) = \cos x$ P

$= \ln(\cos(-2x))$

$= \ln(\cos(2x)) = g(x)$ f. parna ✓

$\neq -g(x)$ nije neparna

- f periodična zbog trigonometrijske f.

$g(x) = \pi \rightarrow$ period ✓

* DRUGA DERIVACIJA

$g(x) = \ln(\cos(2x))$

$g'(x) = \left[\ln(\cos(2x))\right]'$

$g'(x) = \frac{1}{\cos(2x)} \cdot \underbrace{(\cos 2x)'}_{(-\sin 2x)\,2 \,=\, -2\sin 2x}$

$g'(x) = \frac{-2\sin 2x}{\cos(2x)}$ // ✓

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(5.) $h(x) = \frac{x^2+1}{x^2+2}$

$D_f = ?$

$x^2+2 \neq 0$

$x^2 \neq -2$

kvadrat ne može biti negativan, prema tome

$D_f : x \in \mathbb{R}$ ✓

P/N

$h(-x) = \frac{(-x)^2+1}{(-x)^2+2} = \frac{x^2+1}{x^2+2} = h(x)$ f. parna ✓

$\neq -h(x)$ nije neparna

H.A.

$\lim_{x\to\infty} \frac{x^2+1}{x^2+2} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{x^2+1}{x^2+2} /:x^2 = \lim_{x\to\infty} \frac{1+\frac{1}{x^2}}{1+\frac{2}{x^2}} = \lim_{x\to\infty} \frac{1}{1} = 1$

$\lim_{x\to-\infty} \frac{x^2+1}{x^2+2} = |x\to(-x)| = \lim_{x\to\infty} \frac{(-x)^2+1}{(-x)^2+2} = \lim_{x\to\infty} \frac{x^2+1}{x^2+2} /:x^2 = \lim_{x\to\infty} \frac{1+\frac{1}{x^2}}{1+\frac{2}{x^2}} = 1$

H.A. $y = 1$ ✓

- N.K.A

V.A

$\lim_{x\to 0^+} \frac{x^2+1}{x^2+2} = \lim_{x\to 0^+} \frac{1^+}{2^+} = \frac{1}{2}$

$\lim_{x\to 0^-} \frac{(0^-)^2+1}{(0^-)^2+2} = \lim_{x\to 0^-} \frac{1}{2}$

N.V.A ✓

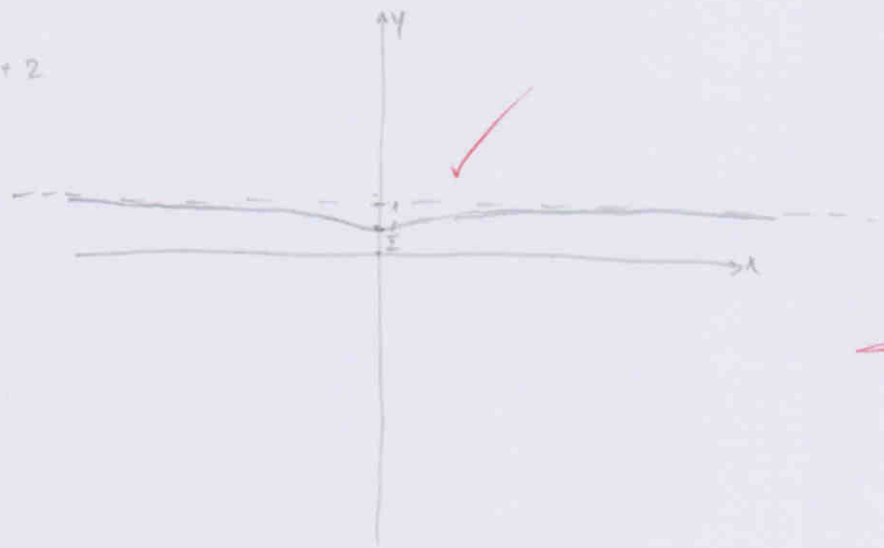
$(0, \frac{1}{2})$

$\frac{x^2+1}{x^2+2} = 0 \quad / \cdot x^2+2$

$x^2+1 = 0$

$x^2 = -1 / \sqrt{\ }$

$x = \pm\sqrt{-1}$



- f. parna
- nije periodična

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5)

ekstremi

$$h(x) = \frac{x^2+1}{x^2+2}$$

$$h'(x) = \frac{(x^2+1)' \cdot (x^2+2) - (x^2+1) \cdot (x^2+2)'}{(x^2+2)^2}$$

$(x^2+1)' = 2x+0 = 2x$; $(x^2+2)' = 2x+0 = 2x$

$$h'(x) = \frac{2x \cdot (x^2+2) - (x^2+1) \cdot 2x}{(x^2+2)^2}$$

$$h'(x) = \frac{2x^3 + 4x - 2x^3 - 2x}{(x^2+2)^2}$$

$$h'(x) = \frac{2x}{(x^2+2)^2} = 0 \quad / \cdot (x^2+2)^2$$

$$2x = 0 \quad / : 2$$

$$x = \frac{0}{2}$$

$x = 0$ → STACIONARNA KRITIČNA T.

Npr: -1 $h'(x) < 0$ ↙
1 $h'(x) > 0$ ↗ } MIN

✓

„SKRAĆENA" DRUGA IZV.

$$h''(x) = (2x)' = 2$$

$h''(x) > 0$ → MIN

$$h(0) = \frac{x^2+1}{x^2+2} = \frac{1}{2} \rightarrow \left(0, \frac{1}{2}\right)$$

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③ KONVERGENCIJA

$$\sum \left(\frac{2n^2+3n+4}{\frac{1}{n}+2n+3n^2} \right)^n$$

$$\left(3n^2+2n+\frac{1}{n}\right) : \left(2n^2+3n+4\right) = 1$$
$$\pm 2n^2$$