Popuniti odmah!

IME I PREZIME: Domagoj Mehić BROJ INDEKSA: 17-2-0028-2018

DATUM: VRIJEME: OD 13:20 DO 15:20

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

oooo
Broj ↓ bodova

1. Koju relaciju zadovoljava inverz matrice? Provjeriti tu relaciju za inverz matrice (ako postoji)

$$A = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}.$$

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2. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3i + 4| = \frac{5}{i^{233}}$.

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3. Odrediti domenu i sve asimptote funkcije $f(x) = \ln\left(\frac{x+5}{1-x}\right)$.

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4. Ispitati domenu, periodičnost, (ne)parnost i prvu derivaciju funkcije $g(x) = \cos(\sin(3x))$.

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5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije f iz zadatka 3.

UKUPNO: 50

1) $\left[\begin{array}{cccc|cccc} 2&0&0&1&1&0&0&0 \\ 0&0&1&0&0&1&0&0 \\ 0&1&0&0&0&0&1&0 \\ 1&0&0&2&0&0&0&1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1&0&0&2&0&0&0&1 \\ 0&1&0&0&0&0&1&0 \\ 0&0&1&0&0&1&0&0 \\ 2&0&0&1&1&0&0&0 \end{array}\right] R4-2R1 \sim$

$\left[\begin{array}{cccc|cccc} 1&0&0&2&0&0&0&1 \\ 0&1&0&0&0&0&1&0 \\ 0&0&1&0&0&1&0&0 \\ 0&0&0&-3&1&0&0&-2 \end{array}\right] R3\cdot(-\frac{1}{3}) \sim \left[\begin{array}{cccc|cccc} 1&0&0&2&0&0&0&1 \\ 0&1&0&0&0&0&1&0 \\ 0&0&1&0&0&1&0&0 \\ 0&0&0&1&-\frac{1}{3}&0&0&\frac{2}{3} \end{array}\right] R1-2R4$

$\sim \left[\begin{array}{cccc|cccc} 1&0&0&0&\frac{1}{3}&0&0&-\frac{1}{3} \\ 0&1&0&0&0&0&1&0 \\ 0&0&1&0&0&1&0&0 \\ 0&0&0&1&-\frac{1}{3}&0&0&\frac{2}{3} \end{array}\right]$

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$\begin{bmatrix} 2&0&0&1 \\ 0&0&1&0 \\ 0&1&0&0 \\ 1&0&0&2 \end{bmatrix} \times \begin{bmatrix} 2/3&0&0&-1/3 \\ 0&0&1&0 \\ 0&1&0&0 \\ -1/3&0&0&2/3 \end{bmatrix} = \begin{bmatrix} 4/3+0+0-\frac{1}{3}, & 0+0+0+0, & 0+0+0+0, & -2/3+0+0+2/3 \\ 0+0+0+0, & 0+0+1+0, & 0+0+0+0, & 0+0+0+0 \\ 0+0+0+0, & 0+0+0+0, & 0+1+0+0, & 0+0+0+0 \\ 2/3+0+0-2/3, & 0+0+0+0, & 0+0+0+0, & -1/3+0+0+4/3 \end{bmatrix}$

$A \qquad A^{-1} \qquad = \qquad I$

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$A^{-1} = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix}$

$A = A^{-1}$ ✗ ✓

$A\,A^{-1} = I$

$A * A^{-1}$

IME I PREZIME: Damnjanović Nikola BROJ INDEKSA: 17-2-0028-2010

2) $z^3 + |3i+4| = \frac{5}{i^{233}}$

$233 : 4 = 58$ (33)

$i^1 = i$

$z^3 = \frac{5}{i} - |3i+4|$

$z^3 = \frac{5}{i} - |\sqrt{4^2+3^2}|$

$z^3 = \frac{5}{i} - |\sqrt{16+9}|$

$z^3 = \frac{5}{i} - 5$

$z = \sqrt[3]{-5+5i}$

$\frac{5}{i} = \frac{5}{i} \cdot \frac{-i}{-i} = \frac{-5i}{1} = -5i$

$z = \sqrt[3]{-5-5i} \Rightarrow \varphi = \frac{5\pi}{4}$

$x = -5$

$y = 5$ ✗

$r = \sqrt{(5)^2+5^2}$

$r = \sqrt{25} = 5\sqrt{2}$?

$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \approx 7.07$ ✓

$tg\varphi = \frac{y}{x} = \frac{5}{-5} = -1$

$\alpha = \frac{1}{4}\pi$

$\varphi = \pi - \alpha = \frac{3}{4}\pi$

$z = \sqrt[3]{5\sqrt{2}} \left(\cos \frac{\varphi + 2k\pi}{3} + i \sin \frac{\varphi + 2k\pi}{3} \right)$

$\sqrt[3]{5\sqrt{2}} = \sqrt[3]{7.07} = 1.91$

$k = 0, 1, 2$

k_0

$z_0 = 1.919 \left(\cos \frac{\frac{3\pi}{4}}{3} + i \sin \frac{\pi}{4} \right) = 1.919 \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \right)$

$= 1.356 + 1.356\,i$

k_1

$z_1 = 1.919 \left(\cos \frac{\frac{3\pi}{4} + 2\pi}{3} + i \sin \frac{11}{12}\pi \right) = 1.919 \left(\frac{-\sqrt{6}-\sqrt{2}}{4} + \frac{\sqrt{6}-\sqrt{2}}{4} i \right)$

$= -1.853 + 0.496\,i$

k_2

$z_2 = 1.919 \left(\cos \frac{\frac{3\pi}{4} + 4\pi}{3} + i \sin \frac{19}{12}\pi \right) = 1.919 \left(\frac{\sqrt{6}-\sqrt{2}}{4} + \left(\frac{-\sqrt{6}-\sqrt{2}}{4} \right) i \right)$

$= 0.496 - 1.853\,i$

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POGREŠNO JE ODREĐEN Im(z), ALI OSTATAK JE UREDAN.

IME I PREZIME: Domagoj Nikić BROJ INDEKSA: 17-2-0028-2010

3). $f(x) = \ln\left(\frac{x+5}{1-x}\right)$

$D(f) = [-5, 1] \setminus \{1\}$ ✗

$D(f) = \langle -5, 1 \rangle$

$f(-5)$ NE POSTOJI.

$\frac{x+5}{1-x} > 0 \quad / \cdot 1-x \qquad 1-x \neq 0$

$-x \neq -1$

$x \neq 1$

$(x+5)(1-x) > 0$

$x - x^2 + 5 - 5x$

$-x^2 - 4x + 5 \quad / \cdot (-1)$

$x^2 + 4x - 5$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 + 20}}{2}$

$= \frac{-4 \pm 6}{2}$

$x_1 = 1 \qquad x_2 = -5$

	$-\infty$		-6		-5		0		1		2		$+\infty$
$f(x)$				$-$		$+$		$+$		$-$			

✓

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f nije definirana za $x \geq 1$

NE POSTOJI $\lim_{x \to +\infty} f(x)$

V.A

$\lim_{x \to \infty} \ln\left(\frac{x+5}{1-x}\right) \overset{L'H}{=} \left(\ln\left(\frac{x+5}{1-x}\right)\right)' = \lim_{x \to \infty} \frac{\frac{1}{\frac{x+5}{1-x}} \cdot \frac{1 \cdot (1-x) - (x+5) \cdot (-1)}{(1-x)^2}}{\frac{1}{-1}} = \frac{-1}{1} = -1$

H.A

$\lim_{x \to -5} \ln\left(\frac{x+5}{1-x}\right) = \lim_{x \to -5} \ln\left(\frac{-5+5}{1+5}\right) = \ln \frac{0}{6} = \infty$ ✗

~~$\lim_{x \to 0} f(x) = \infty$~~

$\lim_{x \to 0} \ln(x) = -\infty$

Nema kose

IME I PREZIME: Dominoggj Nehmić BROJ INDEKSA: 17-2-0028-2010

4) $g(x) = \cos(\sin(3x))$

$D(f) = \langle \frac{\pi}{3}; -\frac{\pi}{3} \rangle$ X

1° $\sin(3x) > 0$

$3x \in \langle \pi; -\pi \rangle \; / : 3$

$x \in \langle \frac{\pi}{3}; -\frac{\pi}{3} \rangle$

2° f-ja je periodična jer je trigonometrijska, ponavlja se S KOJIM PERIODOM?

3° cos je parna f-ja

$\sin(-x) = -\sin(x)$ je neparna f-ja KAKAV JE g(x)?

4° $g(x) = \cos(\sin(3x))$

$g'(x) = -\sin(\cos(3x)) \cdot (3x)'$

$= -\sin(\cos(3x)) \cdot 3$

$= -3\sin(\cos(3x))$ ✓

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IME I PREZIME: Damoegoj Nekić BROJ INDEKSA: 17 - 2 - 0028 - 2010

5)

$$f(x) = \ln\left(\frac{x+5}{1-x}\right)$$

$$f'(x) = \frac{1}{\frac{x+5}{1-x}} \cdot \frac{(x+5)'\cdot(1-x) + (x+5)(1-x)'}{(1-x)^2}$$

$$= \frac{1}{\frac{x+5}{1-x}} \cdot \frac{1\cdot(1-x) + (x+5)(-1)}{(1-x)^2}$$ ✓

$$= \frac{1}{\frac{x+5}{1-x}} \cdot \frac{1-x+(-x-5)}{(1-x)^2} = \frac{1}{\frac{x+5}{1-x}} \cdot \frac{1-x-x-5}{(1-x)^2}$$

$$= \left[\frac{1}{\frac{x+5}{1-x}}\right] \cdot \frac{-x^2-4}{(1-x)^2} = \frac{1-x}{x+5} \cdot \frac{-x^2-4}{(1-x)^2} = \frac{-x^2-4+x^3+4x}{(x+5)(1-x)^2}$$

$$= \frac{x^3-x^2+4x-4}{(x+5)(1-x)^2}$$

Nultočke

$f(x) = 0$ (0

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5) $f'(x) = \dfrac{x^3-x^2+4x-4}{(x+5)(1-x)^2}$ ✗

$$f''(x) = \frac{(x^3-x^2+4x-4)' \cdot ((x+5)(1-x)^2) + (x^3-x^2+4x-4) \cdot ((x+5)(1-x)^2)' \cdot (1-x)'}{((x+5)(1-x)^2)^2}$$

$$= \frac{(3x^2-2x+4) \cdot ((x+5)(1-x)^2) + (x^3-x^2+4x-4)(1 \cdot 2(1-x)) \cdot (-1)}{((x+5)(1-x)^2)^2}$$

$$= \frac{\qquad\qquad}{\qquad\qquad}$$

Popuniti odmah!

IME I PREZIME: MATE IVIĆ BROJ INDEKSA: 17-2-0008-2010

DATUM: VRIJEME: OD 14:00 DO 15:50

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

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~~0~~ 10

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4. Ispitati domenu, periodičnost, (ne)parnost i prvu derivaciju funkcije $g(x) = \cos(\sin(3x))$.

~~0~~

5. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije f iz zadatka 3.

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1.

$$A = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

$$\det A \begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{vmatrix} \leftarrow \begin{matrix} \\ \\ \\ (-2) \end{vmatrix} \sim \begin{vmatrix} 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{vmatrix} \sim 1 \cdot (-1)^5 \begin{vmatrix} 0 & 0 & -3 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} \sim$$

$$\sim -1 \cdot (-1)^4 \begin{vmatrix} 0 & -3 \\ 1 & 0 \end{vmatrix} = -(0 \cdot 0 - (1 \cdot (-3))) = -(0 - (-3)) = -(0 + 3) = -3 \neq 0$$

REGULARNA MATRICA POSTOJI INVERZ (A^{-1})

$$\left[\begin{array}{cccc|cccc} 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \leftarrow \begin{matrix} \\ \\ \\ (-2) \end{matrix} \sim \left[\begin{array}{cccc|cccc} 0 & 0 & 0 & -3 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \leftarrow \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 & 0 & -2 \end{array}\right] \leftarrow \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 & 0 & -2 \end{array}\right] \cdot \left(-\frac{1}{3}\right) \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 & 0 & \frac{2}{3} \end{array}\right] \leftarrow \begin{matrix} \\ \\ \\ (-2) \end{matrix} \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & 0 & 0 & \frac{2}{3} \end{array}\right]$$

A^{-1} INVERZ ✓

NASTAVAK →

$-\frac{4}{3}+\frac{3}{3}=-\frac{1}{3}$

IME I PREZIME: MATE IVIĆ BROJ INDEKSA: 17-1-0008-2010

NASTAVAK (1.) ZADATKA

$A \cdot A^{-1}$

$$\begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{3} & 0 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{3} & 0 & 0 & \frac{2}{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{3}+0+0-\frac{1}{3} & 0+0+0+0 & -0+0+0+0 & -\frac{2}{3}+0+0+\frac{2}{3} \\ 0+0+0+0 & 0+0+1+0 & 0+0+0+0 & 0+0+0+0 \\ 0+0+0+0 & 0+0+0+0 & 0+1+0+0 & 0+0+0+0 \\ \frac{2}{3}+0+0-\frac{2}{3} & 0+0+0+0 & 0+0+0+0 & -\frac{1}{3}+\frac{4}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

JEDINIČNA MATRICA

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

✓

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IME I PREZIME: MATE IVIĆ BROJ INDEKSA:

(2) $|z^3 + |3i + 4| = \frac{5}{i \cdot 333}$

$z^3 = \frac{5}{i \cdot 333} - |3i + 4|$

$z^3 = \frac{5}{i \cdot 333} - \left(\sqrt{(4)^2 + (3)^2}\right)$

$z^3 = \frac{5}{i \cdot 333} - \left(\sqrt{16 + 9}\right)$

$z^3 = \frac{5}{i \cdot 333} - \left(\sqrt{25}\right)$

$z^3 = \frac{5}{i \cdot 333} - 5$

$z^3 = \frac{5}{i} - 5 \quad / \sqrt[3]{}$

$z = \sqrt[3]{\frac{5}{i} - 5}$

TREBALO JE RAČUNATI $\frac{5}{i} \cdot \frac{-i}{-i} = -5i$

$z = \sqrt[3]{-5 - 5i} = \ldots$

IME I PREZIME: BROJ INDEKSA:

NASTAVAK (2)

$$r = \sqrt{|5|^2 + |-5|^2} = \sqrt{25+25} = \sqrt{50}$$

$$\operatorname{tg}\varphi = \left|\frac{y}{x}\right|$$

$$\operatorname{tg}\varphi = \left|\frac{5}{-5}\right| = \frac{5}{5} = 1$$

$x = -5$
$y = -5$

$$\operatorname{arctg} 1 = 0.78539$$

$$\varphi = \pi - \varphi_0$$

$$\varphi = 3.1416 - 0.78539$$

$$\varphi = 2.35621$$

$$k = 0 \quad \sqrt[n]{r}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right)$$

$\sqrt[3]{\sqrt{50}}$
$\sqrt[6]{50}$

$$\sqrt[6]{50}\left(\cos\frac{2.35621 + 2\cdot 0\cdot\pi}{3} + i\sin\frac{2.35621 + 2\cdot 0\cdot\pi}{3}\right)$$

$$1.91938\left(\cos\frac{2.35621}{3} + i\sin\frac{2.35621}{3}\right)$$

$$1.91938\left(\cos(0.78540) + i\sin(0.78540)\right)$$

$$1.91938\,(0.70710 + i\,0.70710)$$

$$1.35719 + i\,1.35719$$

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POGREŠNO JE ODREĐEN $\operatorname{Im}(z)$, ALI OSTATAK JE UREDAN.

IME I PREZIME: MATE IVIĆ BROJ INDEKSA:

NASTAVAK (2)

$k=1 \quad 1.91938\left(\cos\frac{2.35621+2\cdot\pi}{3}+i\sin\frac{2.35621+2\cdot\pi}{3}\right)$

$1.91938\left(\cos(2.87980)+i\sin(2.87980)\right)$

$1.91938\left(-0.95892+i\,0.25881\right)$

$-1.85396+i\,0.49675$

$k=2 \quad 1.91938\left(\cos\frac{2.35621+4\cdot\pi}{3}+i\sin\frac{2.35621+4\cdot\pi}{3}\right)$

$1.91938\left(\cos(4.97420)+i\sin(4.97420)\right)$

$1.91938\left(0.25883+i(-0.96592)\right)$

$0.49679+i(-1.85397)$

IME I PREZIME: IVIĆ BROJ INDEKSA:

③ $f(x) = \ln\left|\frac{x+5}{1-x}\right|$

$\frac{x+5}{1-x} > 0$

$1 - x \neq 0$

$-x \neq -1$

$x \neq 1$

$(x+5)(1-x) > 0$

$x - x^2 + 5 - 5x$

$-x^2 - 4x + 5 \;/\cdot(-1)$

$x^2 + 4x - 5$

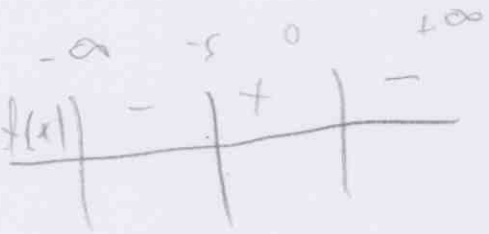
$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-4 \pm \sqrt{16 - 4 \cdot (-5)}}{2}$

$x_{1,2} = \frac{-4 \pm \sqrt{36}}{2}$

$x_{1,2} = \frac{-4 \pm 6}{2}$

$x_1 = -5$ $x_2 = 1$



$D(f) = [-5, 1] \setminus \{1\}$ ✗

$D(f) = \langle -5, 1 \rangle$

$f(-5)$ NE POSTOJI.

KOJE SU SVE ASIMPTOTE OVE FUNKCIJE?

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IME I PREZIME: MATE IVIĆ

BROJ INDEKSA:

PRVA DERIVACIJA

$g(x) = \cos(\sin(3x))$

$(3x)' = 3$

$g'(x) = \cos(\sin(3))$ X

NAUČITI DERIVACIJU KOMPOZICIJE FUNKCIJA.

VIDI LALIĆ