

IME I PREZIME:

UKE SVILICIC

BROJ INDEKSA:

17-2-0043-2010

DATUM:

VRJEME: OD 13:00

DO 14:52

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaci pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljšavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

OXOX

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Broj ↓  
bodova

1. Pronaci sve kompleksne brojeve z takve da vrijedi:  $2 + 2i = \frac{z^3}{z^3 - i^{99}}$ .

2. Odrediti inverz matrice i provjeriti matrični umnožak  $AA^{-1}$  ako je zadano:

$$A = \begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$$

3. Istražiti konvergenciju reda:  $\sum_{n=1}^{\infty} \left(\frac{2+2n}{3n+5}\right)^{n^2}$

4. Odrediti period i prvu derivaciju funkcije:  $g(x) = \frac{\sin(2x)}{\cos(3x)}$ .

5. Ispitati domenu, periodičnost, parnost i pronaci prvu derivaciju funkcije:  $f(x) = \frac{x^2}{x-1}$ .

~~0~~

~~0~~

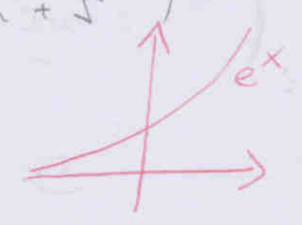
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3.  $\sum_{n=1}^{\infty} \left(\frac{2+2n}{3n+5}\right)^{n^2}$

NUŽNI UVJET

$$\lim_{x \rightarrow \infty} \left(\frac{2+2n}{3n+5}\right)^{n^2} = \lim_{x \rightarrow \infty} \left(1 + \frac{2+2n}{3n+5} - 1\right)^{n^2} = \lim_{x \rightarrow \infty} \left(1 + \frac{-3-n}{3n+5}\right)^{n^2}$$
$$= \lim_{x \rightarrow \infty} \left[1 + \frac{1}{\frac{3n+5}{-3-n}}\right]^{n^2} = e^{-\infty} = ?$$



CAUCHY

$$\sqrt[n]{\left(\frac{2+2n}{3n+5}\right)^{n^2}} = \left(\frac{2+2n}{3n+5}\right)^n = \left(1 + \frac{2+2n}{3n+5} - 1\right)^n = \left(1 + \frac{1}{\frac{3n+5}{-3-n}}\right)^n$$

$$= e^{-\infty} = ?$$

$e^{-\infty} = 0$  ZBOG  $\lim_{x \rightarrow -\infty} e^x = 0$

ZAKLJUČAK?

VIDI GRAF IZNAD.

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$$A \begin{bmatrix} 0 & 3 & 0 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 0 & 1 & 0 \\ 1 & 0 & 3 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 3 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 8 & 0 & 0 & -1 & 0 & 3 \\ 0 & 3 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ 3R2-2R4 \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 3 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 8 & 0 & 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 8 & -1 & 0 & 3 & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \frac{1}{8}R3 \sim \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{8} & 0 & \frac{3}{8} \\ 0 & 0 & 0 & 8 & -1 & 0 & 3 & 0 \end{bmatrix} \begin{matrix} 3R3-2R1 \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -\frac{3}{8} & 0 & \frac{1}{8} \\ 0 & 1 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{8} & 0 & \frac{3}{8} \\ 0 & 0 & 0 & 8 & -1 & 0 & 3 & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \frac{1}{8}R4 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -\frac{3}{8} & 0 & \frac{1}{8} \\ 0 & 1 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{8} & 0 & \frac{3}{8} \\ 0 & 0 & 0 & 1 & -\frac{1}{8} & 0 & \frac{3}{8} & 0 \end{bmatrix} \begin{matrix} \\ 3R4-R2 \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -\frac{3}{8} & 0 & \frac{1}{8} \\ 0 & 1 & 0 & 0 & -\frac{3}{8} & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{8} & 0 & \frac{3}{8} \\ 0 & 0 & 0 & 1 & -\frac{1}{8} & 0 & \frac{3}{8} & 0 \end{bmatrix} \times$$

$$A^{-1} = \begin{bmatrix} \frac{1}{8} & 0 & -\frac{3}{8} & 0 \\ 0 & \frac{1}{8} & 0 & -\frac{3}{8} \\ \frac{3}{8} & 0 & -\frac{1}{8} & 0 \\ 0 & \frac{3}{8} & 0 & -\frac{1}{8} \end{bmatrix}$$

PROVJERA  
 $A A^{-1} = ?$



2.

$$\begin{bmatrix} 0 & 3 & 0 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 0 & 1 & 0 \\ 1 & 0 & 3 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \text{BR-1-13}$$

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 3 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

~~TRICA NEMA  
INVERZ~~

4.

$$g(x) = \frac{\sin(2x)}{\cos(3x)}$$

$$\begin{array}{l} \sin x = \cos x \\ \cos x = -\sin x \end{array}$$

$$g'(x) = \left( \frac{\sin(2x)}{\cos(3x)} \right)'$$

$$g'(x) = \frac{(\sin(2x))' \cdot \cos(3x) - \sin(2x) \cdot (\cos(3x))'}{(\cos(3x))^2} \quad \checkmark$$

$$g'(x) = \frac{\cos(2x) \cdot 2 \cdot \cos(3x) - \sin(2x) \cdot [-\sin(3x) \cdot 3]}{(\cos(3x))^2} \quad \checkmark$$

$$g'(x) = \frac{2 \cos(5x) + 3 \sin(5x)}{[\cos(3x)]^2} \quad \times$$

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$$\cos(2x) \cdot \cos(3x) \neq \cos(5x)$$

$$5. f(x) = \frac{x^2}{x-1}$$

$$1^\circ D(f) \left. \begin{array}{l} x-1 \neq 0 \\ x \neq 1 \end{array} \right\} D(f) \mathbb{R} \setminus \{1\} \quad \checkmark$$

$$\langle -\infty, 1 \rangle \cup \langle 1, +\infty \rangle \quad \checkmark$$

2° funkcija NIJE PERIODIČNA jer ne sadrži trigonometrijske funkcije  $\checkmark$

3° PARNOST

$$f(x) = f(-x)$$

$$\frac{x^2}{x-1} = \frac{-x^2}{-x-1} = \frac{x^2}{-x-1} \quad \left. \begin{array}{l} \text{FUNKCIJA} \\ \text{NIJE PARNA} \end{array} \right\} \checkmark$$

$$\frac{x^2}{x-1} \neq \frac{x^2}{-x-1}$$

$$4^\circ f'(x) = \left( \frac{x^2}{x-1} \right)' = \frac{(x^2)' \cdot (x-1) - x^2 \cdot (x-1)'}{(x-1)^2} =$$

$$= \frac{2x(x-1) - x^2 \cdot 1}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$$

$$f'(x) = \frac{x^2 - 2x}{(x-1)^2} \quad \checkmark \quad \text{GRAF?} \quad \underline{20}$$