

IME I PREZIME: NINO MIKULANORA

BROJ INDEKSA: 57645

DATUM: VRIJEME: OD 12h30min DO 13h47

MATEMATIKA 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

OXOX

Broj ↓
bodova

1. Pronaći sve kompleksne brojeve z takve da vrijedi: $\overline{2+2i} = \frac{z^3}{z^3 - i^{99}}$.

2. Odrediti inverz matrice i provjeriti matricni umnožak AA^{-1} ako je zadano:

$$A = \begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$$

3. Istražiti konvergenciju reda: $\sum_{n=1}^{\infty} \left(\frac{2+2n}{3n+5} \right)^{n^2}$

4. Odrediti period i prvu derivaciju funkcije: $g(x) = \frac{\sin(2x)}{\cos(3x)}$.

5. Ispitati domenu, periodičnost, parnost i pronaći prvu derivaciju funkcije: $f(x) = \frac{x^2}{x-1}$.

2.)

$$\det A = \begin{vmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{vmatrix} \begin{matrix} (-3) \\ \leftarrow + \\ (-3) \end{matrix} \sim \begin{vmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & -8 & 0 & 0 \\ 1 & 0 & 3 & 0 \end{vmatrix} =$$

$$= a_{14} A_{14} = (-1)^5 \begin{vmatrix} 3 & 0 & 1 \\ 0 & -8 & 0 \\ 1 & 0 & 3 \end{vmatrix} \begin{matrix} (-3) \\ \leftarrow + \end{matrix} \sim \begin{vmatrix} 3 & 0 & 1 \\ 0 & -8 & 0 \\ -8 & 0 & 0 \end{vmatrix} =$$

$$= -a_{13} A_{13} = \begin{vmatrix} 0 & -8 \\ -8 & 0 \end{vmatrix} = -(0 - 64) = 64 //$$

$$A_{31} = \begin{vmatrix} \cancel{3} & \cancel{0} & \cancel{1} \\ 0 & 1 & 0 \\ 0 & 3 & 0 \end{vmatrix} = a_{13} A_{13} = \begin{vmatrix} 0 & \cancel{1} & \cancel{0} \\ 0 & \cancel{3} & \cancel{0} \end{vmatrix} = 0 //$$

$$A_{32} = \begin{vmatrix} \cancel{0} & \cancel{0} & \cancel{1} \\ 3 & 1 & 0 \\ 1 & 3 & 0 \end{vmatrix} = -a_{13} A_{13} = - \begin{vmatrix} 3 & \cancel{1} & \cancel{0} \\ 1 & \cancel{3} & \cancel{0} \end{vmatrix} - (9 - 1) = -8 //$$

$$A_{33} = \begin{vmatrix} \cancel{0} & \cancel{3} & \cancel{1} \\ 3 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix} = a_{13} A_{13} = \begin{vmatrix} 3 & \cancel{0} & \cancel{0} \\ 1 & \cancel{0} & \cancel{0} \end{vmatrix} = 0 //$$

$$A_{34} = \begin{vmatrix} 0 & 3 & 0 \\ 3 & 0 & 1 \\ 1 & 0 & 3 \end{vmatrix} \begin{matrix} (-3) \\ (+) \end{matrix} \sim \begin{vmatrix} 0 & 3 & 0 \\ \cancel{3} & \cancel{0} & \cancel{1} \\ -8 & 0 & 0 \end{vmatrix} = a_{23} A_{23} = (-1)^5 \begin{vmatrix} 0 & 3 \\ -8 & 0 \end{vmatrix} \\ = - (0 + 24)$$

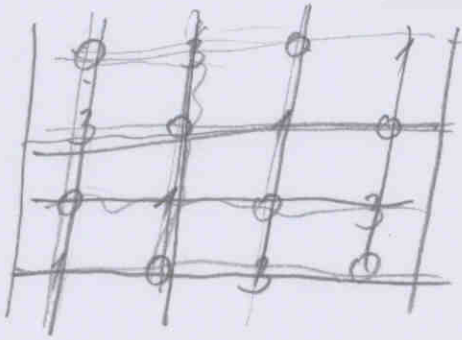
$$A_{41} = \begin{vmatrix} 3 & 0 & 1 \\ \cancel{0} & \cancel{1} & \cancel{0} \\ 1 & 0 & 3 \end{vmatrix} = -a_{22} A_{22} = \begin{vmatrix} 3 & \cancel{0} \\ 1 & \cancel{3} \end{vmatrix} = 24 //$$

$$= - (9 - 1) = -8 //$$

$$A_{42} = \begin{vmatrix} \cancel{0} & \cancel{0} & \cancel{1} \\ 3 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} = a_{13} A_{13} = \begin{vmatrix} 3 & \cancel{1} \\ 0 & \cancel{0} \end{vmatrix} = 0 //$$

$$A_{43} = \begin{vmatrix} 0 & 3 & 1 \\ 3 & 0 & 0 \\ 0 & 1 & 3 \end{vmatrix} \begin{matrix} (-3) \\ (+) \end{matrix} \sim \begin{vmatrix} \cancel{0} & \cancel{3} & \cancel{1} \\ 3 & 0 & 0 \\ 0 & -8 & 0 \end{vmatrix} = -a_{13} A_{13} = \begin{vmatrix} 3 & 0 \\ 0 & -8 \end{vmatrix} = -(-24 - 9) \\ = 24 //$$

$$A_{44} = \begin{vmatrix} 0 & 3 & 0 \\ \cancel{3} & \cancel{0} & \cancel{1} \\ 0 & 1 & 0 \end{vmatrix} = a_{23} A_{23} = \begin{vmatrix} 0 & \cancel{3} \\ 0 & \cancel{1} \end{vmatrix} = 0 //$$



$$A_{11} = (-1)^2 \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 3 \\ 0 & 3 & 0 \end{vmatrix} = a_{21} A_{21} = (-1)^3 \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = -(0 - 0) = \underline{\underline{0}}$$

$$A_{12} = (-1)^3 \begin{vmatrix} 3 & 1 & 0 \\ 0 & 0 & 3 \\ 1 & 3 & 0 \end{vmatrix} \xrightarrow{(-3)} \begin{vmatrix} 3 & 1 & 0 \\ 0 & 0 & 3 \\ -8 & 0 & 0 \end{vmatrix} = -a_{12} A_{12} = \begin{vmatrix} 0 & 3 \\ -8 & 0 \end{vmatrix} = (0 - (-24)) = \underline{\underline{24}}$$

$$A_{13} = (-1)^4 \begin{vmatrix} 3 & 0 & 0 \\ 0 & 1 & 3 \\ 1 & 0 & 0 \end{vmatrix} = a_{22} A_{22} = (-1)^4 \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = \underline{\underline{0}}$$

$$A_{14} = (-1)^5 \begin{vmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{vmatrix} = -a_{22} A_{22} = -(-1)^4 \begin{vmatrix} 3 & 0 \\ 1 & 3 \end{vmatrix} = -(9 - 0) = \underline{\underline{-9}}$$

$$A_{21} = \begin{vmatrix} 3 & 0 & 1 \\ 1 & 0 & 3 \\ 0 & 3 & 0 \end{vmatrix} \xrightarrow{(-3)} \begin{vmatrix} 3 & 0 & 1 \\ -8 & 0 & 0 \\ 0 & 3 & 0 \end{vmatrix} = -a_{13} A_{13} = \begin{vmatrix} -8 & 0 \\ 0 & 3 \end{vmatrix} = -(-24 - 0) = \underline{\underline{24}}$$

$$A_{22} = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 3 \\ 1 & 3 & 0 \end{vmatrix} = a_{31} A_{31} \begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix} = -(0 - 0) = \underline{\underline{0}}$$

$$A_{23} = \begin{vmatrix} 0 & 3 & 1 \\ 0 & 1 & 3 \\ 1 & 0 & 0 \end{vmatrix} = -a_{24} A_{24} = - \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = -(9 - 1) = \underline{\underline{-8}}$$

$$A_{24} = \begin{vmatrix} 0 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{vmatrix} = a_{31} A_{31} = - \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$A^{-1} \left| \begin{array}{cccc|c} 0 & -24 & 0 & -8 & 0 \\ 24 & 0 & -8 & 0 & 0 \\ 0 & -8 & 0 & 24 & 0 \\ -8 & 0 & 24 & 0 & 0 \end{array} \right| \quad \text{inverza}$$

$$A \cdot A^{-1} \left| \begin{array}{cccc|c} 0 & 3 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 1 & 0 & 3 & 0 & 0 \end{array} \right| \times \left| \begin{array}{cccc|c} 0 & -24 & 0 & -8 & 0 \\ 24 & 0 & -8 & 0 & 0 \\ 0 & -8 & 0 & 24 & 0 \\ -8 & 0 & 24 & 0 & 0 \end{array} \right| = ?$$

$$\left| \begin{array}{l} 0 + 72 + 0 + (-8) \\ 0 + 0 + 0 + 0 \\ 0 + 24 + 0 + (-24) \\ 0 + 0 + 0 + 0 \end{array} \right|$$

$$\left| \begin{array}{l} 0 + 0 + 0 + 0 \\ -72 + 0 + (-8) + 0 \\ 0 + 0 + 0 + 0 \\ -24 + 0 + 24 + 0 \end{array} \right|$$

$$\left| \begin{array}{l} 0 + 0 + 0 + 0 \\ 0 + 0 + 0 + 0 \\ 0 + (-8) + 0 + 72 \\ 0 + 0 + 0 + 0 \end{array} \right|$$

$$\left| \begin{array}{l} 0 + 0 + 0 + 0 \\ (-24) + 0 + (-24) + 0 \\ 0 + 0 + 0 + 0 \\ (-8) + 0 + 72 + 0 \end{array} \right|$$

$$A \cdot A^{-1} = \begin{vmatrix} 64 & 0 & 0 & 0 \\ 0 & 64 & 0 & 0 \\ 0 & 0 & 64 & 0 \\ 0 & 0 & 0 & 64 \end{vmatrix}$$

MORA BITI $AA^{-1} = I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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$$A^{-1} = \frac{1}{\det A} (A_{ij}) = \frac{1}{64} \begin{pmatrix} 0 & 24 & 0 & -8 \\ 24 & 0 & -8 & 0 \\ 0 & -8 & 0 & 24 \\ -8 & 0 & 24 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{3}{8} & 0 & -\frac{1}{8} \\ \frac{3}{8} & 0 & -\frac{1}{8} & 0 \\ 0 & -\frac{1}{8} & 0 & \frac{3}{8} \\ -\frac{1}{8} & 0 & \frac{3}{8} & 0 \end{pmatrix}$$

TREBATE NAUČITI I OSTALO GRADIVO
A NE SAMO CRAMEROVO PRAVILO.