OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO .

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. oox

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IME I PREZIME: FRANE ŠIMIČEV **BROJ INDEKSA:** 56637-0069042991

1. Odrediti duljinu 5 navoja zavojnice s parametrizacijom $x = \frac{1}{5}\cos(t)$, $y = \frac{1}{5}\sin(t)$ i $z = \frac{t}{10}$. $(t \in [0, 10\pi])$

2. Izračunati $\iint\limits_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x+y \\ z-\sin x \\ z+\cos x \end{pmatrix}\begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix}$ i ∂K rub kugle K radijusa 2 s centrom u ishodištu, a koji je orijentiran vanjskom normalom.

$\iint\limits_{\partial K} f_1\, dy dz + f_2\, dx dz + f_3\, dx dy$

3. Izračunati volumen tijela omeđenog plohama: $z = x^2 + y^2$, $z = 2$.

4. Neka je točkama $A(1,0)$, $B(1,4)$, $C(-2,2)$ $D(-2,0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral 10

$$\oint\limits_C 2(x+y^2)\mathrm{d}x + (x+y)^2\mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$f'''(t) - f''(t) + f'(t) - f(t) = e^t, \quad f'(0) = f''(0) = 0,\ f(0) = 1.$$

1.

$x = \frac{1}{5}\cos(t)$

$y = \frac{1}{5}\sin(t)$

$z = \frac{t}{10}$

$(t \in [0, 10\pi])$

$25, 10 \mid 5$
$5, 2 \mid 5$
$\mid 2$

$= \sqrt{\frac{1}{5}\sin t^2 - \frac{1}{5}\cos t^2 - \frac{1}{10}}$

$\begin{bmatrix} \frac{1}{5}\sin t \\ -\frac{1}{5}\cos t \\ \frac{1}{10} \end{bmatrix}$

$= \sqrt{\frac{1}{25} - \frac{1}{10}} = \sqrt{\frac{2-5}{50}} = \sqrt{-\frac{3}{50}}$

$\sqrt{-\frac{3}{50}} \int_0^{10\pi} dt =$

VIDI ROKO LENKIĆ

IME I PREZIME: FRANE ŠIMIČEV BROJ INDEKSA: 56637

(5.) $x'''(t) - x''(t) + x'(t) - x(t) = e^t$

$x'(0) = 0$

$x''(0) = 0$

$x(0) = 1$

$s^3 x(t) \cancel{-sx(0)} \cancel{+sx(0)} - x(1) = \frac{1}{s-1}$

$s^3 x(t) - \frac{1}{s} = \frac{1}{s-1}$

$$x(t) = \frac{1}{s^3(s-1)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{Ds+E}{s-1}$$

VIDI ANTE ŠIMIĆEVIĆ

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(4)

A (1, 0)
B (1, 4)
C (-2, 2)
D (-2, 0)

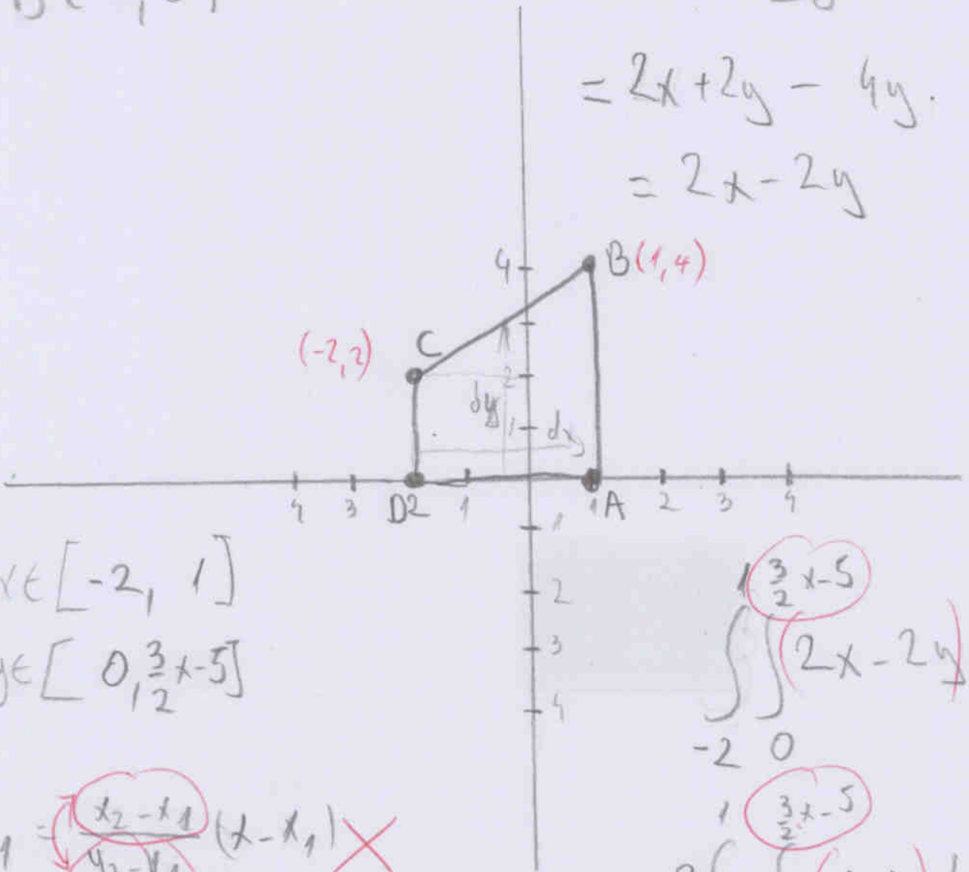
$$\oint_C \underbrace{2(x+y^2)}_{P}dx + \underbrace{(x+y)^2}_{Q}dy$$

$$= \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

$$= 2x + 2y - 4y$$

$$= 2x - 2y$$

10



$x \in [-2, 1]$

$y \in [0, \frac{3}{2}x - 5]$

$$\int_{-2}^{1}\int_{0}^{\frac{3}{2}x-5}(2x-2y)\,dx\,dy$$

$$2\int_{-2}^{1}\int_{0}^{\frac{3}{2}x-5}(x-y)\,dx\,dy = 2\int_{-2}^{1}\left(xy - \frac{y^2}{2}\right)\Big|_0^{\frac{2}{3}x+\frac{10}{3}} dx$$

$$= 2\int_{-1}\left(x\cdot\left(\tfrac{2}{3}x+\tfrac{10}{3}\right) - \frac{\left(\frac{2}{3}x+\frac{10}{3}\right)^2}{2}\right)dx$$

$$= \_\_\_$$

$$y - y_1 = \frac{x_2 - x_1}{y_2 - y_1}(x - x_1)$$

$$y + 4 = \frac{-2-1}{2-4}(x-1)$$

$$y + 4 = \frac{-3}{-2}(x-1)$$

$$y + 4 = \frac{3}{2}(x-1)$$

$$y + 4 = \frac{3}{2}x - 1 - \frac{3}{2}$$

$$y = \frac{3}{2}x - 1 - 4 \quad \frac{3}{2}x - \frac{3}{2} + 4$$

$$y = \frac{3}{2}x - 5 \quad \frac{3}{2}x$$

$$\cancel{y = \frac{3}{2}x + 3}$$

$$= 2\int_{-2}^{1} x\,dx - 1\int_{0}^{\frac{3}{2}x-5} y\,dy$$

$$= 2\left[\frac{x^2}{2}\right]_{-2}^{1} - 1\left[\frac{y^2}{2}\right]_{0}^{\frac{3}{2}x-5}$$

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y = \frac{2}{3}x + \frac{10}{3}$$

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(3)

$z = x^2 + y^2$ $z = 2$

$x = 0 \left[2 = 0^2 + y^2 \right.$

$2 = y^2$

$y = \sqrt{2}$

$y = 0 \left[2 = x^2 + 0 \right.$

$x = \sqrt{2}$

$$\int_0^{\sqrt{2}} dx \int_0^{\sqrt{2}} dy \int_0^{2} dz =$$

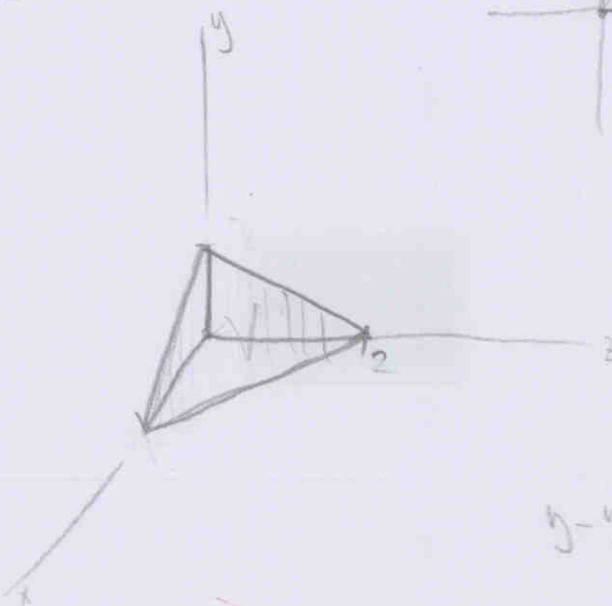
x	y
0,	$\sqrt{2}$
0,	$\sqrt{2}$

$r^2 = z$

$z = r^2$

$r^2 = 2$

$2 = 2$



$x \in (0, \sqrt{2})$

$z \in (0, 2)$

$y \in (0,$

0

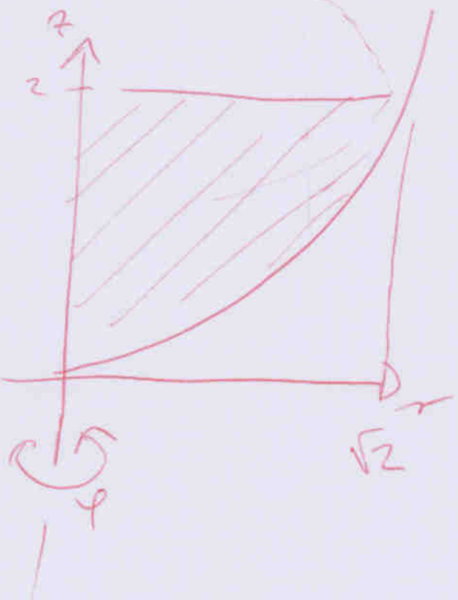
$y - y_1 = \frac{x_2 - x_1}{y_2 - y_1} (x - x_1)$

$y - \sqrt{2} = \frac{0 - 0}{} \cdot 0$

$y = \sqrt{2}$

$z = r^2$

$z = 2$



$$\boxed{V = \int_0^{2\pi} \int_0^{\sqrt{2}} \int_{r^2}^{2} r \, dz \, dr \, d\varphi}$$

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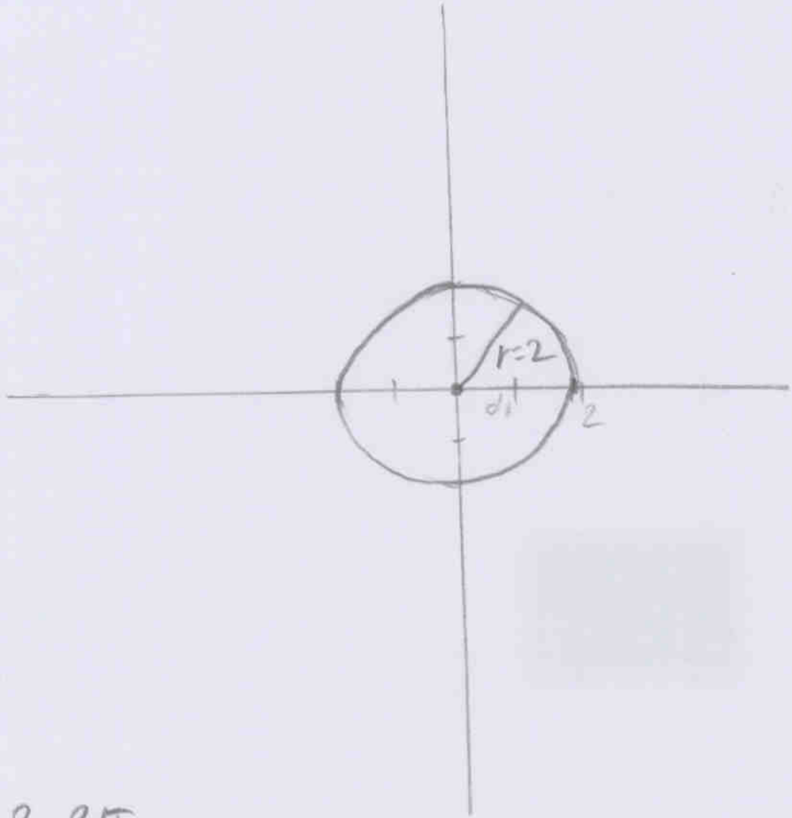
(2)

$$\iint_{\partial K} F\cdot dS \quad \text{gdje je } F\begin{pmatrix} x+y \\ z-\sin x \\ z+\cos x \end{pmatrix} \qquad r=2$$



$x = \cos f$
$y = \sin f$

$x_1 = x+2$
$y_1 = y+2$
$x_1 = r\cos f + 2$
$y_1 = r\sin f + 2$

$r \in [0,2)$
$f \in (0, 2\pi$

$$\int_0^2\int_0^{2\pi} x+y \, dy\, dz + z - \sin x \, dx\, dz + z + \cos x \, dx\, dy$$

∅

$$\iint_{\partial K} F\cdot dS = \iiint_K \operatorname{div} F \, dx\,dy\,dz = \iiint_K 2 \, dx\,dy\,dz = 2V(K)$$

$$\boxed{\operatorname{div} F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 1 + 0 + 1 = 2}$$