OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO .

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. oxo

20

IME I PREZIME: JOSIP VLASTELIĆ **BROJ INDEKSA:** 54951-2007

1. Izračunati dvostruki integral:

$$\iint_S x \,\mathrm{d}x\mathrm{d}y\,,$$

gdje je S područje gornje poluravnine ($y \geq 0$) omeđeno kružnicom $(x-1)^2 + y^2 = 4$.

2. Izračunati $\int_{\widehat{ABC}} z\mathrm{d}x + y\mathrm{d}y + x\mathrm{d}z$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(1,0,0)$, $B(0,1,0)$, $C(0,0,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

3. Izračunati volumen tijela omeđenog valjkom $x^2 + 4y^2 = 4$ i ravninama $z = y$ i $z = -2$.

4. Izračunati

$$\int_{(0,\pi)}^{(2,2\pi)} 2x \sin y \,\mathrm{d}x + (x^2+1)\cos y \,\mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$2x'''(t) - 4\,x''(t) = \cos(2t)\,, \quad x(0) = x''(0) = 0,\ x'(0) = 4.$$

4. $\displaystyle\int_{(0,\pi)}^{(2,2\pi)} 2x \sin y\, dx + (x^2+1)\cos y\, dy$

$\varnothing$

$W = \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} 2x\sin y \\ (x^2+1)\cos y \end{bmatrix} - \text{grad } f$

$d_x f = 2x \sin y \quad / \int dx$ $\qquad \int 2x \sin y\, dx = \sin y \int 2x\,dx = (\sin y)\cdot x^2 = x^2 \sin y$

$f = 2\cos y + C(y)$ $\times$

$d_y f = (x^2+1)\cos y$

$d_y\left(2\cos y + C(y)\right) = x^2\cos y + \cos y$ $\qquad \int x^2 \cos y\, dy = x^2 \sin y$

$-2\sin y + C'(y) = x^2 \cos y + \cos y$

$C'(y) = x^2\cos y + \cos y + 2\sin y \;/\int dy \Rightarrow C(y) = \int \frac{x^3}{3}\sin y\left(+\sin y - 2\cos y\right) dy$ $\times$

IME I PREZIME: JOSIP VLASTELIĆ   BROJ INDEKSA: 54951-2007

4. nastavak

$$\int_{(0,\pi)}^{(2,2\pi)} \frac{x^3}{3}\sin y\left(+\sin y-2\cos y\right)dy = \int_{(0,\pi)}^{(2,2\pi)}\frac{x^3}{3}\sin y\,dy+\int_{(0,\pi)}^{(2,2\pi)}\sin y\,dy-2\int_{(0,\pi)}^{(2,2\pi)}\cos y\,dy$$

$$=\int_{(0,\pi)}^{(2,2\pi)}\frac{x^3}{3}\sin y\,dy+\int_{(0,\pi)}^{(2,2\pi)}\sin dy-2\left(\underbrace{\cos(2,2\pi)-\cos(0,\pi)}_{0}\right)=\int_{(0,\pi)}^{(2,2\pi)}\frac{x^3}{3}\sin y\,dy+\int_{(0,\pi)}^{(2,2\pi)}\sin dy-0$$

$$=\int_{(0,\pi)}^{(2,2\pi)}\frac{x^3}{3}\sin y\,dy+\left(\underbrace{\sin(2,2\pi)-\sin(0,\pi)}_{0}\right)=\frac{\overbrace{\left(\sin(2,2\pi)-\sin(0,\pi)\right)}^{0}{}^4}{4}=\underline{0}$$

2. $\displaystyle\int_{ABC} z\,dx+y\,dy+x\,dz$

$A(\overset{x}{1},\overset{y}{0},\overset{z}{0}),\ B(\overset{x}{0},\overset{y}{1},\overset{z}{0})\ C(\overset{x}{0},\overset{y}{0},\overset{z}{0})$

$$\iint W\,dx+W\,dy+W\,dz$$

$$W\begin{bmatrix}dx\\dy\\dz\end{bmatrix}=\begin{bmatrix}z\\y\\x\end{bmatrix}\qquad \text{rot}=\nabla\times W$$

$$\text{rot}\begin{bmatrix}dx\\dy\\dz\end{bmatrix}\times\begin{bmatrix}z\\y\\x\end{bmatrix}=\begin{bmatrix}0-0\\1-1\\0-0\end{bmatrix}=0$$

dx z
dy y

$$\iint_{ABC}\text{rot}\,ds=\underline{0}$$

20

IME I PREZIME: JOSIP VLASTELIĆ   BROJ INDEKSA: 54951 - 2007

3. $x^2 - 4y^2 = 4$ $z = y$ i $z = -2$

$x^2 + 4y^2 = 4$

$(r\cos\varphi)^2 - (4r\sin\varphi)^2 = 4$

$r^2\cos^2\varphi - 16r^2\sin^2\varphi = 4$

$16r^2\underbrace{(\cos^2\varphi - \sin^2\varphi)}_{1} = 4$

$16r^2 = 4$

$r^2 = \frac{4}{16}$

$r^2 = \frac{2}{4}$

$r = \frac{1}{2}$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$r \in [0, \frac{1}{2}]$

$\varphi \in [0, 2\pi]$

$z \in [-2, y]$

$x = 2r\cos\varphi$

$y = r\sin\varphi$ $d(\varphi) = 2r$

$z = z$

$\int_0^{2\pi}\int_0^{1}\int_{-2}^{r\sin\varphi} 2r\,dz\,dr\,d\varphi$

$(2r\cos\varphi)^2 + 4(r\sin\varphi)^2 = 4$

$4r^2\cos^2\varphi + 4r^2\sin^2\varphi = 4$

$4r^2(\cos^2\varphi + \sin^2\varphi) = 4 \Rightarrow r^2 = 1$

$\int_0^{2\pi} d\varphi \int_0^{\frac{1}{2}} r\,dr \int_{-2}^{y} dz = \int_0^{2\pi} d\varphi \int_0^{\frac{1}{2}} r\,dr\,(y+2) = \int_0^{2\pi} d\varphi \int_0^{\frac{1}{2}} yr\,dr + 2\int_0^{\frac{1}{2}} r\,dr =$

$= \int_0^{2\pi} d\varphi\, y\int_0^{\frac{1}{2}} r\,dr + 2\left(\frac{(\frac{1}{2})^2}{2} + \frac{(0)^2}{2}\right) = \int_0^{2\pi} d\varphi\, y\int_0^{\frac{1}{2}} r\,dr + \frac{2}{8} = \frac{1}{4}y\int_0^{2\pi} d\varphi\, \frac{1}{8}$

$= \frac{1}{4}\cdot\frac{1}{8}y\,2\pi = \frac{1}{32}y\,2\pi$

$\varnothing$

NE MOŽE PISATI GRANICA U Y KAD U INTEGRACIJI VIŠE NEMA VARIJABLE Y VEĆ SAMO φ, r, z.

KAKAV JE OVO VOLUMEN TIJELA OVISAN O VARIJABLI Y?

IME I PREZIME: JOSIP VLASTELIĆ

BROJ INDEKSA: 54951 - 2007

$$\iint_S x\,dx\,dy \qquad y \geq 0$$

$$(x-1)^2+y^2=4 \qquad \varphi\in[0,2\pi]$$

$$r=\left[0,\frac{3}{r-2\cos\varphi}\right]$$

$$x=r\cos\varphi$$
$$y=r\sin\varphi$$

$$\int_0^{2\pi}d\varphi\int_0^{\frac{3}{r-2\cos\varphi}}r\,dr=\int_0^{2\pi}d\varphi\left(\frac{\left(\frac{3}{r-2\cos\varphi}\right)^2}{2}\right)$$

$$=\int_0^{2\pi}d\varphi\,\frac{\frac{9}{r^2-4\cos^2\varphi}}{2}$$

$$=\frac{9}{2r^2-8\cos^2\varphi}\int_0^{2\pi}d\varphi=\frac{9}{2r^2-8\cos^2\varphi}\,2\pi$$

$$(r\cos\varphi-1)^2+(r\sin\varphi)^2=4$$

$$r^2\cos^2\varphi-2r\cos\varphi+1+r^2\sin^2\varphi=4$$

$$r^2(\cos^2\varphi+\sin^2\varphi)-2r\cos\varphi=3$$

$$r^2-2r\cos\varphi=3 \quad ✓$$

$$r(r-2\cos\varphi)=3 \quad ✓ \quad \Big| :(r-2\cos\varphi)$$

$$r=\frac{3}{(r-2\cos\varphi)}$$

NIŠTA NE ZNAČI IZRAZITI r POMOĆU r.

MOGLO SE IZRAZITI φ UZ POMAĆ r:

$$\cos\varphi=\frac{r^2-3}{2r}$$

$$\varphi=\arccos\left(\frac{r^2-3}{2r}\right)$$

VIDI KARLO PERKOVIĆ

IME I PREZIME: JOSIP VLASTELIĆ   BROJ INDEKSA: 54951 - 2007

5. $2x'''(t) - 4x''(t) = \cos(2t)$ $x(0) = x''(0) = 0 \quad x'(0) = 4$

$$L[2x'''(t)] - L[4x''(t)] = L[\cos(2t)]$$

$$2[s^3X(t) - s^2\cancel{x(0)}^{0} - s\cancel{x'(0)}^{4} - \cancel{x''(0)}^{0}] - 4[s^2X(t) - s\cancel{x(0)}^{0} - \cancel{x'(0)}^{4}] = \left[\frac{s}{s^2+2^2}\right]$$

$$2(s^3X(t) - 4s) - 4s^2X(t) + 4\cdot 4 = \frac{s}{s^2+4}$$

Ø

JOŠ MALO VJEŽBE I NADAM SE DA ĆETE NA SLJEDEĆEM ROKU BITI USPJEŠNIJI.

VIDJETI BARIČEVIĆ

DUNATOV