

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pišaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posledicu imati udaljšavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

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1.  $X$  je zadan kao paralelogram s vrhovima  $O(0,0)$ ,  $A(1,0)$ ,  $B(2,1)$  i  $C(1,1)$ . Skicirati taj paralelogram i izračunati dvostruki integral 20

$$\iint_X y^2 dx dy$$

2. Neka je  $K$  kvadar ( $0 \leq x \leq 1$ ,  $0 \leq y \leq 2$ ,  $0 \leq z \leq 3$ ) i  $\partial K$  rub te kocke. Izračunati plošni integral 20

$$\iint_{\partial K} x dy dz + zy dx dz + xy dx dy$$

3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle  $x^2 + y^2 + z^2 = 9$  za koji vrijedi  $z \geq 2$ . 20

4. Izračunati

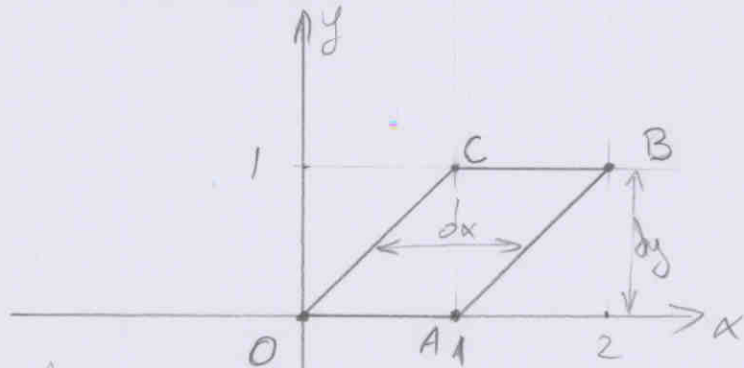
$$\int_{(3,2)}^{(1,1)} (x+y)(dy+dx)$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačbu:

$$y'''(t) - 2y''(t) = e^t, \quad y(0) = y''(0) = 1, \quad y'(0) = 2.$$

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- 1)  $O(0,0)$   
 $A(1,0)$   
 $B(2,1)$   
 $C(1,1)$



pravci:

$$y-0 = \frac{1-0}{1-0}(x-0)$$

$$\underline{y=x}$$

$OC \Rightarrow x=y$   
 $CB \Rightarrow y=1$   
 $OA \Rightarrow y=0$

$AB \Rightarrow$

$$y-0 = \frac{1-0}{2-1}(x-1)$$

pa je  $y = x-1$   
 $x = y+1$

$$\int_0^1 dy \int_y^{y+1} y^2 dx = \int_0^1 y^2 (y+1-y) dy =$$

$$= \int_0^1 y^2 dy = \left( \frac{y^3}{3} \right) \Big|_0^1 = \frac{1}{3}$$

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$$2) \quad \begin{aligned} 0 &\leq x \leq 1 \\ 0 &\leq y \leq 2 \\ 0 &\leq z \leq 3 \end{aligned}$$

$$\iiint \alpha \, dy \, dz + zy \, dx \, dz + xy \, dx \, dy$$

$$w = \begin{bmatrix} x \\ zy \\ xy \end{bmatrix} \quad \text{div } w = \begin{bmatrix} 1 \\ z \\ 0 \end{bmatrix} = (1+z) \quad \checkmark$$

$$\int_0^3 (1+z) \, dz \int_0^2 dy \int_0^1 dx = \int_0^3 (1+z) \, dz \int_0^2 (1-0) \, dy \quad \checkmark$$

$$= \int_0^3 (1+z) \, dz \int_0^2 dy = \int_0^3 (1+z) \, dz (2-0) = 2 \int_0^3 (1+z) \, dz \quad \checkmark$$

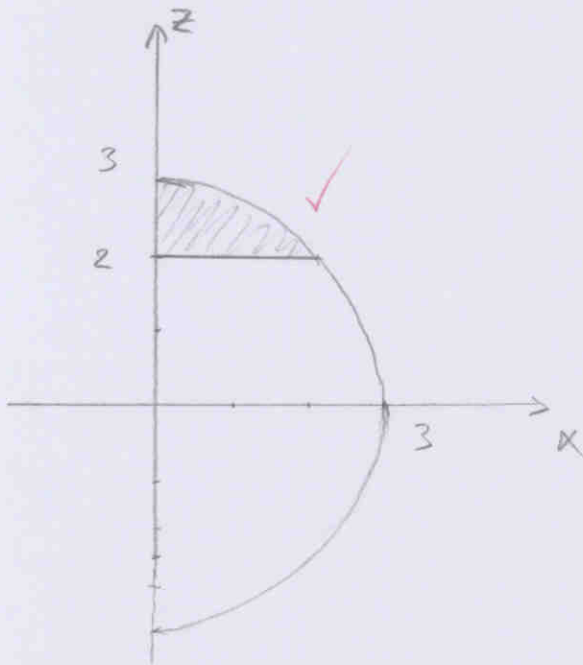
$$= 2 \left[ \int_0^3 dz + \int_0^3 z \, dz \right] = 2 \left[ 3 + \left. \left( \frac{z^2}{2} \right) \right|_0^3 \right]$$

$$= 2 \left[ 3 + \frac{9}{2} \right] = 2 \left[ \frac{15}{2} \right] = \underline{\underline{15}} \quad \checkmark \quad \underline{\underline{20}}$$

3)

$$x^2 + y^2 + z^2 = 9$$

$$z \geq 2$$



$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

$$z = z$$

$$x^2 + y^2 + z^2 = 9$$

$$r^2 \cos^2 \varphi + r^2 \sin^2 \varphi + z^2 = 9$$

$$r^2 = 9 - z^2$$

$$r = \sqrt{9 - z^2}$$

$$\varphi \in [0, 2\pi]$$

$$r \in [0, \sqrt{9 - z^2}]$$

$$z \in [2, 3]$$

$$\int_0^{2\pi} d\varphi \int_2^3 dz \int_0^{\sqrt{9-z^2}} r dr$$

$$= \int_0^{2\pi} d\varphi \int_2^3 \left[ \left( \frac{r^2}{2} \right) \Big|_0^{\sqrt{9-z^2}} \right] dz =$$

$$= \int_0^{2\pi} d\varphi \int_2^3 \left( \frac{(\sqrt{9-z^2})^2}{2} \right) dz = \int_0^{2\pi} d\varphi \int_2^3 \left( \frac{9-z^2}{2} \right) dz =$$

$$= \int_0^{2\pi} d\varphi \left[ \int_2^3 \frac{9}{2} dz - \int_2^3 \frac{z^2}{2} dz \right] = \int_0^{2\pi} d\varphi \left[ \underbrace{\frac{9}{2}(3-2)}_{\frac{9}{2}} - \frac{1}{2} \left( \frac{z^3}{3} \right) \Big|_2^3 \right]$$

$$= \int_0^{2\pi} \left( \frac{9}{2} + \frac{19}{6} \right) d\varphi = \int_0^{2\pi} \left( \frac{23}{3} \right) d\varphi = \frac{23}{3} \cdot 2\pi = \frac{46\pi}{3}$$

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$\frac{27 \cdot 4}{3}$

$= \frac{9}{6}$

~~$\frac{46\pi}{3}$~~

$$4) \int_{(3,2)}^{(1,1)} (x+y)(dy+dx)$$

$$W \begin{bmatrix} x+y \\ x+y \end{bmatrix} = -\text{grad } f$$

$$dx f = -x-y \quad / \int dx \Rightarrow f = \int (-x-y) dx = -\frac{x^2}{2} - xy + C(y)$$

$$f = -\frac{x^2}{2} + C(y) \quad \times$$

$$dy f = -x-y \quad \Rightarrow \quad \frac{\partial}{\partial y} \left( -\frac{x^2}{2} - xy + C(y) \right) = -x-y$$

$$-x + \frac{\partial C(y)}{\partial y} = -x-y$$

$$\frac{dy}{dy} \left( -\frac{x^2}{2} + C(y) \right) = -x-y$$

$$= 0 + C'(y)$$

$$C(y) = \int -y dy = -\frac{y^2}{2}$$

$$-\frac{x^2}{2} + \frac{C(y)}{dy} = -x-y$$

$$\Rightarrow f = -\frac{x^2}{2} - xy - \frac{y^2}{2}$$

$$\frac{C(y)}{dy} = \frac{x^2}{2} - x - y \quad / \int dy$$

$$C(y) = -\frac{y^2}{2} + C$$

$$f = -\frac{x^2}{2} - \frac{y^2}{2}$$

$$f(A) - f(B) = \left( -\frac{1}{2} - \frac{1}{2} \right) - \left( -\frac{9}{2} - 2 \right)$$

$$= \frac{11}{2} = \underline{\underline{5,5}}$$

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$$5) y'''(t) - 2y''(t) = e^t$$

$$y(0) = 1$$

$$y'(0) = 2$$

$$y''(0) = 1$$

$$\Delta^3 Y(\Delta) - \Delta^2 y(0) - \Delta y'(0) - y''(0) - 2(\Delta^2 Y(\Delta) - \Delta y(0) - y'(0)) = \frac{1}{s-1}$$

$$\Delta^3 Y(\Delta) - \Delta^2 - 2\Delta - 1 - 2\Delta^2 Y(\Delta) + 2\Delta + 2 = \frac{1}{\Delta-1}$$

$$Y(\Delta) (\Delta^3 - 2\Delta^2) = \frac{1}{\Delta-1} + \Delta^2 - 1$$

OVA GREŠKA SE  
PROVLAČI DALJE,  
ALI POSTUPAK JE  
ISPRAVAN DO KRAJA.

$$Y(\Delta) = \frac{1 + \Delta^3 - \Delta^2 - \Delta + 1}{(\Delta^3 - 2\Delta^2)(\Delta-1)}$$

$$Y(\Delta) = \frac{\Delta^3 - \Delta^2 - \Delta + 2}{\Delta^2(\Delta-2)(\Delta-1)}$$

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$$\frac{\Delta^3 - \Delta^2 - \Delta + 2}{\Delta^2(\Delta-2)(\Delta-1)} = \frac{A}{\Delta^2} + \frac{B}{\Delta} + \frac{C}{\Delta-2} + \frac{D}{\Delta-1}$$

$$\Delta^3 - \Delta^2 - \Delta + 2 = A(\Delta^2 - \Delta - 2\Delta + 2) + B(\Delta^3 - \Delta^2 - 2\Delta^2 + 2\Delta) + C(\Delta^3 - \Delta^2) + D(\Delta^3 - 2\Delta^2)$$

$$\Delta^3 - \Delta^2 - \Delta + 2 = A(\Delta^2 - 3\Delta + 2) + B(\Delta^3 - 3\Delta^2 + 2\Delta) + C(\Delta^3 - \Delta^2) + D(\Delta^3 - 2\Delta^2)$$

$$\Delta^3 \dots 1 = B + C + D$$

$$\Delta^2 \dots 1 = A - 3B - C - 2D$$

$$\Delta \dots 1 = -3A + 2B \implies 1 = -3 + 2B \implies 4 = 2B$$

$$C \dots 2 = 2A \implies \boxed{A = 1} \quad \boxed{B = 2}$$

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NASTAVAK (B)

$$1 = B + C + D$$

$$1 = A - 3B - C - 2D$$

uvrška sem  
o obič  
A i B

$$1 = 2 + C + D$$

$$1 = 1 - 6 - C - 2D$$

$$-1 = C + \frac{5}{2} \Rightarrow \boxed{C = -\frac{7}{2}}$$

$$\begin{array}{l} -1 = C + D \\ -4 = -C - 2D \end{array} \Bigg] +$$

$$-5 = -2D \Rightarrow \boxed{D = \frac{5}{2}}$$

$$y(t) = 1 \cdot \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}}_t + 2 \cdot \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}}_1 - \frac{7}{2} \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}}_{e^{2t}} + \frac{5}{2} \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}}_{e^t}$$

$$y(t) = t + 2 - \frac{7}{2}e^{2t} + \frac{5}{2}e^t$$