OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO .

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. xoo

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1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - 2\,y''(t) = \cos(2t)\,, \quad y(0) = y''(0) = 0,\ y'(0) = -1.$$

2. Zadana je kružnica u prostoru: $K = \{(\cos t,\, 2 + \sin t, -\cos t) \in \mathbb{R}^3 \mid t \in [0, 2\pi]\}$ i vektorska funkcija $\mathbf{w}(x, y, z) = (0,\, 3z - 3x,\, 3x - 3y)$. Izračunati $\oint_{\widehat{K}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. 10

3. Izračunati 20

$$\int_{(0,1,0)}^{(1,0,1)} (x^2\,\mathrm{d}x + y\,\mathrm{d}y + 2z\,\mathrm{d}z)$$

4. Izračunati volumen područja između plašta stošca $x^2 + y^2 = z^2$ i plašta paraboloida $x^2 + y^2 = 3z$. 20

5. Odrediti duljinu 10 navoja zavojnice s parametrizacijom $x = \frac{1}{2}\cos(2t)$, $y = \frac{1}{2}\sin(2t)$ i $z = \frac{t}{10}$. ($t \in [0, 10\pi]$). 20

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$y'''(t) \Rightarrow s^3 Y(s) - s^2 \cdot 0 - s \cdot (-1) - 0 \Rightarrow s^3 Y(s) + s$

$y''(t) \Rightarrow s^2 Y(s) - s \cdot 0 - (-1) = s^2 Y(s) + 1$

$\cos(2t) \Rightarrow \dfrac{s}{s^2+4}$

$s^3 Y(s) + s - 2\left(s^2 Y(s) + 1\right) = \dfrac{s}{s^2+4}$

$s^3 Y(s) + s - 2s^2 Y(s) - 2 = \dfrac{s}{s^2+4}$

$Y(s)\left(s^3 - 2s^2\right) = \dfrac{s}{s^2+4} - s + 2$ ✓

OSIM OVE GREŠKE POSTUPAK JE DOBAR DO KRAJA.

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$Y(s) = \dfrac{s - s(s^2+4) + 2(s^2+4)}{s^2+4} \cdot \dfrac{1}{(s^3-2s^2)}$

$Y(s) = \dfrac{s - s^3 + 4s + 2s^2 + 8}{s^5 - 2s^4 + 4s^3 - 8s^2} = \dfrac{-s^3 + 2s^2 \overset{-3s}{+4s} + 8}{s^2(s^2+4)(s-2)}$ ✗

$\dfrac{-s^3 + 2s^2 + 4s + 8}{s^2(s^2+4)(s-2)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{Cs+D}{s^2+4} + \dfrac{E}{s-2} \Big/ \cdot s^2(s^2+4)(s-2)$

① NASTAVAK:

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$$-s^3+2s^2+4s+8 = As(s^2+4)(s-2) + B(s^2+4)(s-2) + (C\cdot s+D)s^2(s-2) + Es^2(s^2+4)$$

$$-s^3+2s^2+4s+8 = As^4 - \boxed{2As^3} + 4As^2 - 8As + \boxed{Bs^3} - 2Bs^2 + 4Bs - 8B + Cs^4 - \boxed{2Cs^3} + \boxed{Ds^3} - 2Ds^2 + Es^4 + 4Es^2$$

$$-s^3+2s^2+4s+8 = s^4(A+C+E) - s^3(2A-B+2C-D) + s^2(4A-2B-2D+4E) - s(8A-4B) - 8B$$

$A+C+E=0$

$2A-B+2C-D=-1$

$4A-2B-2D+4E=2 \Rightarrow 2A-B-D+2E=1$

$8A-4B=4 \Rightarrow 2A-B=1$

$\boxed{B=-1}$ $\boxed{A=0}$ $\boxed{E=\frac{1}{5}}$ $\boxed{C=-\frac{1}{5}}$ $\boxed{D=\frac{1}{10}}$

$$Y(s) = \frac{-1}{s^2} - \frac{\frac{1}{5}s-\frac{1}{10}}{s^2+4} + \frac{\frac{1}{5}}{s-2} = \frac{-1}{s^2} - \frac{2s-1}{10(s^2+4)} + \frac{1}{5(s-2)}$$

$$y(t) = -t - \frac{1}{5}\cos(2t) + \frac{1}{10}\cdot\frac{1}{2}\sin(2t) + \frac{1}{5}e^{2t} = -t - \frac{1}{5}\cos(2t) + \frac{1}{20}\sin(2t) + \frac{1}{5}e^{2t}$$

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③

$$\int_{(0,1,0)}^{(1,0,1)} (x^2dx + ydy + 2zdz)$$

$$w = \begin{bmatrix} x^2 \\ y \\ 2z \end{bmatrix} = -\operatorname{grad} f$$

$d_x f = -x^2 \;/\int dx$

$f = -\frac{x^3}{3} + C(y,z)$

$d_y f = -y$

$d_y\left(\frac{-x^3}{3} + C(y,z)\right) = -y$

$\frac{dC(y,z)}{dy} = -y \;/\int dy$

$C(y) = -\frac{y^2}{2} + C(z)$

$d_z f = -2z$

$d_z\left(\frac{-x^3}{3} - \frac{y^2}{2} + C(z)\right) = -2z \;/\int dz \Rightarrow$

$\Rightarrow \frac{dC(z)}{dz} = -2z \;/\int dz$

$C(z) = -z^2$

$f = \frac{-x^3}{3} - \frac{y^2}{2} - z^2$ ✓

$f(0,1,0) - f(1,0,1) =$

$= \left(0 - \frac{1}{2} - 0\right) - \left(\frac{-1}{3} - 0 - 1\right) =$

$= -\frac{1}{2} + \frac{4}{3} = \frac{-3+8}{6} = \frac{5}{6}$ ✓

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(5)

$$x=\frac{1}{2}\cos(2t),\ y=\frac{1}{2}\sin(2t),\ z=\frac{t}{10},\ t\in[0,10\pi]$$

$$L=\int_0^{10\pi}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt$$

$$\frac{dx}{dt}=-\frac{1}{2}\sin(2t)\cdot 2=-\sin(2t)$$

$$\frac{dy}{dt}=\frac{1}{2}\cos(2t)\cdot 2=\cos(2t)$$

$$\frac{dz}{dt}=\frac{1}{10}$$

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$$L=\int_0^{10\pi}\sqrt{\underbrace{\sin^2(2t)+\cos^2(2t)}_{=1}+\frac{1}{100}}\,dt=\int_0^{10\pi}\sqrt{\frac{101}{100}}\,dt=\frac{\sqrt{101}}{\cancel{10}}\cdot\cancel{10}\pi=\pi\cdot\sqrt{101}$$ ✓

(4)

$$x^2+y^2=z^2$$

$$x^2+y^2=3z \Rightarrow r^2=3z \Rightarrow r=\pm\sqrt{3z}$$

$$r^2=z^2 \qquad\qquad z=\frac{r^2}{3}$$

$$r=\pm z$$

$$z=\pm r$$

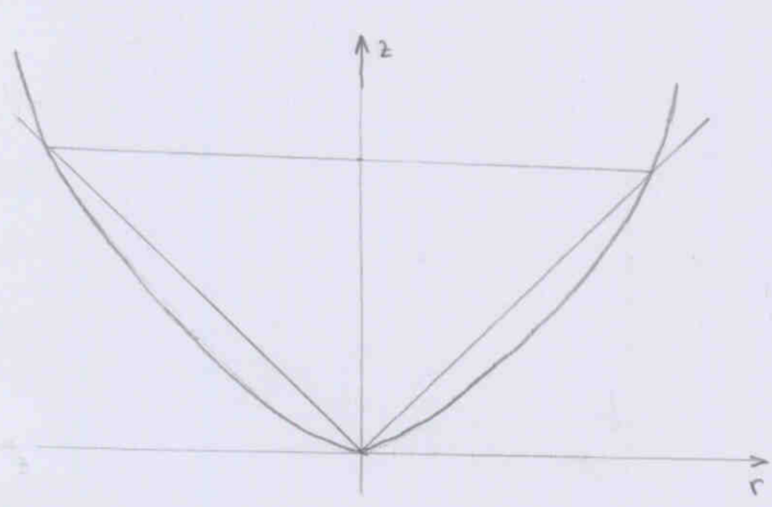
SJECIŠTE:

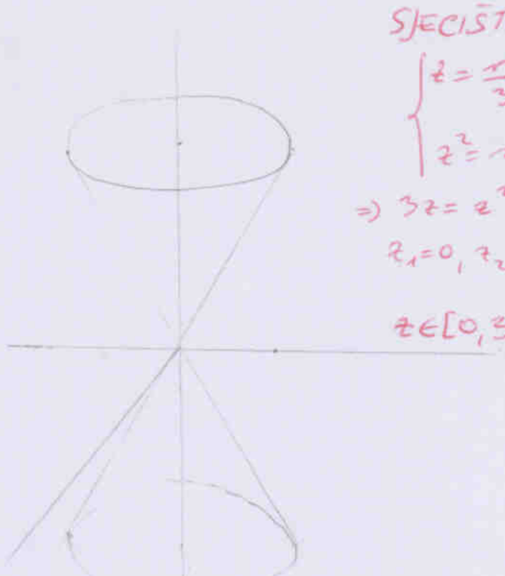
$$\begin{cases} z=\frac{r^2}{3} \\ z^2=r^2 \end{cases}$$

$$\Rightarrow 3z=z^2$$

$$z_1=0,\ z_2=3$$

$$z\in[0,3]$$





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④ NASTAVAK:

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$$V = \int_0^{2\pi} d\varphi \int_0^3 dz \int_z^{\sqrt{3z}} r\,dr = 2\pi \int_0^3 \left(\frac{3z}{2} - \frac{z^2}{2}\right) dz = 2\pi \left[\frac{3z^2}{4} - \frac{z^3}{6}\right]_0^3 =$$

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$$= 2\pi\left(\frac{27}{4} - \frac{27}{6}\right) = 2\pi \cdot \frac{162-108}{24} = \frac{54}{12}\cdot\pi = \frac{27}{6}\pi = \frac{9\pi}{2}$$

②

$$w = \begin{bmatrix} 0 \\ 3z-3x \\ 3x-3y \end{bmatrix} \Rightarrow \mathrm{rot}\,w = \nabla \times w = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \times \begin{bmatrix} 0 \\ 3z-3x \\ 3x-3y \end{bmatrix} = \begin{bmatrix} -3-3 \\ 0-3 \\ -3-0 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ -3 \end{bmatrix}$$

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$$\vec{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

KRUŽNICA K JE NASTALA OD KRUŽNICE U RAVNINI $\overline{K} = \{(\cos t, \sin t, 0) : t \in [0, 2\pi]\}$ NAKON PRESLIKAVANJA $T(x,y,z) = \begin{bmatrix} x \\ 2+y \\ -x \end{bmatrix}$

STOGA JE

$$\vec{n} = \frac{\partial T}{\partial x} \times \frac{\partial T}{\partial y} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} +1 \\ 0 \\ 1 \end{bmatrix}$$

ZATO

$$(\mathrm{rot}\,w \,|\, \vec{n}) = -3$$

$$\iint \mathrm{rot}\,w\,dS = \iint_K \begin{bmatrix} 0 \\ -3 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} dx\,dy = \iint_K -3\,dx\,dy$$

$$\int_0^{2\pi} d\varphi \int_0^1 -3r\,dr = 2\pi \cdot (-3) \cdot \frac{1}{2} = -3\pi$$

OVAJ INTEGRAL $\int_{\vec{K}} (w \,|\, dr)$ MOŽE SE JEDNOSTAVNO RIJEŠITI KAO KRIVULJNI INTEGRAL DRUGE VRSTE BEZ UPOTREBE STOKESOVE FORMULE.

$$\Rightarrow \oint_{\vec{K}} (w \,|\, dr) = \iint_{\overline{K}} (\mathrm{rot}\,w \,|\, n)\,dx\,dy$$

$$= \int_0^{2\pi}\int_0^1 -3r\,dr = -3\pi$$