

IME I PREZIME: Cvor SilvijanBROJ INDEKSA: 17-2-0066-2010

xoxo

MATEMATIKA 1: KOLOKVIJ 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uredaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uredaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno medusobno posudivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. **ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOJ STRANICI I PREDLOŠCIMA ZA PISANJE KOJE MOŽETE DOBITI OD NASTAVNIKA.**

Broj ↓
bodova

(50)

1. Neka su z_1 i z_2 rješenja jednadžbe $z^2 - 4z + 5 = 0$. Izračunati vrijednost izraza $\left(\frac{1-i}{z_1} - \overline{z_2} \right)$.

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2. Odrediti inverz i determinantu matrice:

$$A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Izračunati matrični umnožak $A A^{-1}$.

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3. Za funkciju $f(x) = e^x$ nacrtati graf i navesti: domenu, kodomenu, periodičnost, (ne)parnost, ograničenost, rast ili pad; da li je injekcija, surjekcija ili bijekcija; da li postoji inverz i ako postoji koja je to funkcija.

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4. Gaussovom metodom riješiti sustav:

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= -4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

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5. Pokazati da li točke $A(2, -1, -2)$, $B(2, 2, 1)$, $C(2, 3, -1)$ i $D(5, 0, -6)$ pripadaju istoj ravnini.

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VĀŽNO JE POZNAVATI ELEMENTARNE FUNKCIJE!

UVJET ZA OVU OCJENU (ROK 15.01.2011.)

PODΝIJETI ASISTENTU U TERMINU KONZULTACIJA
ISPUNJENE MATERIJALE SA SEMINARA 3, GDJE
ĆE UZ SVAKU FUNKCIJU SOŠ BITI NAZNAČENO
SVE ŠTO SE TRAŽI U ZADATKU 3 OVOG
KOLOKVIJA.

→

- ⑤
 A(2, -1, -2)
 B(2, 2, 1)
 C(2, 3, -1)
 D(5, 0, -6)

$$\vec{AB} = \vec{v}_1 = \begin{bmatrix} 2 & -2 \\ 2 & -(-1) \\ 1 & -(-2) \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$$

$$\vec{AC} = \vec{v}_2 = \begin{bmatrix} 2 & -2 \\ 3 & -(-1) \\ -1 & -(-2) \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$$

$$\vec{AD} = \vec{v}_3 = \begin{bmatrix} 5 & -2 \\ 2 & -(-1) \\ -6 & -(-2) \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$$

$$(\vec{v}_1 \times \vec{v}_2) \cdot \vec{v}_3$$

$$(\vec{AB} \times \vec{AC}) \cdot \vec{AB}$$

$$\left(\begin{bmatrix} 0 & 0 \\ 3 & 4 \\ 3 & 1 \\ 0 & 0 \\ 3 & 4 \end{bmatrix} \right) \cdot \begin{bmatrix} 5 \\ 1 \\ -4 \end{bmatrix}$$

$$\left(\begin{bmatrix} 3 \cdot 1 - 4 \cdot 3 \\ 3 \cdot 0 - 1 \cdot 0 \\ 0 \cdot 1 - 0 \cdot 3 \end{bmatrix} \right) \cdot \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 3 - 12 \\ 0 - 0 \\ 0 - 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} -9 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} -27 \\ 0 \\ 0 \end{bmatrix}$$

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⑥ $\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= -4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$

$$\left(\begin{array}{cccc|c} 2 & -3 & -1 & 2 & 3 \\ 4 & -4 & -1 & 4 & 11 \\ 2 & -5 & -2 & 2 & -1 \\ 0 & 2 & 1 & 0 & 4 \end{array} \right) \xrightarrow{\frac{1}{2}} \left(\begin{array}{cccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & \frac{3}{2} & \frac{3}{2} \\ 0 & 2 & 1 & 0 & 4 \\ 0 & -2 & -1 & 0 & -4 \\ 0 & 1 & 0 & 0 & 2 \end{array} \right) \xrightarrow{\text{R2} - 2\text{R1}}$$

$= -27 \rightarrow$ Tođe ve pripadaju istoj ravnini ✓

$$\left(\begin{array}{cccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & \frac{3}{2} & \frac{3}{2} \\ 0 & 2 & 1 & 0 & 4 \\ 0 & -2 & -1 & 0 & -4 \\ 0 & 1 & 0 & 0 & 2 \end{array} \right) \xrightarrow{\text{R3} + \text{R2}}$$

Prema tablici, rezultujući rješenje je:

TREBAJU BITI RAZLICITE PARAMETRI

parametri su razliciti

$$x = \frac{119}{4} - \frac{1}{4}n - \frac{1}{4} = \frac{119 - n - 1}{4} = \frac{118 - n}{4}$$

$$y = \frac{12}{4} - \frac{1}{4}n = \frac{12 - n}{4}$$

$$z = \frac{-7}{4} - \frac{1}{4}n = \frac{-7 - n}{4}$$

$$w = \frac{0}{4} - \frac{1}{4}n = \frac{0 - n}{4}$$

$$v = \frac{0}{4} - \frac{1}{4}n = \frac{0 - n}{4}$$

$$\left(\begin{array}{cccc|c} 2 & -3 & -1 & 2 & 3 \\ 4 & -4 & -1 & 4 & 11 \\ 2 & -5 & -2 & 2 & -1 \\ 0 & 2 & 1 & 0 & 4 \end{array} \right) \xrightarrow{\text{R2} - 2\text{R1}, \text{R3} - \text{R1}}$$

$$\left(\begin{array}{cccc|c} 2 & -3 & -1 & 2 & 3 \\ 0 & 2 & 1 & 0 & 4 \\ 0 & -3 & -3 & 0 & -5 \\ 0 & 1 & 0 & 0 & 2 \end{array} \right) \xrightarrow{\text{R3} + 3\text{R2}}$$

$$\left(\begin{array}{cccc|c} 2 & -3 & -1 & 2 & 3 \\ 0 & 2 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 \end{array} \right) \xrightarrow{\text{R3} \leftrightarrow \text{R4}}$$

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$$\left(\begin{array}{cccc|c} 2 & -3 & -1 & 2 & 3 \\ 0 & 2 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 \end{array} \right) \xrightarrow{\text{R4} \leftrightarrow \text{R3}}$$

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$$\begin{aligned}
 ② A &= \begin{vmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{vmatrix} = -1 \cdot \begin{vmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} \\
 &= -1 \cdot \left(1 \cdot \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} \right) \\
 &= -1 \cdot \left(1 \cdot (-1 \cdot 0 - 1 \cdot 1) \right) \\
 &= -1 \cdot (1 \cdot (-1)) \\
 &= -1 \cdot (1 \cdot (-1)) \\
 &= -1 \cdot (-1)
 \end{aligned}$$

D $\boxed{A^{-1}}$ regularna ✓

$$\begin{array}{c}
 \left[\begin{array}{cccc|cccccc} 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\text{R1}} \left[\begin{array}{cccc|cccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\text{R2}} \left[\begin{array}{cccc|cccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \\
 \text{A} \quad \text{I} \quad \text{II} \quad \text{III} \quad \text{IV}
 \end{array}$$

$$\xrightarrow{\text{R3}} \left[\begin{array}{cccc|cccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\text{R1} \leftrightarrow \text{R4}} \left[\begin{array}{cccc|cccccc} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{R1} \rightarrow \text{R1} + \text{R2}} \left[\begin{array}{cccc|cccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\boxed{A \cdot A^{-1} = I} \\
 \left[\begin{array}{cccc} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{array} \right] \cdot \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right] = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad \boxed{I}$$

$$\begin{aligned}
 A_1 &= 0+0-0+0=0 \\
 A_2 &= 0+0-0+0=0 \\
 A_3 &= 0+0-1+1=0 \\
 A_4 &= 0+0-1+1=0
 \end{aligned}$$

$$\begin{aligned}
 A_{11} &= 0+0+0+0=0 \\
 A_{12} &= 0+0+0+0=0 \\
 A_{13} &= 0+0+0+0=0 \\
 A_{14} &= 0+0+0+0=0
 \end{aligned}$$

$$\boxed{A^{-1} \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right]}$$

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BROJ INDEKSA: 17-2-0066-2010

$$\textcircled{1} \quad z^2 - 4z + 5 = 0$$

$$a = 1$$

$$b = -4$$

$$c = 5$$

$$z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$

$$= \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2} \rightarrow \sqrt{-4} = \sqrt{4 \cdot (-1)} = \pm 2i$$

$$z_{1,2} = \frac{4 \pm 2i}{2}$$

$$z_1 = \frac{4+2i}{2} \quad z_2 = \frac{4-2i}{2}$$

$$\boxed{z_1 = 2+i}$$

$$\boxed{z_2 = 2-i}$$



$$\begin{aligned}
 \text{a) } & \left(\overline{\frac{1-i}{z_1} - \frac{1}{z_2}} \right) = \left(\overline{\frac{1-i}{2+i} - \frac{1}{2-i}} \right) = \left(\overline{\frac{1-i}{2+i} - \frac{2+i}{1}} \right) \quad \checkmark \\
 & = \left(\frac{1-i}{2+i} + \frac{2+i}{1} \right) = \frac{(1-i)(2+i) + (2+i)}{2+i} \\
 & \quad \swarrow \qquad \searrow \qquad \downarrow \\
 & = \frac{1-i+2i+i^2+4i+2i+i^2}{2+i} \\
 & = \frac{1-i+4+4i+(-1)}{2+i} \\
 & = \frac{4+3i-1}{2+i} \\
 & = \frac{4+3i}{2+i} \cdot \frac{2-i}{2-i} = \frac{8-4i+6i-3i^2}{4-2i+2i-i^2} \\
 & = \frac{8+2i-3(-i)}{4-(i^2)} = \frac{8+2i+3}{5} = \frac{11+2i}{5} \\
 & = \boxed{\frac{11}{5} + \frac{2}{5}i}
 \end{aligned}$$

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MATEMATIKA 1: KOLOVIJ 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uredaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno medusobno posudivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOJ STRANICI I PREDLOŠCIMA ZA PISANJE KOJE MOŽETE DOBITI OD NASTAVNIKA.

1. Neka su z_1 i z_2 rješenja jednadžbe $z^2 - 4z + 5 = 0$. Izračunati vrijednost izraza $\left(\frac{1-i}{z_1} - \overline{z_2} \right)$.

2. Odrediti inverz i determinantu matrice:

$$A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Izračunati matrični umnožak $A A^{-1}$.

3. Za funkciju $f(x) = e^x$ nacrtati graf i navesti: domenu, kodomenu, periodičnost, (ne)parnost, ograničenost, rast ili pad; da li je injekcija, surjekcija ili bijekcija; da li postoji inverz i ako postoji koja je to funkcija.

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5. Pokazati da li točke $A(2, -1, -2)$, $B(2, 2, 1)$, $C(2, 3, -1)$ i $D(5, 0, -6)$ pripadaju istoj ravnini.

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Broj ↓ bodova

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VĀŽNO JE POZNAVATI
ELEMENTARNE FUNKCIJE.

BEZ TOGA ZNANJA

NEREALNO JE NADATI
SE PROLASKU ISPITA,

JOŠ HALO BOJE NAUČITE RIJEŠAVANJE SUSTAVA
I TRAŽENJE INVERZA.

MNOGO VJEŽBAJTE RIJEŠAVANJE ZADATAKA,
NPR. PREKO MOODLE PROVJERA ZNANJA.

IME I PREZIME: GREGOR

HAMARIC

BROJ INDEKSA: 57650

X

1. $z^2 - 4z + 5 = 0$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$z = \frac{4 \pm \sqrt{-4}}{2}$$

$$z = \frac{4 \pm 2i}{2}$$

$$z = 2 \pm i$$

$$z_1 = 2+i$$

$$z_2 = 2-i$$

✓ ✓

$$\begin{aligned} \overline{\left(\frac{1-i}{z_1} - \frac{2}{z_2} \right)} &= \left(\overline{\frac{1-i}{2+i}} - \overline{\frac{2}{2-i}} \right) = \boxed{\left(\overline{\frac{1-i}{2+i}} - \overline{(2+i)} \right) / \overline{(2+i)}} \\ &= \overline{(1-i+4+4i-1)} = \overline{4+3i} = 4-3i \quad (2+i)(2-i) = 4+2i+2i-1 \end{aligned}$$

KADA NEKI IZRAS POMNOŽITE

SA BROJEM RAZLIČITIM OD 1

NPR. SA $(2+i)$ TADA SE
VRIJEDNOST IZRASA MIJENJA.VIDI CVAR, BEKAVAC

2.

$$det A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot (-1)^5 = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \cdot (-1) \cdot (-1)^4 = -1 \cdot (-1) = 1 \quad \checkmark$$

S

INVERZ

$$A_{11} = 0$$

$$A_{12} = 0$$

$$A_{13} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot (-1) = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \cdot (-1)^4 \cdot -1 = 0 \cdot -1 = 0$$

$$A_{14} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot 1 = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \cdot (-1)^4 = -2$$

$$A_{21} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot 0 = 0$$

$$A_{22} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot -1 = 0$$

$$A_{23} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$A_{24} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ \hline 1 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{31} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -1 \cdot \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} = 0 \quad A_{33} = 0$$

$$A_{32} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot 1 = -1 \quad A_{34} = 0$$

$$A_{41} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = -1 \quad A_{42} = 0 \quad A_{43} = 0 \quad A_{44} = 0$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix} \rightarrow \text{INVERZ } \cancel{\textcircled{O}}$$

$$AA^{-1} = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ \hline 1 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \neq I.$$

AKO JE $B = A^{-1}$ TADA $AB = I$, $BA = I$
 UVIJEK VRIJEDI $AA^{-1} = I$. DAKLE, VAS
 IZRACUNATI A^{-1} NIJE INVERZ OD A.

IME I PREZIME: GREGOR HAHARIC

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$$5. \quad A(2, -1, -2), B(2, 2, 1), C(2, 3, -1), D(5, 0, 6)$$

$$\vec{a} = \vec{AB} = (2-2)x + (2+1)y + (-2-1)z \quad \text{X} \quad \underline{\text{VIDI CVAR}}$$

$$\vec{b} = \vec{AC} = (2-2)x + (3+1)y + (-1+2)z$$

$$b = 0x + 4y + 1z$$

$$\vec{c} = \vec{AD} = (5-2)x + (0+1)y + (-6+2)z$$

$$c = 3x + 1y - 4z$$

$$V = \begin{vmatrix} 2 & 3 & -3 \\ 0 & 4 & 1 \\ 3 & 2 & -4 \end{vmatrix} = (0+36) + (9-0) + (0-0) = 45$$

TOČKE NE TRIPADAJU ISTOJ RAVNINI.

$$4. \quad \left| \begin{array}{cccc|cc} 1 & 1 & 1 & 1 \\ 2 & -3 & -1 & 2 & 3 & 1 & 4 \\ 4 & -4 & -1 & 4 & 11 & 1 & -4 \\ 2 & -5 & -2 & 2 & -1 & 1 & 9 \\ 0 & 2 & 1 & 0 & 4 & | & -5 \end{array} \right| = \left| \begin{array}{cccc|cc} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 2 & 1 & 0 & 5 & 1 & -2 \\ 0 & 2 & -1 & 0 & -4 & 1 & 5 \\ 0 & 2 & 1 & 0 & 4 & | & -5 \end{array} \right| \xrightarrow[-2]{\text{R}_2} \left| \begin{array}{cccc|cc} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ 0 & 0 & -2 & 0 & -9 & 1 & -4 \\ 0 & 0 & 0 & 0 & -1 & | & 17 \end{array} \right| \quad \begin{array}{cccc|cc} x & y & z & w & v \\ -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ \frac{1}{2} & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ -2 & 0 & -9 & 1 & -4 & \\ 0 & 0 & -1 & | & 17 & \end{array}$$

$$V = \underline{17}$$

$$y + \frac{1}{2}z + \frac{5}{2}v = -6$$

$$2z + 9v = 7$$

$$y + \frac{73}{2} + \frac{85}{2} = -6$$

$$2z + 153 = 7$$

$$y + 79 = -6$$

$$2z = 7 - 153$$

$$y = \underline{-85}$$

$$2z = -146$$

$$y = \underline{-85}$$

$$z = \underline{\underline{73}}$$

$$x + \frac{255}{2} - \frac{73}{2} + \frac{51}{2} + w = 2$$

$$x + w = 2 - \underline{\underline{233}}$$

$$x + w = -\frac{229}{2}$$

$$\begin{array}{cccc|cc} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ 0 & 0 & 1 & 0 & \frac{7}{2} & 1 & -4 \\ 0 & 0 & 0 & 0 & -1 & | & 17 \end{array}$$

$$\begin{array}{cccc|cc} x & y & z & w & v \\ -\frac{3}{2} & -\frac{1}{2} & 3 & 2 & 1 & 2 \\ 1 & \frac{1}{2} & \frac{1}{2} & \frac{5}{2} & 1 & -6 \\ 0 & 0 & 1 & \frac{7}{2} & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & | & 17 \end{array}$$

PARAMETARSKI ZAPIS?
PROVJERA?

IME I PREZIME: MATE BALŠAK

BROJ INDEKSA: 57715

MATEMATIKA 1: KOLOKVIJ 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uredaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posudivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. **ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOJ STRANICI I PREDLOŠCIMA ZA PISANJE KOJE MOŽETE DOBITI OD NASTAVNIKA.**

xoxo

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Broj ↓
bodova1. Neka su z_1 i z_2 rješenja jednadžbe $z^2 - 4z + 5 = 0$. Izračunati vrijednost izraza $\overline{\left(\frac{1-i}{z_1} - \bar{z}_2\right)}$.

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2. Odrediti inverz i determinantu matrice:

$$A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

15

Izračunati matrični umnožak $A A^{-1}$.

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3. Za funkciju $f(x) = e^x$ nacrtati graf i navesti: domenu, kodomenu, periodičnost, (ne)parnost, ograničenost, rast ili pad; da li je injekcija, surjekcija ili bijekcija; da li postoji inverz i ako postoji koja je to funkcija.

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4. Gaussovom metodom rješiti sustav:

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= -4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

5. Pokazati da li točke $A(2, -1, -2)$, $B(2, 2, 1)$, $C(2, 3, -1)$ i $D(5, 0, -6)$ pripadaju istoj ravnini.

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2.)

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 1 & 1 \\ 0 & -1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-1 \cdot R_1} \left[\begin{array}{cccc|c} 1 & -1 & -2 & 1 & 1 \\ 0 & -1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 1 & 1 \\ 0 & -1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{cross out}} \left[\begin{array}{cccc|c} 1 & -1 & -2 & 1 & 1 \\ 0 & -1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$-1 \cdot (-1 - (1)) = -1 \cdot -2 = 2 \quad \times$$

$$\left[\begin{array}{cccc|c} 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right] \xrightarrow{R_1 \sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_4} \left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

3. $\overline{AB} \quad \overline{AD} \quad \overline{BC} = \frac{1}{2}$ -32 4 -4 -75 + 6

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$\begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 2 & 1 & 0 & 5 & -12 \\ 0 & -2 & -1 & -6 & -23 & 77 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{bmatrix} : \frac{1}{2}$

$\begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -6 \\ 0 & -2 & -1 & -6 & -23 & 77 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{bmatrix} \xrightarrow{\cdot 2} \begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 2 & 1 & 0 & 5 & -12 \\ 0 & -2 & -1 & -6 & -23 & 77 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{bmatrix} \xrightarrow{\frac{20}{4} + \frac{4}{4}} \begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 2 & 1 & 0 & 5 & -12 \\ 0 & -2 & -1 & -6 & -23 & 77 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{bmatrix} \xrightarrow{\frac{10}{4} + \frac{4}{4}} \begin{bmatrix} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 2 & 1 & 0 & 5 & -12 \\ 0 & -2 & -1 & -6 & -23 & 77 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{bmatrix} \xrightarrow{\frac{-78}{2} + \frac{5}{2} = -\frac{73}{2}}$

$\begin{bmatrix} 1 & 0 & -\frac{5}{4} & 1 & -\frac{9}{4} & 1 & 77 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -6 \\ 0 & 0 & 1 & 6 & -15 & -71 \\ 0 & 0 & 2 & 1 & 4 & -5 \end{bmatrix} \xrightarrow{\cdot (-1)} \begin{bmatrix} 1 & 0 & 0 & -\frac{34}{4} & \frac{81}{4} & 1 & \frac{79}{4} \\ 0 & 1 & 0 & -3 & -\frac{13}{2} & 1 & -\frac{7}{2} \\ 0 & 0 & 1 & 6 & -78 & -5 \\ 0 & 0 & 0 & -11 & -32 & 5 \end{bmatrix} \xrightarrow{\frac{34}{4} + \frac{90}{4}} \begin{bmatrix} 1 & 0 & 0 & -\frac{34}{4} & \frac{81}{4} & 1 & \frac{79}{4} \\ 0 & 1 & 0 & -3 & -\frac{13}{2} & 1 & -\frac{7}{2} \\ 0 & 0 & 1 & 6 & -78 & -5 \\ 0 & 0 & 0 & -11 & -32 & 5 \end{bmatrix} \xrightarrow{\frac{81}{4} - \frac{25}{4} + \frac{44}{4}} \begin{bmatrix} 1 & 0 & 0 & -\frac{34}{4} & \frac{81}{4} & 1 & \frac{79}{4} \\ 0 & 1 & 0 & -3 & -\frac{13}{2} & 1 & -\frac{7}{2} \\ 0 & 0 & 1 & 6 & -78 & -5 \\ 0 & 0 & 0 & -11 & -32 & 5 \end{bmatrix}$

5. $\overline{AB} (0, 3, 3)$

$\begin{bmatrix} 0 & 3 & 3 & 1 & 0 & 3 \\ 3 & 1 & -4 & 1 & 3 & 1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} = -9 \quad \checkmark$

$\overline{AD} (3, 1, -4)$

$\overline{BC} (0, 1, 2)$

Nepotpunost u trij ravnini

~~$\begin{bmatrix} 0 & 3 & 3 & 1 & 0 & 3 \\ 0 & 4 & 1 & 1 & 0 & 4 \\ 3 & -2 & 7 & 1 & 3 & -2 \end{bmatrix}$~~

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$76 + 9 = 27$

IME I PREZIME: MATE BALJAK

BROJ INDEKSA: 57715

(2)

$$\left[\begin{array}{cc|cc} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array} \right] \xrightarrow{e_1} \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{e_1} \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{e_1} \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{array} \right] \xrightarrow{e_1} \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{e_1} \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{e_1}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad \checkmark$$

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$$\left[\begin{array}{cc|cc} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right] \times \left[\begin{array}{cc|cc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array} \right] = \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad \checkmark$$

5.)

$$\left[\begin{array}{ccccc} 2 & -3 & -1 & 2 & 3 \\ 4 & -4 & -1 & 4 & 1 \\ 2 & -5 & -2 & 2 & -7 \\ 0 & 0 & 0 & 0 & 5 \end{array} \right] \cdot \frac{1}{2} \left[\begin{array}{ccccc} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} \\ 4 & -4 & -1 & 4 & 1 \\ 2 & -5 & -2 & 2 & -7 \\ 0 & 0 & 2 & 1 & 4 \end{array} \right] \xrightarrow{e_4} \left[\begin{array}{ccccc} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} \\ 4 & -4 & -1 & 4 & 1 \\ 2 & -5 & -2 & 2 & -7 \\ 0 & 0 & 2 & 1 & 4 \end{array} \right] \xrightarrow{e_4} \left[\begin{array}{ccccc} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} \\ 4 & -4 & -1 & 4 & 1 \\ 2 & -5 & -2 & 2 & -7 \\ 0 & 0 & 2 & 1 & 4 \end{array} \right] \xrightarrow{\cdot(-2)} \left[\begin{array}{ccccc} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} \\ 4 & -4 & -1 & 4 & 1 \\ 2 & -5 & -2 & 2 & -7 \\ 0 & 0 & 2 & 1 & 4 \end{array} \right] \xrightarrow{\cdot(-5)}$$

(4)

IME I PREZIME: MATE BALJAK

BROJ INDEKSA: 57115

$$\left[\begin{array}{cccc|c} 1 & -3 & -1 & 2 & 3 & 4 \\ 4 & -4 & -1 & 4 & 11 & -4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \cdot \frac{1}{2} \left[\begin{array}{cccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 4 & -4 & -1 & 4 & 11 & -4 \\ 2 & -5 & -2 & 2 & -1 & 9 \\ 0 & 2 & 1 & 0 & 4 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\substack{-4 \\ -2}} \left[\begin{array}{cccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -6 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\substack{\frac{3}{2} \\ 2 \\ (-2)}} \left[\begin{array}{cccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -6 \\ 0 & 0 & 0 & 1 & -4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\frac{3}{7} - \frac{3}{4}$$

$$\frac{75}{4} + \frac{3}{2} = \frac{21}{4}$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -6 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \cdot \frac{1}{2} \left[\begin{array}{cccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -6 \\ 0 & -2 & -1 & 0 & -4 & 5 \\ 0 & 2 & 1 & 0 & 4 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\substack{\frac{3}{2} \\ 2 \\ (-2)}} \left[\begin{array}{cccc|c} 1 & 0 & \frac{1}{4} & 1 & \frac{21}{4} & -7 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & -6 \\ 0 & 0 & 0 & 1 & -7 & 7 \\ 0 & 0 & 0 & 0 & -1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Cvij: nultom množi rješenja X

VIDI CVAR

IME I PREZIME: Ivana Blavac

BROJ INDEKSA: 17-2-0022-2010

xoxo

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MATEMATIKA 1: KOLOKVIJ 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisači pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uredaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uredaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posudivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOJ STRANICI I PREDLOŠCIMA ZA PISANJE KOJE MOŽETE DOBITI OD NASTAVNIKA.

Broj ↓
bodova

1. Neka su z_1 i z_2 rješenja jednadžbe $z^2 - 4z + 5 = 0$. Izračunati vrijednost izraza $\left(\frac{1-i}{z_1} - \overline{z_2} \right)$.

20

2. Odrediti inverz i determinantu matrice:

$$A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

5

Izračunati matrični umnožak $A A^{-1}$.

20

3. Za funkciju $f(x) = e^x$ načrtati graf i navesti: domenu, kodomenu, periodičnost, (ne)parnost, ograničenost, rast ili pad; da li je injekcija, surjekcija ili bijekcija; da li postoji inverz i ako postoji koja je to funkcija.

20

4. Gaussovom metodom riješiti sustav:

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= -4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

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20

5. Pokazati da li točke $A(2, -1, -2)$, $B(2, 2, 1)$, $C(2, 3, -1)$ i $D(5, 0, -6)$ pripadaju istoj ravnini.

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~~DVA ZADATKA NISTE MTI ZAPOČELI
A KAKO STE SLENE ZADATKE
RIJESILI NA MOODLE PROVJERI?~~

VIDI CVAR

IME I PREZIME: Luka Bekavac

BROJ INDEKSA: 17-2-0022-2010

4.

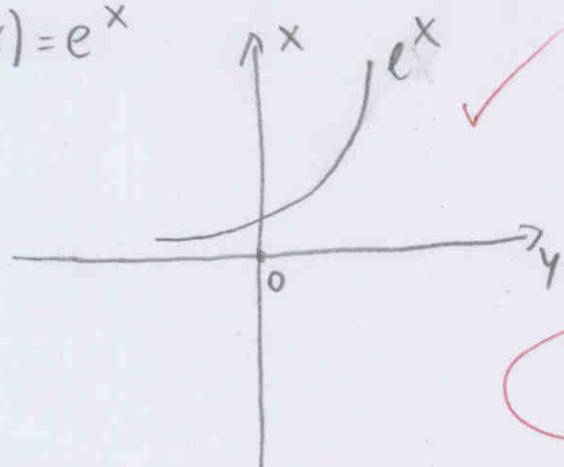
$$A = \begin{bmatrix} 2 & -3 & -1 & 2 & 3 & 1 & 2 & -3 \\ 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 \\ 2 & -5 & 2 & 2 & 1 & 1 & 2 & -5 \\ 0 & 2 & 1 & 0 & 3 & 0 & 2 & 1 \end{bmatrix}$$

SARRUSOV PRAVILO
VRIJEDI SAMO ZA
 3×3 MATRICE

$$\begin{aligned} &= (2 \cdot (-4) \cdot (-2) \cdot 0 + (-3) \cdot (-1) \cdot 2 \cdot 1 + (-1) \cdot 1 \cdot (-1) \cdot 0) + 2 \cdot 11 \cdot 2 \cdot 2 \\ &\quad - (-3) \cdot 1 \cdot (-1) \cdot 0 - 2 \cdot 11 \cdot 2 \cdot 1 - 3 \cdot 1 \cdot (-2) \cdot 2 - (2 \cdot (-1) \cdot -5 \cdot 0) \\ &= -8 \cdot 0 + 3 \cdot 8 + 0 + 88 - 44 + 48 - 82 - 0 - 11 + 4 \\ &= 24 - 44 + 48 + 88 \\ &= 116 \end{aligned}$$

TRAŽILO SE RJEŠENJE
SUSTAVA, NE DETERMINANTA

3) $f(x) = e^x$



Rastuća, bijekcija,
Invanz je lin X , nije periodična,
nije parna ili neparna,
donja meda je 0, GORNJA MEDA?
 $\langle 0, +\infty \rangle \rightarrow \mathbb{R}$?

DOMENA ? INJEKCIJA ?

KODOMENA ? SURJEKCIJA ?

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IME I PREZIME: Luka Bakavac

BROJ INDEKSA: 17-2-0022-2010

1.

$$z^2 - 4z + 5 = 0$$

$$z_{1,2} = \frac{4 \pm \sqrt{16-20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

$$z_1 = 2+i$$

$$z_2 = 2-i$$

$$\left(\frac{1-i}{z_1} - \bar{z}_2 \right) = \left(\frac{1-i}{2+i} - \frac{2+i}{1} \right)$$

$$\left(\frac{1-i}{2+i} \right) = \left(\frac{1-i}{2+i} \cdot \frac{2-i}{2-i} \right) = \left(\frac{2-1+2i-i}{4+1} \right) = \frac{1}{5}(1+3i)$$

$$\frac{1-i}{2-i} = \frac{1+i}{2-i} \cdot \frac{2+i}{2+i} = \frac{2-1+2i+i}{5} = \frac{1}{5}(1+3i)$$

$$\left(\frac{1+i}{2-i} - \frac{2-i}{1} \right) = \left(\frac{1+i-(2-i)^2}{2-i} \right)$$

$$= \left(\frac{1+i-4+4i-1}{2-i} \right) = \left(\frac{-4+5i}{2-i} \cdot \frac{2+i}{2+i} \right) =$$

$$\left(\frac{-4+5i}{2-i} \cdot \frac{2+i}{2+i} \right) = \left(\frac{-8-4i+10i-5}{4+2i-2i+1} \right) = \left(\frac{-13+6i}{5} \right)$$

$$= \left(\frac{(-2-2i)(2+i)}{5-2i+2i+1} \right) = \left(\frac{-4-2i-10i-5}{5} \right) = \frac{-13+6i}{5} = \frac{-9-8i}{5} = -\frac{9-8i}{5}$$

$$(2+i)^2 = 4+4i-1$$

$$+(2-i)^2 = 4-4i-1$$

$$-(2-i)^2 = -4+4i+1$$

$$\operatorname{Re}(z) = \frac{-13}{5}$$

$$\operatorname{Im}(z) = \frac{6i}{5}$$

IME I PREZIME: Luka Belavac

BROJ INDEKSA: 17-2-0022-2010

$$② A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Det}(A) = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = -1 \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = -1 \left(1 \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \right) = -1(-1) =$$

$$\begin{array}{cccc} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{array}$$

$$\text{Det}(A) = 1 \checkmark \underline{5} = \boxed{1}$$

$$\begin{array}{ccc} + & - & + \\ - & + & - \\ + & - & + \end{array}$$

INVERZ?

VIDI KOLEGA

ŠTETA ~~DA~~ MISTE RIJEŠILI OSTATAK ZADATKA.
STO

$$⑤ A(2, -1, -2), B(2, 2, 1), C(2, 3, -1), D(5, 0, -6)$$

$$\vec{T}_A \vec{T}_B = \begin{bmatrix} 2 & -2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$$

$$\vec{T}_A \vec{T}_C = \begin{bmatrix} 2 & -2 \\ 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = (\vec{T}_A \vec{T}_B \times \vec{T}_A \vec{T}_C) \cdot \vec{T}_A \vec{T}_D$$

$$\vec{T}_A \vec{T}_D = \begin{bmatrix} 5 & -2 \\ 0 & 1 \\ -6 & 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix} = \left(\begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right) \cdot \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$$

$$\begin{array}{|cc|} \hline 0 & 0 \\ 3 & 1 \\ 3 & 1 \\ 3 & 1 \\ 0 & 0 \\ 3 & 1 \\ \hline \end{array} \quad 3 - 12 = -9$$

$$= \begin{bmatrix} -9 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix} = -27 + 0 + 0$$

$$\boxed{-27} \quad \underline{20}$$

Ne pripadaju istoj ravnini ✓

IME I PREZIME: LOURE KOLEGA

BROJ INDEKSA:

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Broj ↓
bodova

1. Neka su z_1 i z_2 rješenja jednadžbe $z^2 - 4z + 5 = 0$. Izračunati vrijednost izraza $\left(\frac{1-i}{z_1} - \overline{z_2} \right)$.

20

2. Odrediti inverz i determinantu matrice:

$$A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

10

Izračunati matrični umnožak $A A^{-1}$.

20

3. Za funkciju $f(x) = e^x$ nacrtati graf i navesti: domenu, kodomenu, periodičnost, (ne)parnost, ograničenost, rast ili pad; da li je injekcija, surjekcija ili bijekcija; da li postoji inverz i ako postoji koja je to funkcija.

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4. Gaussovom metodom riješiti sustav:

$$\begin{aligned} 2x - 3y - z + 2w + 3v &= 4 \\ 4x - 4y - z + 4w + 11v &= -4 \\ 2x - 5y - 2z + 2w - v &= 9 \\ 2y + z + 4v &= -5 \end{aligned}$$

20

5. Pokazati da li točke $A(2, -1, -2)$, $B(2, 2, 1)$, $C(2, 3, -1)$ i $D(5, 0, -6)$ pripadaju istoj ravnini.

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5. $A(2, -1, -2)$
 $B(2, 2, 1)$
 $C(2, 3, -1)$
 $D(5, 0, -6)$

$$\vec{AB} = \vec{OB} - \vec{OA} \quad \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$$

$$\begin{vmatrix} 0 & 0 & 3 \\ 3 & 4 & 1 \\ 3 & 1 & 8 \end{vmatrix} =$$

$$\begin{aligned} D &= 0(32-1) - 0(24-3) + 3(3-12) \\ &= 0(31) - 0(21) + 3(-9) \end{aligned}$$

$$D = -27 \neq 0$$

$$\vec{AC} = \vec{OC} - \vec{OA} \quad \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$$

Točke ne pripadaju istoj ravni
jer determinanta nije 0. ✓

$$\vec{AD} = \vec{OD} - \vec{OA} \quad \begin{bmatrix} 5 \\ 0 \\ 6 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix}$$

20

IME I PREZIME: LOVRE KOLEGA

BROJ INDEKSA:

2.

$$A = \left[\begin{array}{cc|cc} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \middle| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_1 \leftrightarrow R_4 \\ R_2 + R_1 \\ R_3 + R_1}} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & -1 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{array} \right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{array} \right] \xrightarrow{R_4 + R_3}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{array} \right] \rightarrow \text{INVERZ}$$

~~10~~ 10

DETERMINANTA?

IZRAČUNATI $A A^{-1} = ?$

IME I PREZIME: LOURE KOLEGA

BROJ INDEKSA:

1.

$$z_1 z_2 = ?$$

$$z^2 - 4z + 5 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$x_{1,2} = \frac{4 \pm \sqrt{-4}}{2}$$

$$x_{1,2} = \frac{4 \pm 2i}{2} = \frac{2(2 \pm 2i)}{2}$$

$$z_1 = 2 + 2i$$

$$z_2 = 2 - 2i$$

$$\frac{1-i}{z_1} - \overline{z_2}$$

$$\frac{1 - \frac{4}{3}}{2 + \frac{8}{3}} - 2 + \frac{8}{3}$$

$$(2+2i)(2-2i)$$

$$\sqrt{-4} = \sqrt{4 \cdot -1} = 2i$$

$$4 - 4i + 4i - 4i^2$$

$$\frac{1-i}{2+2i} - \frac{(2+2i)}{1} = 0$$

$$\frac{1-i-(4-4i+4i^2)}{2+2i} = \frac{1-i-4+4i^2}{2+2i} = \frac{4i^2-i-3}{2+2i}$$

$$\frac{4i^2-i-3}{2+2i} \cdot \frac{2-2i}{2-2i} = \frac{8i^2-8i^3-2i+2i^2-6+6i}{2-2i}$$

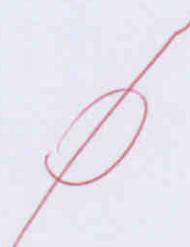
$$= -8i^3 + 10i^2 + 4i - 6 =$$

$$= 8i - 10 + 4i - 6 = 0$$

$$= 12i - 16 = 0$$

$$12i = 16 \quad | :12$$

$$i = \frac{16}{12} = \boxed{\frac{4}{3}}$$



101 BEKAVAC

-5+3-

-4+6

IME I PREZIME:

LOVRE KOLEGA

BROJ INDEKSA:

11--

$$4. \left[\begin{array}{cccccc|c} 2 & -3 & -1 & 2 & 3 & 1 & 4 \\ 4 & -4 & -1 & 4 & 11 & 1 & -4 \\ 2 & -5 & -2 & 2 & -1 & & 9 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{\cdot \frac{1}{2}} \left[\begin{array}{cccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 4 & -4 & -1 & 4 & 11 & 1 & -4 \\ 2 & -5 & -2 & 2 & -1 & 1 & 9 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{R_2 - 4R_1} \left[\begin{array}{cccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 0 & 1 & 0 & \frac{11}{2} & 1 & -4 \\ 2 & -5 & -2 & 2 & -1 & 1 & 9 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{R_3 - 2R_1} \left[\begin{array}{cccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 0 & 1 & 0 & \frac{11}{2} & 1 & -4 \\ 0 & 0 & 0 & 1 & \frac{1}{2} & 1 & 9 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{-2+1=2-2} \left[\begin{array}{cccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 0 & 1 & 0 & \frac{11}{2} & 1 & -4 \\ 0 & 0 & 0 & 1 & \frac{1}{2} & 1 & 9 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right]$$

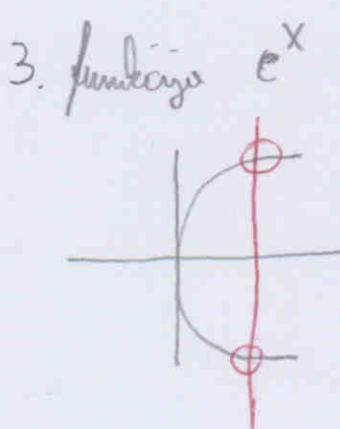
$$\left[\begin{array}{cccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 2 & 1 & 0 & 5 & 1 & -12 \\ 0 & -2 & -1 & 0 & -4 & 1 & 5 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{\cdot \frac{1}{2}} \left[\begin{array}{cccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ 0 & -2 & -1 & 0 & -4 & 1 & 5 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{R_1 - R_2} \left[\begin{array}{cccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ 0 & -2 & -1 & 0 & -4 & 1 & 5 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{R_3 + 2R_2} \left[\begin{array}{cccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{5-12} \left[\begin{array}{cccccc|c} 1 & -\frac{3}{2} & -\frac{1}{2} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right]$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -\frac{1}{4} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ 0 & 0 & 0 & 0 & -1 & 1 & -7 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{(-\frac{1}{4})} \left[\begin{array}{cccccc|c} 1 & 0 & -\frac{1}{4} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ 0 & 0 & 0 & 0 & 1 & -1 & 7 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right] \xrightarrow{(\cdot \frac{1}{4})} \left[\begin{array}{cccccc|c} 1 & 0 & -\frac{1}{4} & 1 & \frac{3}{2} & 1 & 2 \\ 0 & 1 & \frac{1}{2} & 0 & \frac{5}{2} & 1 & -6 \\ 0 & 0 & 0 & 0 & 1 & -1 & 7 \\ 0 & 0 & 2 & 1 & 4 & & -5 \end{array} \right]$$

?



$$\begin{aligned} \frac{3}{2} - \frac{1}{2} &= -\frac{3}{4} \\ -\frac{1}{2} + \frac{3}{4} &= \frac{-2+3}{4} = \frac{1}{4} \\ \frac{3}{2} + \frac{3}{2} \cdot \frac{5}{2} &= \frac{15}{4} \\ 6 + 15 &= 21 \end{aligned}$$



domena

$$[0, +\infty)$$

codomena

$$\mathbb{R}$$



VERTIKALNI PRAVAC
BIJEĆE GRAF FUNKCIJE
SAMO U JEDNOJ TOČKI

VAŽNO SE POZNAVATI ELEMENTARNE FUNKCIJE. BEZ TOGA JE TEŠKO SAVLADATI OVAJ KOLEGIJ.