OBAVEZNO ODMAH POPUNITI! DATUM: VRIJEME: OD DO

IME I PREZIME: MARIN MAGAŠ BROJ INDEKSA: 17-2-0061-2010

MATEMATIKA 1: KOLOKVIJ 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOJ STRANICI I PREDLOŠCIMA ZA PISANJE KOJE MOŽETE DOBITI OD NASTAVNIKA. ooxo

56

Broj ↓ bodova

1. Neka su z_1 i z_2 rješenja jednadžbe $z^2 + 3z + 3 = 0$. Izračunati vrijednost izraza $Re\left(\overline{z_1 - i} + \frac{|z_2|}{z_1 + i}\right) = ?$. ~~20~~ 0

2. Odrediti inverz i determinantu matrice:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Izračunati matrični umnožak $A\,A^{-1}$. 20

3. Za eksponencijalnu funkciju $f(x) = e^x$ nacrtati graf i navesti: domenu, kodomenu, periodičnost, (ne)parnost, ograničenost, rast ili pad; da li je injekcija, surjekcija ili bijekcija; da li postoji inverz i ako postoji koja je to funkcija. 16 ~~20~~

4. Gaussovom metodom riješiti sustav jednadžbi: ~~20~~ 0

$$\begin{aligned} 5x_1 + x_2 + x_3 - x_4 &= 3 \\ x_1 + x_2 - x_3 + 2x_4 &= -10 \\ -2x_1 - x_2 + x_3 + x_4 &= -10 \\ x_2 + x_3 \quad &= 4 \end{aligned}$$

5. Odrediti da li točke $A(2, -1, 2)$, $B(1, 2, 1)$, $C(2, 3, 0)$ i $D(5, 0, -6)$ pripadaju istoj ravnini. 20

1. $z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 - 4 \cdot 1 \cdot 3}}{2} = \frac{-3 \pm \sqrt{-3}}{2} = \frac{-3 \pm 3i}{2}$

$z_1 = -\frac{3}{2} + \frac{3}{2}i$

$z_2 = -\frac{3}{2} - \frac{3}{2}i$

$\left(\frac{\quad}{-\frac{3}{2} + \frac{3}{2}i - i} + \frac{\left|-\frac{3}{2} - \frac{3}{2}i\right|}{-\frac{3}{2} + \frac{3}{2}i + i}\right)$

$\frac{\quad}{-\frac{3}{2} + \frac{1}{2}i} + \frac{\frac{9}{4} + \frac{9}{4}}{-\frac{3}{2} + \frac{5}{2}i} \quad -\frac{3}{2} - \frac{1}{2}i + \frac{\frac{18}{4}}{-\frac{3}{2} + \frac{5}{2}i}$

$\frac{3}{2} - 1 = \frac{3-2}{2} = \frac{1}{2}$

$\frac{3}{2} + 1 = \frac{3+2}{2} = \frac{5}{2}$

$= \frac{\left(-\frac{3}{2} - \frac{1}{2}i\right)\left(-\frac{3}{2} + \frac{5}{2}i\right) + 4{,}5}{-\frac{3}{2} + \frac{5}{2}i} = \frac{\frac{9}{4} - \frac{15}{4}i + \frac{3}{4}i - \frac{5}{4}i^2 + 4{,}5}{-\frac{3}{2} + \frac{5}{2}i}$

$= \frac{\frac{9}{4} - \frac{12}{4}i + \frac{5}{4} + \frac{18}{4}}{-\frac{3}{2} + \frac{5}{2}i} = \frac{8 - 3i}{-\frac{3}{2} + \frac{5}{2}i} \cdot \frac{-\frac{3}{2} - \frac{5}{2}i}{-\frac{3}{2} - \frac{5}{2}i} = \frac{-\frac{24}{2} - \frac{40}{2}i + \frac{9}{2}i + \frac{15}{2}i^2}{\frac{9}{4} + \frac{15}{4}i - \frac{15}{4}i - \frac{25}{4}i^2} \rightarrow$

$= \frac{12 - 20i + 4{,}5i - 7{,}5}{2{,}25 + 6{,}25} = \frac{4{,}5 - 15{,}5i}{8{,}5}$ $Re = 0{,}53$

IME I PREZIME: MARIN MAGAJ BROJ INDEKSA: 17-2-0061-2010

1.

$$\frac{-3\pm\sqrt{9-4\cdot 3}}{2}=\frac{-3\pm 3i}{2}\left\langle\begin{array}{l}-\frac{3}{2}+\frac{3}{2}i\\ -\frac{3}{2}-\frac{3}{2}i\end{array}\right.$$

$$\overline{-1,5+1,5i-i}+\frac{\overline{\sqrt{\frac{9}{4}+\frac{9}{4}}}}{-1,5+1,5i+i}=-1,5+0,5i+\frac{4,5}{-1,5+2,5i}$$

$$=\frac{(-1,5+0,5i)(-1,5+2,5i)+4,5}{-1,5+2,5i}=\frac{2,25-3,75i+0,75i-1,25i^2}{-1,5+2,5i}$$

$$=\frac{2,25-3i+1,25}{-1,5+2,5i}=\frac{3,5-3i}{-1,5+2,5i}\cdot\frac{-1,5-2,5i}{-1,5-2,5i}$$

$+7,5i^2$

$$=\frac{-5,25-8,75i+4,5i-7,5}{2,25+\cancel{3,75i}-\cancel{3,75i}+6,25}=\frac{-12,75-4,25}{8,5}$$

$$Re=-1,5=-\frac{3}{2}$$

VIDI PUDELKO

0

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2. $$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = 1\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= 1\left(1\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}\right) - 1\left(1\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}\right)$$

$$+ 1\left(1\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}\right) - 1\left(1\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}\right)$$

$$= 1(0 - 0 + 0) - 1(0 - 0 + 0) + 1(0 - 0 + 0) - 1(0 - 0 - 1)$$

$$0 - 0 + 0 + 1 = 1$$ ✓ 20

VIDI MANDARIĆ

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R2-R1 \\ R3-R1 \\ R4-R1 \end{array} = \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \end{array}\right] \cdot (-1)R2 = \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \end{array}\right] R1-R2$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \end{array}\right] \cdot (-1) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \end{array}\right] R2-R3$$

Ime i prezime: MARIO MAGAŠ

Broj indeksa: 17-2-0061-2010

2 →

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 \end{array}\right] \cdot(-1)$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 \end{array}\right] R3-R4$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 \end{array}\right]$$

$$\qquad I \qquad\qquad\qquad A^{-1}$$

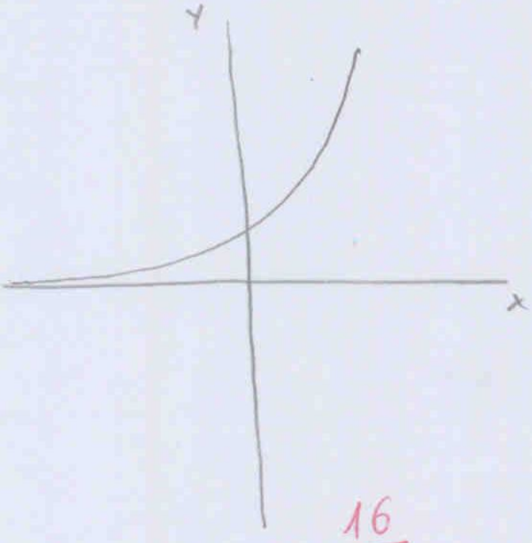
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} = A \cdot A^{-1}$$

$$= \begin{bmatrix} 0+0+0+1 & 0+0 \; 1-1 & 0+1-1+0 & 1-1+0+0 \\ 0+0+0+0 & 0+0+1+0 & 0+1-1+0 & 1-1+0+0 \\ 0+0+0+0 & 0+0+0+0 & 0+1+0+0 & 1-1 \; 0 \; 0 \\ 0+0+0+0 & 0+0+0+0 & 0+0+0+0 & 1+0+0+0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IME I PREZIME: MARIN MAGAŠ   BROJ INDEKSA: 17-2-0061-2010

3.



BIJEKCIJA ✓ SURJEKCIJA

INJEKCIJA ✓

INVERZ -> ln x ✓

RASTUĆA ✓

INFIMUM = 0 ✓ NEMA GORNJU MEĐU

NEMA PERIOD ✓

DOMENA R ✓

KODOMENA ⟨0, +∞⟩ ?

NITI PARNA NITI NEPARNA ✓

16

4.
$$
\begin{aligned}
5x_1 + x_2 + x_3 - x_4 &= 3\\
x_1 + x_2 - x_3 + 2x_4 &= -10\\
-2x_1 - x_2 + x_3 + x_4 &= -10\\
x_2 + x_3 &= 4
\end{aligned}
$$

$$
\begin{array}{c}
\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}\\
\left[\begin{array}{cccc|c}
5 & 1 & 1 & -1 & 3\\
1 & 1 & -1 & 2 & -10\\
-2 & -1 & 1 & 1 & -10\\
0 & 1 & 1 & 0 & 4
\end{array}\right]
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cccc} x_2 & x_1 & x_3 & x_4 \end{array}\\
\left[\begin{array}{cccc|c}
1 & 5 & 1 & -1 & 3\\
1 & 1 & -1 & 2 & -10\\
-1 & -2 & 1 & 1 & -10\\
1 & 0 & 1 & 0 & 4
\end{array}\right]
\end{array}
\begin{array}{l} \\ R2-R1\\ R3+R1\\ R4-R1 \end{array}
=
\left[\begin{array}{cccc|c}
1 & 5 & 1 & -1 & 3\\
0 & -4 & -2 & 3 & -13\\
0 & 3 & 2 & 0 & -7\\
0 & -5 & 0 & 1 & 1
\end{array}\right] 6\cdot\left(-\frac{1}{4}\right)R2
$$

$$
\left[\begin{array}{cccc|c}
1 & 5 & 1 & -1 & 3\\
0 & 1 & 1/2 & -1/2 & 5/2\\
0 & 3 & 2 & 0 & -7\\
0 & -5 & 0 & 1 & 1
\end{array}\right]
\begin{array}{l} R1-5R2\\ \\ R3-3R2\\ R4+5R2 \end{array}
=
\left[\begin{array}{cccc|c}
1 & 0 & -3/2 & 3/2 & -19/2\\
0 & 1 & 1/2 & -1/2 & 5/2\\
0 & 0 & 1/2 & 3/2 & -29/2\\
0 & 0 & 5/2 & -3/2 & 27/2
\end{array}\right]\cdot 2
$$

IME I PREZIME: MARIN MAGAŠ   BROJ INDEKSA: 17-2-0061-2010

4. $\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -3/2 & 3/2 & -19/2 \\ 0 & 1 & 1/2 & -1/2 & 5/2 \\ 0 & 0 & 1 & 3 & -29 \\ 0 & 0 & 5/2 & -3/2 & 27/2 \end{array}\right] \begin{array}{l} R1+\frac{3}{2}R3 \\ R2-\frac{1}{2}R3 \\ \\ R4-\frac{5}{2}R3 \end{array}$

$\frac{3}{2}+\frac{3}{2}3$

$\frac{3}{2}+\frac{9}{2}=\frac{12}{2}$

$-\frac{19}{2}+\frac{3}{2}(-29)$

$-\frac{19}{2}-\frac{87}{2}$

$\frac{5}{2}-\frac{1}{2}(-29)$ $-\frac{1}{2}-\frac{1}{2}3$

$\frac{5}{2}+\frac{29}{2}$ $-\frac{1}{2}-\frac{3}{2}$ $-\frac{4}{2}$

$\frac{5}{2}-\frac{5}{2}$ $-\frac{3}{2}-\frac{5}{2}(3)$

$\frac{27}{2}-\frac{5}{2}(-29)$ $-\frac{3}{2}-\frac{15}{2}$ $-\frac{18}{2}$

$\frac{27}{2}+\frac{145}{2}=$

$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 6 & -53 \\ 0 & 1 & 0 & -2 & 17 \\ 0 & 0 & 1 & 3 & -29 \\ 0 & 0 & 0 & -9 & 86 \end{array}\right] \begin{array}{l} \\ \\ \\ \cdot(-\frac{1}{9}) \end{array}$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 6 & -53 \\ 0 & 1 & 0 & -2 & 17 \\ 0 & 0 & 1 & 3 & -29 \\ 0 & 0 & 0 & 1 & -86/9 \end{array}\right] \begin{array}{l} R1-6R4 \\ R2+2R4 \\ R3-3R4 \\ \\ \end{array}$

$-53-\cancel{6}^{3}\left(-\frac{86}{\cancel{9}_3}\right)$

$-53-3\left(-\frac{86}{3}\right)$

$-53+\frac{258}{3}$

$\frac{-159+258}{3}$

$17+2\cdot\left(-\frac{86}{9}\right)$

$17-\frac{172}{9}$

$\frac{153-172}{9}$

$$\begin{array}{c} \begin{array}{cccc} x_2 & x_1 & x_3 & x_4 \end{array} \\ = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 33 \\ 0 & 1 & 0 & 0 & -19/9 \\ 0 & 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 1 & -86/9 \end{array}\right] \end{array}$$

X $\cancel{0}$

$-29-\cancel{3}^{1}\cdot\left(-\frac{86}{\cancel{9}_3}\right)$

$-29+\frac{86}{3}=\frac{-87+86}{3}$

$=-\frac{1}{3}$

PROVJERA:

$$\begin{pmatrix} 5 & 1 & 1 & -1 \\ 1 & 1 & -1 & 2 \\ -2 & -1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -\frac{19}{9} \\ 33 \\ -\frac{1}{3} \\ -\frac{86}{9} \end{pmatrix} = \begin{pmatrix} -\frac{95}{9}+33-\frac{1}{3}+\frac{86}{9} \\ \\ \\ \\ \end{pmatrix} = \begin{pmatrix} 31\frac{1}{3} \\ \\ \\ \\ \end{pmatrix} \neq \begin{pmatrix} 3 \\ -10 \\ -10 \\ 4 \end{pmatrix}$$

Ime i prezime: MARIN MAGAŠ Broj indeksa: 17-2-0061-2010

5. $A(2,-1,2)$ $B(1,2,1)$ $C(2,3,0)$ $D(5,0,-6)$
(T_1) (T_2) (T_3) (T_4)

$\overrightarrow{T_1T_2} = [-1 \;\; 3 \;\; -1] \cdot \overrightarrow{T_1T_3}[0 \;\; 4 \;\; -2] \quad \overrightarrow{T_1T_4}[3 \;\; 1 \;\; -8]$

$(\overrightarrow{T_1T_2} \times \overrightarrow{T_1T_3}) \cdot \overrightarrow{T_1T_4} = \begin{bmatrix} -2 \\ -2 \\ -4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ -8 \end{bmatrix}$

$$\begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 4 \\ -2 \end{bmatrix}$$

$-1 \quad 0$
$3 \quad 4$

$3 \cdot (-2) - (4 \cdot (-1))$
$-1 \cdot 0 - (-2 \cdot (-1))$
$-1 \cdot 4 - (0 \cdot 3)$

$-6 - (-4)$
$0 - (2)$
$-4 - 0$

-2
-2
-4

$= -2 \cdot 3 + 1 \cdot (-2) + (-4 \cdot (-8))$

$= -6 - 2 + 32 = 24$

NE PRIPADAJU ✓

20